Amplitude Modulation

Objectives:

- To introduce amplitude modulation
- To study double sideband suppressed carrier modulation (DSB-SC) systems
- To describe coherent demodulation method for amplitude modulation
- To assess the influence of carrier phase for coherent demodulation
- To study standard (conventional) amplitude modulation (AM) systems
- To introduce modulation index and illustrate its effect in AM
- To calculate sideband power, carrier power and transmission efficiency in AM
- To briefly explain single sideband modulation (SSB) systems
- To describe envelope demodulation in AM system
- To compare the performance in different amplitude modulation systems

Generally speaking, modulation is the process by which a parameter of a carried wave is varied in proportion to a signal. The precise dependence is determined by the type of modulation employed.

Types of analog modulation

Usually a carrier wave in analog modulation is sinusoidal, given by $A\cos(\omega_c t + \theta)$

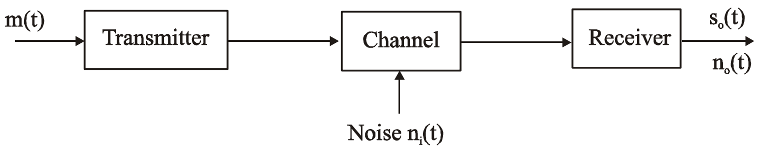
This sinusoidal wave is expressed by three parameters: amplitude A, frequency ω_c (angular frequency), and phase angle θ.

Any of the three parameters can be varied in accordance with the baseband signal. Accordingly, the modulation process is termed as **amplitude modulation**, **frequency modulation**, or **phase modulation**. The frequency modulation and phase modulation both effectively, vary total angle of the carrier wave, and hence are combined in a single heading – **angle modulation**.

The main aim of analog modulation (amplitude or angle) is frequency translation, to achieve frequency division multiplexing and practical antenna design, etc.

Baseband communication systems

In baseband communication systems, the signal is transmitted directly without any modulation.



Such a system is simple, but not widely used, mainly used in short-haul system.

Double sideband suppressed carrier modulation (DSB-SC) systems

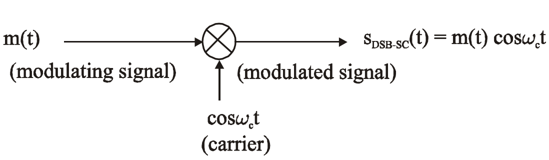
The modulation theorem of Fourier transform states that the spectrum of $g(t)\cos(\omega_c t)$ is the same as that of g(t) except that it is translate by $\pm\ \omega_c$. Mathematically,

If $\quad g(t) \Leftrightarrow G(\omega)$

then $\quad g(t)\cos(\omega_c t) \Leftrightarrow (1/2)[G(\omega + \omega_c) + G(\omega - \omega_c)]$

Therefore, a method for achieving frequency translation is to multiply the modulating signal g(t) with a sinusoidal carrier signal $\cos\omega_c t$.

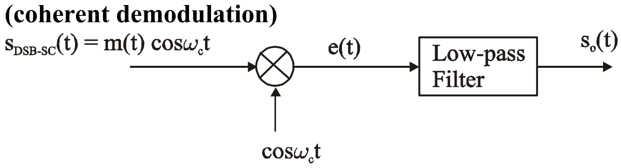
DSB-SC signal generation



Waveform: $\quad s_{DSB-SC}(t) = m(t)\cos\omega_c t$

Spectrum: $\quad S_{DSB-SC}(\omega) = (1/2)[M(\omega + \omega_c) + M(\omega - \omega_c)]$

Demodulation method (coherent demodulation)



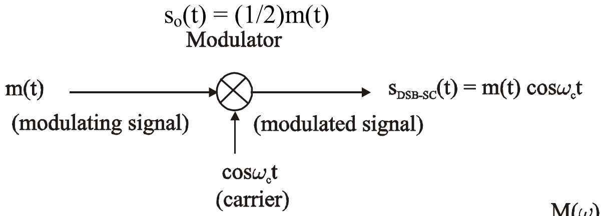
Multiplying the signal $m(t)\cos\omega_c t$ by a local carrier wave $\cos\omega_c t$

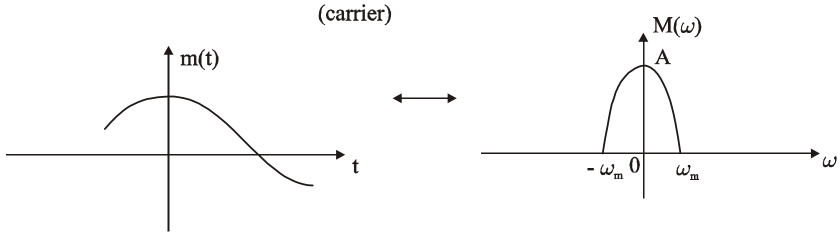
$$e(t) = m(t)\cos^2\omega_c t = (1/2)[m(t) + m(t)\cos 2\omega_c t]$$

