

## **10. Starting Method for Induction Motors**

A 3-phase induction motor is theoretically self starting. The stator of an induction motor consists of 3-phase windings, which when connected to a 3-phase supply creates a rotating magnetic field. This will link and cut the rotor conductors which in turn will induce a current in the rotor conductors and create a rotor magnetic field. The magnetic field created by the rotor will interact with the rotating magnetic field in the stator and produce rotation.

Therefore, 3-phase induction motors employ a starting method not to provide a starting torque at the rotor, but because of the following reasons;

- 1) Reduce heavy starting currents and prevent motor from overheating.
- 2) Provide overload and no-voltage protection.

There are many methods in use to start 3-phase induction motors. Some of the common methods are;

- Direct On-Line Starter (DOL)
- Star-Delta Starter
- Auto Transformer Starter

- Rotor Impedance Starter
- Power Electronics Starter

### **Direct On-Line Starter (DOL)**

The Direct On-Line (DOL) starter is the simplest and the most inexpensive of all starting methods and is usually used for squirrel cage induction motors. It directly connects the contacts of the motor to the full supply voltage. The starting current is very large, normally 6 to 8 times the rated current. The starting torque is likely to be 0.75 to 2 times the full load torque. In order to avoid excessive voltage drops in the supply line due to high starting currents, the DOL starter is used only for motors with a rating of less than 5KW

There are safety mechanisms inside the DOL starter which provides protection to the motor as well as the operator of the motor. The power and control circuits of induction motor with DOL starter are shown in figure(1).