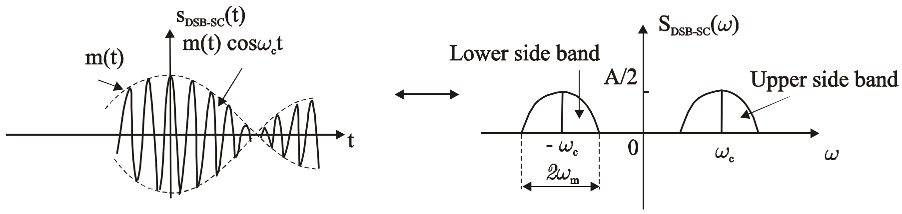
The spectrum of e(t): $E(\omega) = (1/2)M(\omega) + (1/4)[M(\omega + 2\omega_c) + M(\omega - 2\omega_c)]$

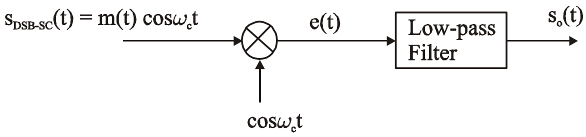
Pass through a low pass filter: $S_o(\omega) = (1/2)M(\omega)$

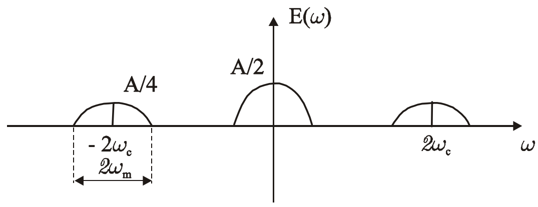
The output signal: $s_o(t) = (1/2)m(t)$

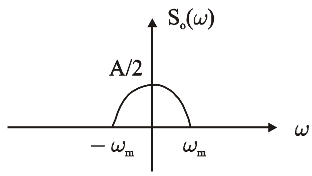












Example: **Tone modulation**

The modulation signal is a pure sinusoidal, $\cos\omega_m t$.

Signal $m(t) = \cos\omega_m t$

Signal spectrum $\pi[\delta(\omega + \omega_m) + \delta(\omega - \omega_m)]$

Modulated signal waveform $s_{DSB\text{-}SC}(t) = \cos\omega_m t \cos\omega_c t = (1/2)[\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t]$

Modulated signal spectrum

$$S_{DSB\text{-}SC}(\omega) = (\pi/2)[\delta(\omega + \omega_c + \omega_m) + \delta(\omega + \omega_c - \omega_m) + \delta(\omega - \omega_c + \omega_m) + \delta(\omega - \omega_c - \omega_m)]$$

When modulated signal multiplied by a carrier $\cos\omega_c t$,

$$e(t) = \cos\omega_m t \cos^2\omega_c t = (1/2)\cos\omega_m t\,(1 + \cos 2\omega_c t) = (1/2)\cos\omega_m t + (1/2)\cos\omega_m t \cos 2\omega_c t$$
$$= (1/2)\cos\omega_m t + (1/4)[\cos(2\omega_c + \omega_m)t + \cos(2\omega_c - \omega_m)t]$$

Spectrum of e(t)

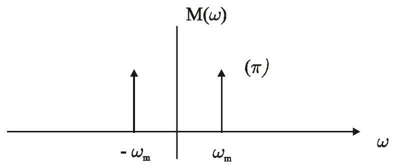
$$E(\omega) = (\pi/2)[\delta(\omega + \omega_m) + \delta(\omega - \omega_m)] +$$
$$(\pi/4)[\delta(\omega + 2\omega_c + \omega_m) + \delta(\omega + 2\omega_c - \omega_m) + \delta(\omega - 2\omega_c - \omega_m) + \delta(\omega - 2\omega_c + \omega_m)]$$

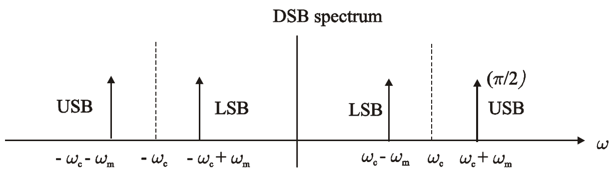
The output signal spectrum after being suppressed by a low pass filter

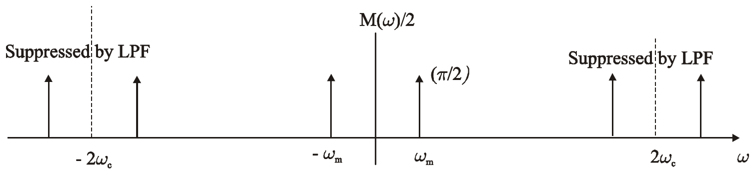
$$S_o(\omega) = (\pi/2)[\delta(\omega + \omega_m) + \delta(\omega - \omega_m)]$$

Output signal waveform

$$s_o(t) = (1/2)\cos\omega_m t$$







The spectrum of the modulated signal consists of frequency components at $\omega_c + \omega_m$ and $\omega_c - \omega_m$, there is no component at carrier frequency, this is the name *suppressed carrier* comes from.

Assignment 9:

An audio signal occupies the frequency range 30 Hz < | f | < 15 kHz. This signal is to be conveyed using DSB-SC amplitude modulation over a bandpass channel which transmits signals in the range 90 kHz to 120 kHz. Determine a suitable carrier frequency to match the audio signal to this channel.

In DSB-SC system, the demodulation process consists of multiplying the incoming signal by a local carrier of the same frequency and phase with the carrier at the modulator. The product is then passed through a low pass filter in order to recover the signal. However, this task is difficult to achieve or quite expensive.

Influence of carrier phase for coherent demodulation

DSB-SC signal: $s_{DSB\text{-}SC}(t) = m(t)\cos\omega_c t$

Local carrier wave : $\cos[(\omega_c + \Delta\omega_c)t + \Delta\theta]$

Where $\Delta\omega_c$ is the frequency error and $\Delta\theta$ is the phase error.

The output from the multiplier :

$$m(t)\cos\omega_c t \cos[(\omega_c + \Delta\omega_c)t + \Delta\theta] = (1/2)m(t)\{\cos[(2\omega_c + \Delta\omega_c)t + \Delta\theta] + \cos(\Delta\omega_c t + \Delta\theta)\}$$

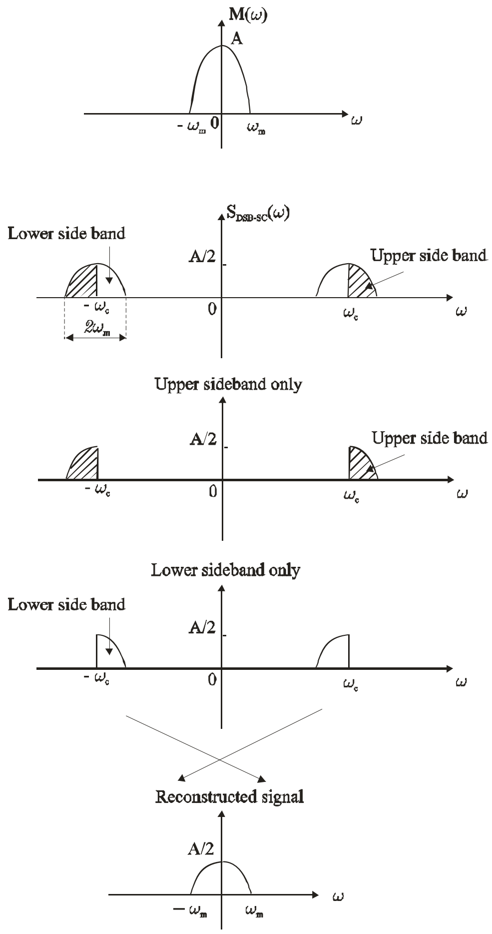
The output from the low-pass filter : $s(t) = (1/2)m(t)\cos(\Delta\omega_c t + \Delta\theta)$

When $\Delta\omega_c = 0$, only phase error exists, $s(t) = (1/2)m(t)\cos\Delta\theta$
if $\Delta\theta = \pm \pi/2$, $s(t) = 0$, there is no component in the output corresponding to the message. Furthermore, if $\Delta\theta$ varies with time, s(t) will appear as the noise.

Assignment 10:
Find the receiver output if the carrier replica at the demodulator is $x_c(t) = A_c \cos(\omega_c + \Delta\omega_c)t$?

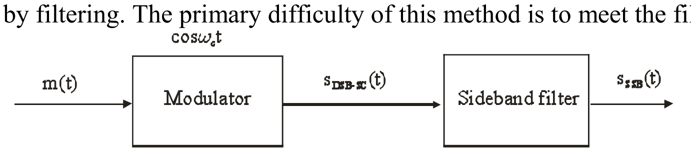
Disadvantages of DSB-SC:
1. It transmits both sidebands which contain identical information and thus waste the channel bandwidth resources;
2. It requires a fairly complicated (expensive) circuitry at a remotely located receiver in order to acquire and maintain the necessary phase synchronization.