\* K1M Main contactor

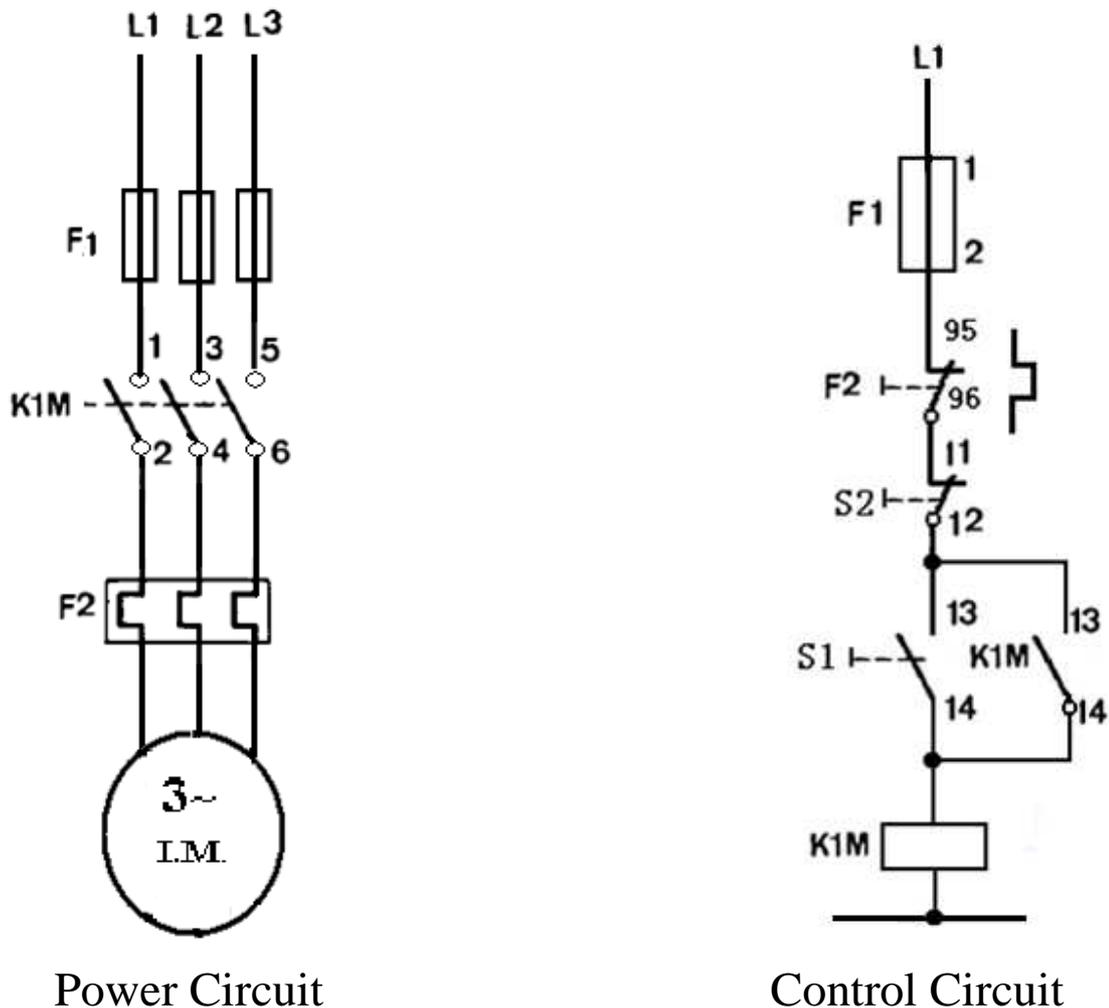


Fig.(1): power and control circuits of I.M. with DOL starter

The DOL starter consists of a coil operated contactor K1M controlled by start and stop push buttons. On pressing the start push button S1, the contactor coil K1M is energized from line L1. The three mains contacts (1-2), (3-4), and (5-6) in fig. (1) are closed. The motor is thus connected to the supply. When the stop push

button S2 is pressed, the supply through the contactor K1M is disconnected. Since the K1M is de-energized, the main contacts (1-2), (3-4), and (5-6) are opened. The supply to motor is disconnected and the motor stops.

## **Star-Delta Starter**

The star delta starting is a very common type of starter and extensively used, compared to the other types of the starters. This method used reduced supply voltage in starting. Figure(2) shows the connection of a 3phase induction motor with a star – delta starter.

The method achieved low starting current by first connecting the stator winding in star configuration, and then after the motor reaches a certain speed, throw switch changes the winding arrangements from star to delta configuration.

By connecting the stator windings, first in star and then in delta, the line current drawn by the motor at starting is reduced to one-third as compared to starting current with the windings connected in delta. At the time of starting when the stator windings are start connected, each stator phase gets voltage  $V_L / \sqrt{3}$ , where  $V_L$  is the line voltage. Since the torque developed

by an induction motor is proportional to the square of the applied voltage, star- delta starting reduced the starting torque to one – third that obtainable by direct delta starting.

- K2M Main Contactor
- K3M Delta Contactor
- K1M Star Contactor
- F1 Thermal Overload Relay