Single sideband modulation (SSB)
The DSB-SC signal spectrum has two sidebandsL the upper sideband (USB) and the lower sideband (LSB), both containing the complete information of the baseband signal, this is unnecessarily increases the bandwidth. Lower the bandwidth of the modulated signal, more is the number of channels that can be accommodated in a given frequency space. It is, therefore, desirable to transmit only one sideband. This type of modulation is called **single sideband modulation** (SSB).
SSB solves the first disadvantage from which DSB-SC suffers by using a highpass filter at the output of the modulator so that either the upper sideband or the lower one is filtered out before transmission. Lower the bandwidth of the modulated signal can increase the number of channels.



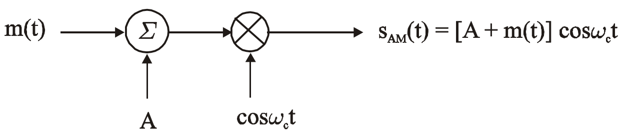
Spectra of DSB and SSB signals

SSB signal generation
The most straightforward way to generate an SSB signal is to first generate a DSB signal and then suppress one of the sidebands by filtering. The primary difficulty of this method is to meet the filter requirement.

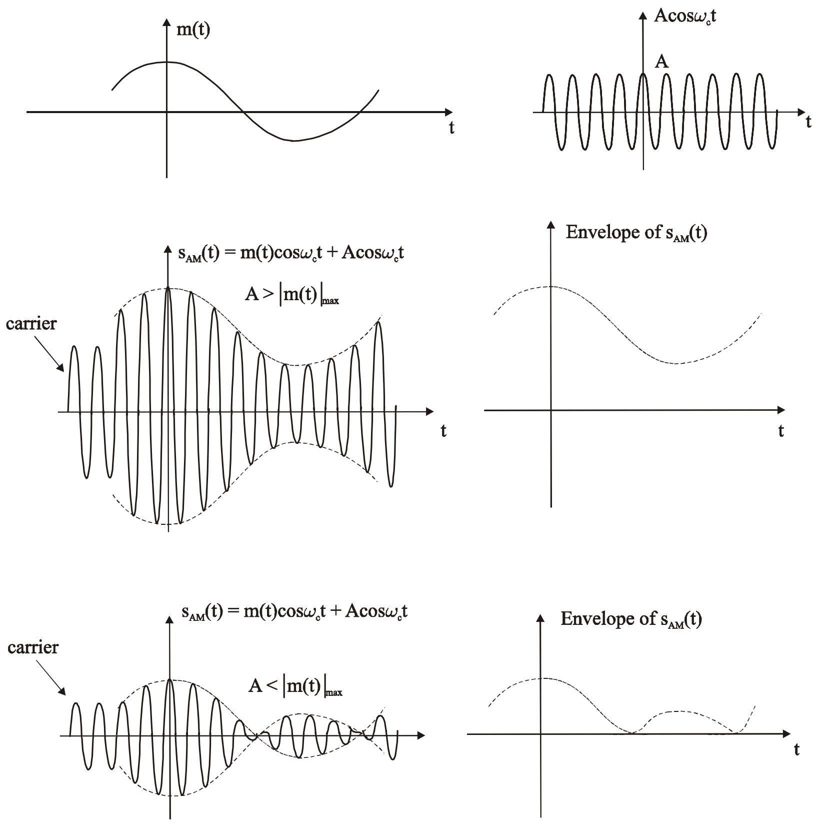


Standard (conventional) amplitude modulation (AM)
The disadvantage of high cost receiver circuit of the DSB-SC system can be solved by use of AM, but at the price of a less efficient transmitter.
The conventional amplitude system transmits a large power carrier wave, $A\cos\omega_c t$, along with the modulated signal, $m(t)\cos\omega_c t$, so that there is no need to generate a carrier at the receiver. The advantage of such a system is that the receiver is made simple and less expensive. In a *broadcast system*, a transmitter has to transmit power for a large number of receivers. In other words, the transmitter is associated with a large number of receivers, and hence a less costly receiver is needed. However, a single large power transmitter can be installed to feed power for a multitude of receivers. The AM system is preferred for this type of application.

AM signal generation:



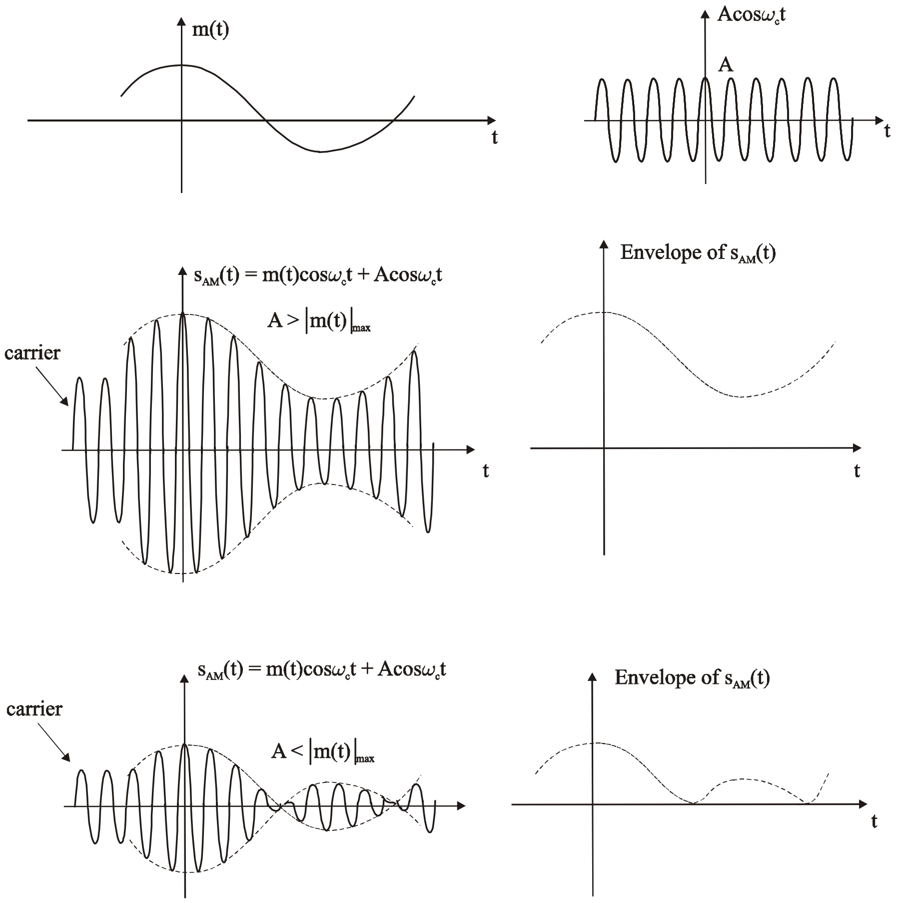
Waveform : $s_{AM}(t) = A\cos\omega_c t + m(t)\cos\omega_c t = [A + m(t)]\cos\omega_c t$
Spectrum : $S_{AM}(\omega) = (1/2)[M(\omega + \omega_c) + M(\omega - \omega_c)] + A\pi[\delta(\omega + \omega_c) + \delta(\omega - \omega_c)]$



The amplitude modulated signal, $s_{AM}(t) = [A + m(t)]\cos\omega_c t$, may be regards as a carrier signal $\cos\omega_c t$ whose amplitude is given by the quantity $[A + m(t)]$, i.e. message + dc component. If the message + dc component is positive, the envelop of the modulated signal then has the same shape as the message, m(t). Demodulation in this case simply reduces to detection of the envelope of a sinusoid, with no dependence on the exact phase or frequency of the sinusoid (i.e. of the carrier). If A is not large enough, however, the envelope of the amplitude modulated signal is not always proportional to m(t).
The carrier amplitude A must be made large enough so that $A + m(t) \geq 0$ at all times, or

$$A \geq |m(t)|_{max}$$

If above condition is not satisfied, the m(t) cannot be recovered by the relatively simply process of envelope detection. Coherent detection, however will still correctly demodulate such signals.



Modulation index
The extent of amplitude variation in AM about a unmodulated carrier amplitude is measured in terms of a factor called *modulation index* defined as

$$m = |m(t)|_{max} / A = \text{maximum excursion in AM amplitude about A / carrier amplitude A}$$

the factor is also known as **depth of modulation**, **degree of modulation** and **modulation factor**.

The baseband signal is preserved in the envelope of the AM signal only if

$$|m(t)|_{max} \leq A,$$

i.e. $$m = |m(t)|_{max} / A \leq 1$$

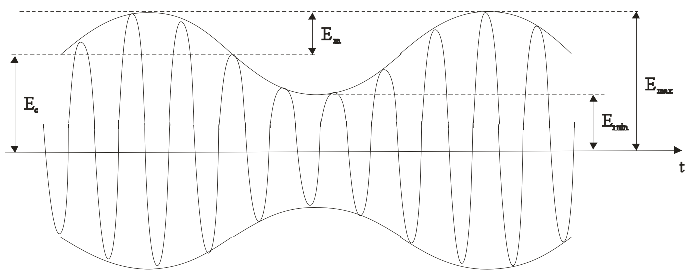
The following three cases arise depending on the amplitude of the baseband signal relative to carrier amplitude A

(1) When m < 1, then $|m(t)|_{max} < A$, the envelope is not reaching the zero-amplitude axis of the AM wave, and the baseband signal is fully preserved in the envelope of the AM wave. An envelope detector can recover the baseband signal without distortion.
(2) When m = 1, then $|m(t)|_{max} = A$, the waveform envelope just touches the zero-amplitude axis. The baseband signal remains preserved in the envelope and can be recovered by using an envelope detector. However, this is a limiting factor as even a small variation in the magnitude of m(t) will cause envelope distortion.
(3) When m >1, then $|m(t)|_{max} > A$, the amplitude of a baseband signal exceeds carrier amplitude. This situation is called **overmodulation**. In overmodulation, the portion of the envelope crosses the zero-amplitude axis, for both positive and negative excursions cancelling each other. The envelope and the baseband signal are not the same. An envelope detector will provide distorted baseband signal. A portion of the detected baseband signal is clipped.