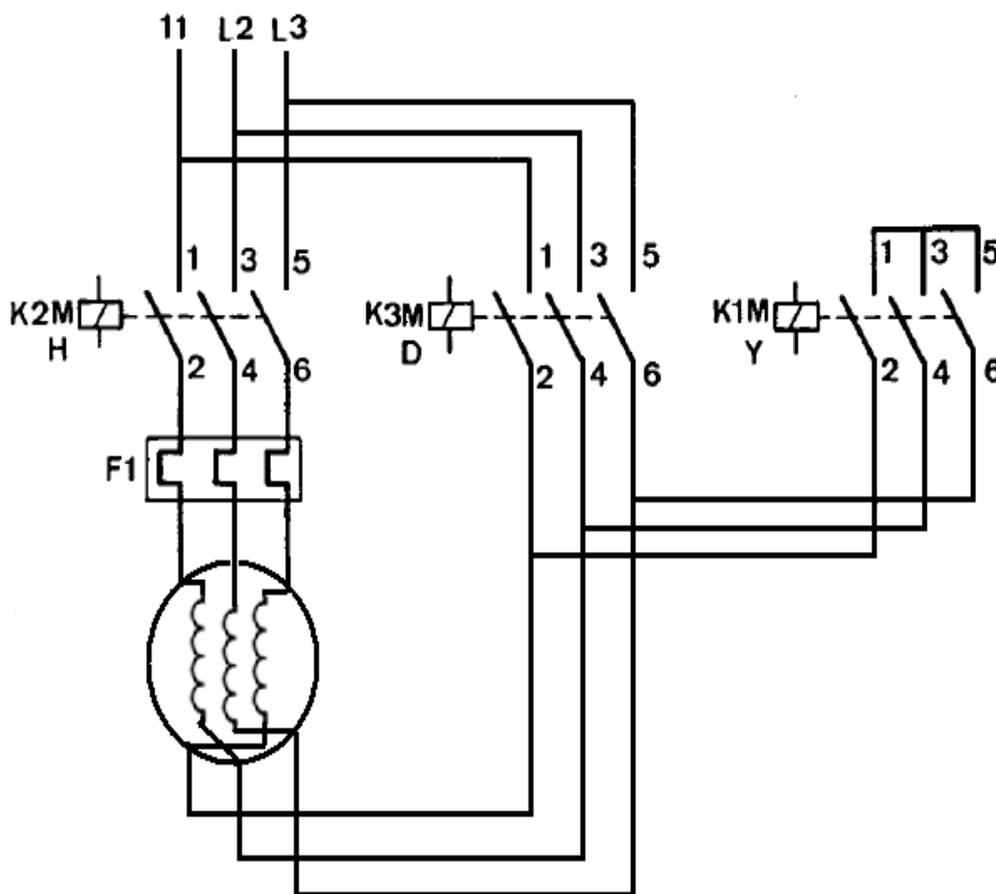


Fig.(2) Induction Motor with Star Delta Starter

## Auto Transformer Starter

The operation principle of auto transformer method is similar to the star delta starter method. The starting current is limited by (using a three phase auto transformer) reduce the initial stator applied voltage.

The auto transformer starter is more expensive, more complicated in operation and bulkier in construction when compared with the star – delta starter method. But an auto transformer starter is suitable for both star and delta connected motors, and the starting current and torque can be adjusted to a desired value by taking the correct tapping from the auto transformer. When the star delta method is considered, voltage can be adjusted only by factor of  $1/\sqrt{3}$ .

Figure (3) shows the connection of a 3phase induction motor with auto transformer starter.

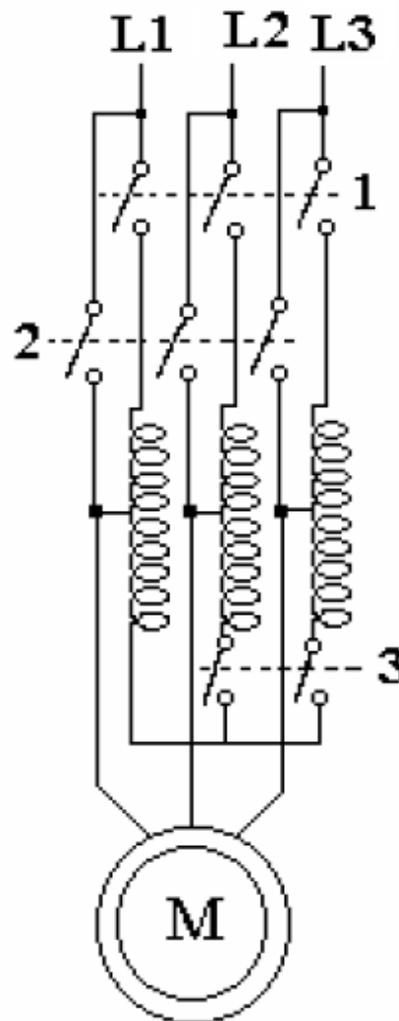


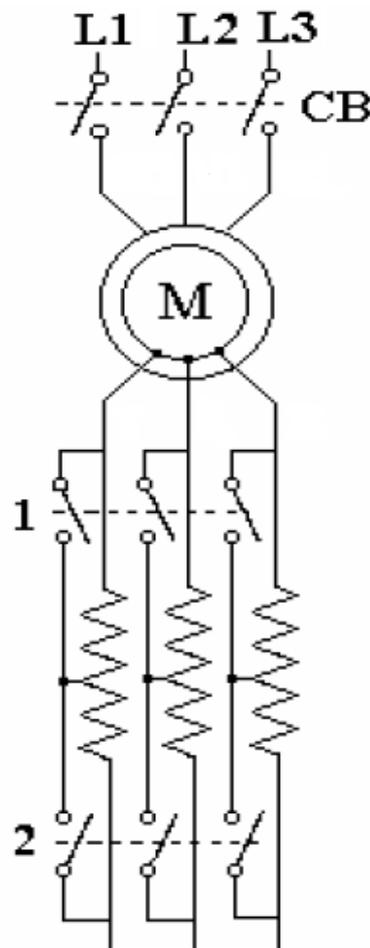
Fig.(3) shows I.M with auto transformer starter.

### Rotor Impedance Starter

This method allows external resistance to be connected to the rotor through slip rings and brushes. Initially, the rotor resistance is set to maximum and is then gradually decreased as the motor speed increases, until it becomes zero.

The rotor impedance starting mechanism is usually very bulky and expensive when compared with other methods. It also has very high maintenance costs. Also, a considerable amount of heat is generated through the resistors when current runs through them. The starting frequency is also limited in this method. However, the rotor impedance method allows the motor to be started while on load. Figure (4) shows the connection of a 3phase induction motor with rotor resistance starter.

Fig. (4) Shows the I.M. with rotor resistance starter.



**Example (9):**

It is desired to install a 3-phase cage induction motor restricting the maximum line current drawn from a 400 V 3-phase supply to 120 A. if the starting current is 6 times full load current, what is the maximum permissible full load kVA of the motor when

- i. It is directly connected to the mains
- ii. It is connected through an auto-transformer with a tapping of 60%
- iii. It is designed for used with star-delta starter.

**Solution:****i. Direct-on-line starting**

Maximum line current,  $I_L = 120\text{A}$

Starting current  $I_{st} = 6 \times \text{full load current} = 6I_{fl}$

Since the maximum line current drawn from the supply is 120A

$$6I_{ft} = 120, \quad I_f = \frac{120}{6} = 20\text{A}$$

Maximum permissible rating of the motor

$$= \sqrt{3}V_L I_{ft} = \sqrt{3} \times 20 \times 400 = 13856 \text{ VA} = 13.856 \text{ k VA}$$

**ii. Auto-transformer starting**

$$I_{st} = x^2 I_{sc} = x^2 (6I_{ft})$$

$$120 = (0.6)^2 (6I_{ft})$$

$$I_{ft} = \frac{120}{6 \times (0.6)^2} = 55.55A$$

Maximum permissible rating of the motor

$$= \sqrt{3}V_L I_{ft} = \sqrt{3} \times 400 \times 55.55 = 38.49 \text{ k VA}$$

**iii. Star-delta starting**

$$I_{st} = \frac{1}{3} (6I_{ft})$$

$$120 = 2I_{ft}, \quad I_{ft} = 60A$$

Maximum permissible kVA rating of the motor

$$= \sqrt{3}V_L I_{ft} = \sqrt{3} \times 400 \times 60 = 41.56 \text{ k VA}$$

## **11. SPEED CONTROL OF INDUCTION MOTORS**

The speed of an induction motor is given as

$$N = 120f/p (1-S).$$

So obviously the speed of an induction motor can be controlled by varying any of three factors namely supply frequency  $f$ , number of pole  $P$  or slip  $S$ .

The main methods employed for speed control of induction motors are as follows:

1. Pole changing
2. Stator voltage control
3. Supply frequency control
4. Rotor resistance control
5. Slip energy recovery.

The basic principles of these methods are described below

### **Pole changing**

The number of stator poles can be change by

- Multiple stator windings
- Method of consequent poles
- Pole amplitude modulation (PWM)

The methods of speed control by pole changing are suitable for cage motors only because the cage rotor automatically develops number of poles equal to the poles of stator winding.

### **1. Multiple stator windings**

In this method the stator is provided with two separate windings which are wound for two different pole numbers. One winding is energized at a time. Suppose that a motor has two windings for 6 and 4 poles. For 50 Hz supply the synchronous speed will be 1000 and 1500 rpm respectively. If the full load slip is 5% in each case, the operating speeds will be 950 rpm and 1425 rpm respectively. This method is less efficient and more costly, and therefore, used only when absolutely necessary.

## 2.Method of consequent poles

In this method a single stator winding is divided into few coil groups. The terminals of all these groups are brought out. The number of poles can be changed with only simple changes in coil connections. In practice, the stator winding is divided only in two coil groups. The number of poles can be changed in the ratio of 2:1.

Fig.(1) shows one phase of a stator winding consisting of 4 coils divided into two groups a – b and c – d. Group a – b consists of odd numbered coils(1,3) and connected in series. Group c – d has even numbered coils (2, 4) connected in series. The terminals a,b,c,d are taken out as shown.