From the above discussion, it becomes clear that a baseband signal can be recovered from AM signal by using a less costly and simple envelope detector, provided the modulation index does exceed 1, i.e., the signal is not overmodulated. The limiting value of m = 1 is also avoided to be on the safer side. However, an overmodulated signal can be recovered using a costly and as well as complex technique: synchronous (coherent) detection.

Assignment 11:
For an AM signal $x(t) = [1 + m(t)]A_c\cos\omega_c t$ but with $m(t) = \cos \omega_m t$, (a) Determine the total power used to transmit the carrier and the sidebands, (b) Find the percentage of transmitted power which is wasted because the carrier does not carry information, (c) Repeat the calculation in (b) if $m(t) = 0.5 \cos \omega_m t$.

Experimental determination of modulation index
The modulation index can be determined experimentally by observing the AM waveform on an oscilloscope.



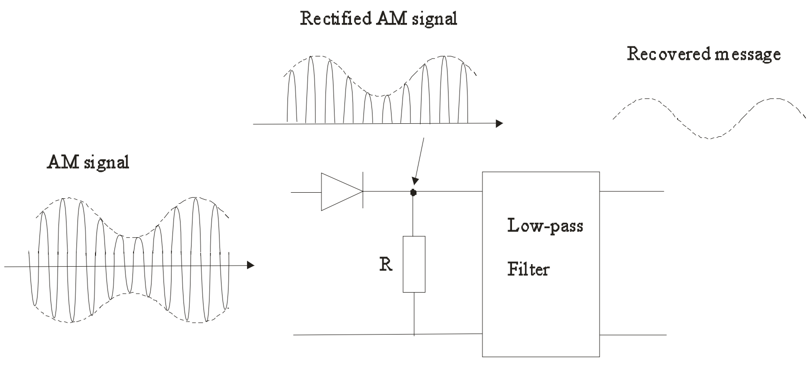
By definition, $m = E_m / E_c$,

from $E_m = (1/2)(E_{max} - E_{min})$

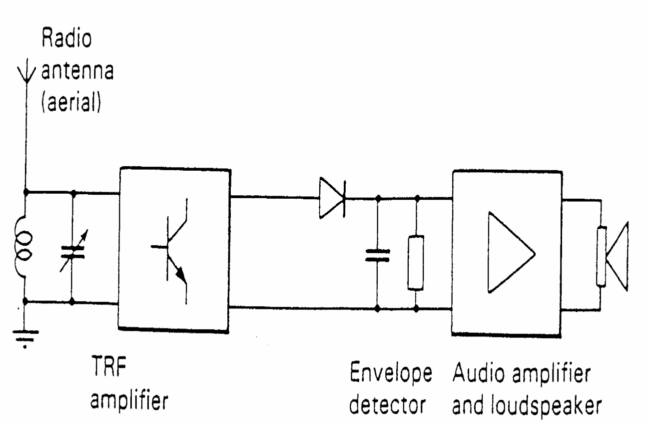
and $E_c = E_{min} + E_m = E_{min} + (1/2)(E_{max} - E_{min}) = (1/2)(E_{max} + E_{min})$

so that $m = (E_{max} - E_{min}) / (E_{max} + E_{min})$

Envelope detector



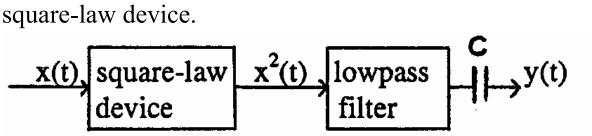
Conventional AM can be demodulated by using very simple (and inexpensive) receivers which use the principle of envelope detection and which don't have local carriers.
In an envelope detection circuit, a diode removes the negative part of the modulated signal, whereas the filter removes the carrier. The dc term is eliminated by using ac coupling. (*AC coupling uses a capacitor to block the dc current*).



TRF receiver. (O'Reilly)

The block diagram of a tuned radio frequency (TRF) receiver using envelope detection is as shown.
The TRF receiver is not very popular (even though it is very simple) because it provides poor selectivity or poor gain. (Selectivity is the ability of a receiver to select a signal of a desired frequency while rejecting those on closely adjacent frequencies.)

Assignment 12:
(a) Show that a conventional AM signal, $x(t) = [A + m(t)] \cos\omega_c t$, where $A >> |m(t)|$, can be demodulated by squaring it and then passing the resulting signal through a lowpass filter as shown below. (This type of detector is known as a square-law detector.)
(b) Find y(t).
(c) Give an example of a square-law device.



Carrier and sideband power in AM
In AM signal waveforms, the carrier terms does not contain any information about the modulating signal m(t). Therefore the power expended in this carrier is wasted for any transfer of information. It is the price one must be willing to pay in order to make cheap receivers available.
A general AM signal waveform can be described by

$$s_{AM}(t) = A\cos\omega_c t + m(t)\cos\omega_c t$$

the total average power is given by the mean-square value of $s_{AM}(t)$.

$$\overline{s^2{}_{AM}(t)} = \overline{A^2\cos^2\omega_c t} + \overline{m^2(t)\cos^2\omega_c t} + \overline{2Am(t)\cos^2\omega_c t}$$

where the bar indicates the time average. We shall assume that m(t) varies slowly with respect to $\cos\omega_c t$. If we also assume that the average value of m(t) is zero (the usual case where there is no dc component in m(t)), then the last term in the above equation is zero so that

$$\overline{s^2{}_{AM}(t)} = \overline{A^2\cos^2\omega_c t} + \overline{m^2(t)\cos^2\omega_c t} = \frac{A^2}{2} + \frac{\overline{m^2(t)}}{2}$$

Thus, the total power can be expressed as the sum of the carrier power, P_c, and sideband power, P_s.

$$P_t = \frac{A^2}{2} + \frac{\overline{m^2(t)}}{2} = P_c + P_s$$

Carrier power

$$P_c = \overline{A^2 \cos^2 \omega_c t} = A^2 \overline{\frac{1+\cos 2\omega_c t}{2}} = \frac{A^2}{2}$$

Sideband power

$$P_s = \overline{m(t)^2 \cos^2 \omega_c t} = \frac{\overline{m^2(t)}}{2}$$

The transmission efficiency of AM signal

The useful message (baseband) power is the sideband power, P_s. The large carrier power, P_c, does not carry any message and is wasted. The amount of useful message power present in AM wave is expressed by a term called transmission efficiency.

$$\eta = \frac{P_s}{P_t} = \frac{P_s}{P_c + P_s} = \frac{\overline{m^2(t)}}{A^2 + \overline{m^2(t)}}$$

Example Tone modulation

Message signal $m(t) = A_m \cos\omega_m t$

AM signal $s_{AM}(t) = A\cos\omega_c t + m(t)\cos\omega_c t = A\cos\omega_c t + A_m \cos\omega_m t \cos\omega_c t$

$= A[1 + (A_m/A) \cos\omega_m t]\cos\omega_c t = A(1 + m \cos\omega_m t)\cos\omega_c t$

$= A\cos\omega_c t + m\, A \cos\omega_m t\cos\omega_c t$

we have $P_t = \overline{s^2{}_{AM}(t)} = \overline{A^2 \cos^2 \omega_c t + 2mA \cos\omega_m t \cos^2 \omega_c t + m^2 A^2 \cos^2 \omega_m t \cos^2 \omega_c t}$

$$= \frac{A^2}{2} + \frac{1}{4} m^2 A^2 \overline{(1+\cos 2\omega_m t)(1+\cos 2\omega_c t)} = \frac{A^2}{2} + \frac{1}{4} m^2 A^2 = \frac{A^2}{2}(1 + \frac{1}{2} m^2) = P_c(1 + \frac{m^2}{2})$$

and $\eta = \frac{P_s}{P_c + P_s} = \frac{m^2}{2 + m^2}$

Because $m \le 1$, the transmission efficiency of an AM system is at best 33%.

Example

A given AM braoadcast station transmits an average carrier power output of 40 kW and uses a modulation index of 0.707 for sine wave modulation. Calculate (a) the total average power output ; (b) the transmission efficiency ; and (c) the peak amplitude of output if the antenna is represented by a 50Ω resistive load.

Solution

(a) $P_t = P_c(1 + m^2/2)$

For m = 0.707, $P_t = P_c(1 + 1/4) = (5/4)P_c = (5/4)\, 40 = 50$ kW

(b) $\eta = \frac{P_s}{P_c + P_s} = \frac{m^2}{2 + m^2} = \frac{(0.707)^2}{2 + (0.707)^2} = \frac{0.50}{2.50} = 20\%$

(c) $P_c = A^2 / 2R$, $A^2 = 2RP_c = 4 \times 10^6$, then $A + A_m = (1 + m)A = 3414$ V

Comparison of various AM systems

Various amplitude systems have been discussed, and it is interesting to compare their performance characteristics.

1. Receiver end

For the receiver, a standard (conventional) AM (large carrier) system has advantage of simple and inexpensive envelope detectors. Therefore, the standard AM system is preferably used in public communication systems where a transmitter is associated with a large number of receivers. The receivers of suppressed carrier system are complex and costly as they need additional synchronizing circuits.