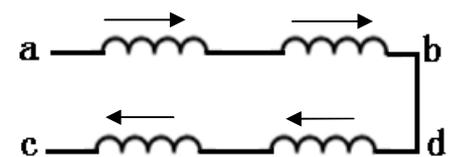
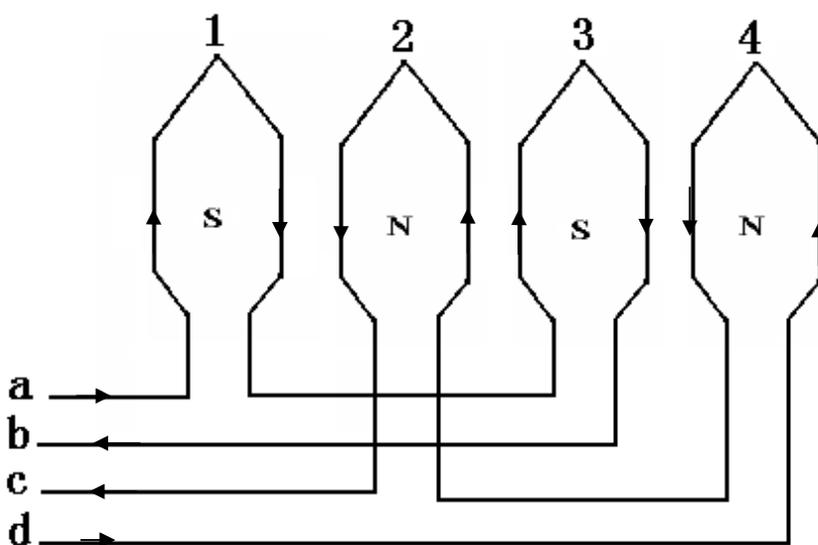


fig. (1-b)

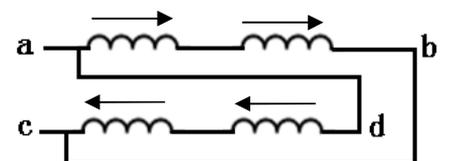


fig. (1-c)

Fig. (1) Stator phase connections for 4 poles

The coils can be made to carry current in given directions by connecting coil groups either in series or parallel shown in fig. (1-b) and fig.(1-c) respectively.

With this connection, there will be a total of 4 poles giving a synchronous speed of 1500 rpm for 50 Hz system. If the current through the coils of group a – b is reversed (fig.2), then all coils will produce north (N) poles.

In order to complete the magnetic path, the flux of the pole groups must pass through the spaces between the groups, thus inducing magnetic poles of opposite polarity (S poles) in the inter – pole spaces.

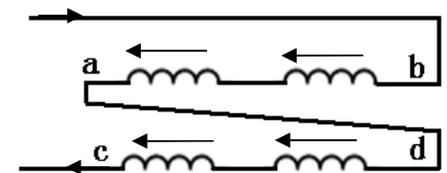
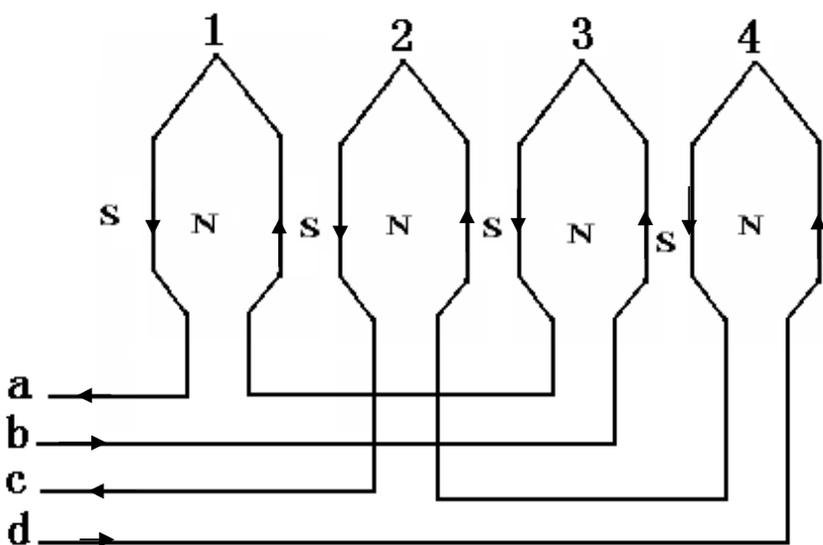


fig. (2-b) series connection

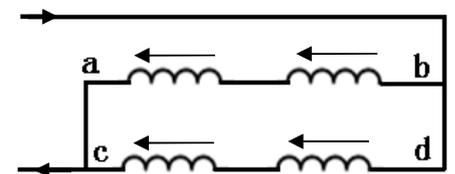


fig. (2-c) parallel connection

Fig. (2) Stator phase connections for 8 poles

## Stator Voltage Control

The torque developed by an induction motor is proportional to the square of the applied voltage. The variation of speed torque curves with respect to the applied voltage is shown in fig.(3). These curves show that the slip at maximum torque  $s_m$  remains same, while the value of stall torque comes down with decrease in applied voltage.