2. Transmitter end

The suppressed carrier system need low power transmitters as they transmit only sideband power and no carrier power is transmitted. The low power transmitters used in suppressed carrier systems are less expensive than the large power AM transmitters. Suppressed carrier system are useful in point to point communication where we need many transmitters but only few receivers per transmitter.

3. Generation of modulated signals

SSB-SC is more difficult to generate than DSB-SC.

4. Bandwidth

SSB-SC system is most advantageous from bandwidth point of view, it has the message bandwidth. SSB-SC is used in long-range high-frequency communications, especially in audio communication, where phase-distortion is not significant. Both the DSB-SC and standard AM systems have twice message bandwidth.

5. Transmission efficiency

Efficiency of a suppressed carrier system is 100%, whereas, in a standard AM system, the maximum efficiency is only 33.3%, corresponding to $m = 1$.

Review Questions

1. Name three parameters of the carrier that can be varied to produce modulation.
2. Define amplitude modulation
3. Is the signal modulated when it transmits through a baseband communication system?
4. How to generate a DSB-SC signal?
5. Draw a time-domain and frequency-domain picture of a DSB-SC signal.
6. What is the demodulation method for DSB-SC signal?
7. What is tone modulation?
8. What is the influence of carrier phase for coherent demodulation?
9. What are the key advantages offered by standard AM over DSB-SC?
10. What is the condition for distortionless modulation?
11. Define modulation index.
12. Explain why 100% modulation is *not* an ideal value.
13. Define *overmodulation*.
14. What happens to the modulation index of an AM signal if the carrier level remains constant and the sideband level increases?
15. How to determine the modulation index experimentally?
16. What is the main demodulation method used for standard AM signal?
17. Write the expression of transmission efficiency for AM signal.
18. What is the merit of SSB-SC system?
19. Compare the characteristics of various AM systems.

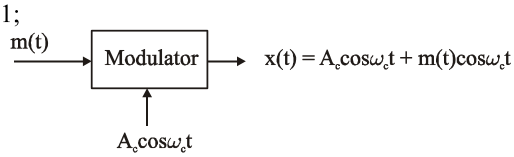
Exercise Problems (Amplitude Modulation)

1. An AM voltage signal consists of a carrier wave $100\cos(2\pi \times 10^6 t)$ and a DSB-SC signal $(20\cos 6.28t + 50\cos 12.56t)\cos(2\pi \times 10^6 t)$,
   (a) Draw the spectrum of the modulated message;
   (b) Determine the carrier power, sideband power and the total power of the modulated signal.

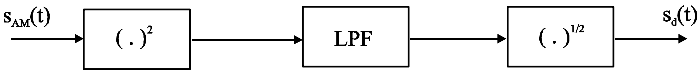
2. An AM transmitter has a carrier power of 30W. The message signal is a sinusoidal signal and the percentage of modulation is 85%, i.e. $m = 0.85$. Calculate
   (a) the total power;
   (b) the power in one sideband.

3. For DSB-SC modulation, message signal $m(t) = 4 + 2\cos(2\pi \times 10^3 t)$, carrier wave $x_c(t) = 8\cos(2\pi \times 10^6 t)$
   (a) Draw the frequency domain representation of $m(t)$, $x_c(t)$ and the modulation output $x(t)$;
   (b) Draw the block diagram of the demodulation system diagram, if low pass filter is used in the demodulator, what is the minimum bandwidth required to fully recover the signal?
   (c) What is the frequency and time domain representation of the demodulation output?

4. The figure below shows an amplitude modulator. Assuming sinusoidal carrier $x_c(t)$ and sinusoidal message signal $m(t)$, i.e., $x_c(t) = A_c\cos\omega_c t$, $m(t) = A_m\cos\omega_m t$
   The modulated signal $x(t)$ can be written as $x(t) = A_c[1 + (A_m/A_c)\cos\omega_m t]\cos\omega_c t$
   (a) For $A_m = A_c$, calculate the modulation index;
   (b) Determine the fraction of total transmitted power concentrated in the modulation sideband for
      (1) $A_m = A_c$;
      (2) $A_m = A_c/2$;
      (3) $A_m = aA_c$, where $|a| < 1$;



5. To prove that the following system can be used for AM signal demodulation and the bandwidth of the low-pass filter must be $2W_m$, where W_m is the highest frequency of the message signal.



6. The following figure shows the output of a conventional amplitude modulator.
   (1) What is the mathematical expression of $x(t)$?
   (2) Calculate the modulation index;
   (3) What is the amplitude of the sideband? What is the amplitude of the unmodulated carrier?
   (4) Draw a two sided frequency domain representation of $x(t)$;
   (5) What is the ratio of power in the sidebands to the total power?