Further, we also note that the starting torque is also lower at lower voltages. Thus, even if a given voltage level is sufficient for achieving the running torque, the machine may not start. This method of trying to control the speed is best suited for loads that require very little starting torque, but their torque requirement may increase with speed  $T \propto \omega^2$ .

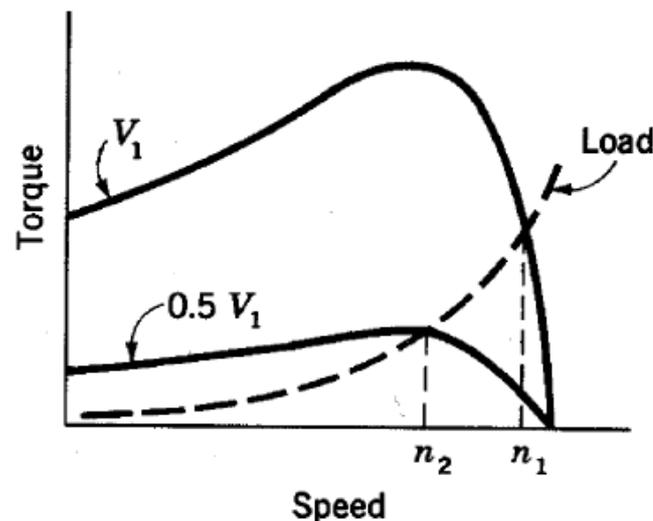


Fig. (3): Torque - speed curves for various terminal voltages

## Supply Frequency Control

The synchronous speed of an induction motor is given by

$$N_s = \frac{120f_s}{P}$$

The synchronous speed and, therefore, the speed of motor can be controlled by varying the supply frequency.

The emf induced in the stator of an induction motor is given by

$$E = 4.44k\Phi_m f_s N_1$$

Therefore, if the supply frequency is change,  $E_1$  will also change to maintain the same air gap flux. If the stator voltage drop is neglected the terminal voltage  $V_1$  is equal to  $E_1$  . in order to avoid saturation and to minimize losses, motor is operated at rated air gap flux by varying terminal voltage with frequency so as to maintain  $(V/f)$  ratio constant at rated value.

This type of control is known as constant volt in per hertz. Thus, the speed control of an induction motor using variable frequency supply requires a variable voltage power source.

## Rotor Resistance Control

In wound rotor induction motor, it is possible to change the shape of the torque – speed curve by inserting extra resistance into rotor circuit of the machine. The resulting torque – speed characteristic curves are shown in fig.(4).

This method of speed control is very simple. It is possible to have a large starting torque and low starting current at small value of slip.

The major disadvantage of this method is that the efficiency is low due to additional losses in resistors connected in the rotor circuit. Because of convenience and simplicity, it is often employed when speed is to be reduced for a short period only (cranes).

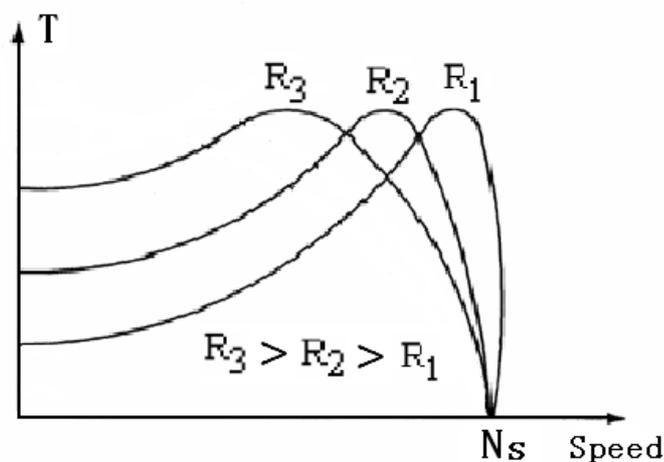


Fig.(4) Torque – speed curve for rotor resistance variation

## **Single Phase Motors**

As the name suggests, these motors are used on single – phase supply. Single phase motors are the most common type of electric motors, which finds wide domestic, commercial and industrial applications. Single phase motors are small size motors of fraction – kilowatt ratings. Domestic applications like fans, hair driers, washing machines, mixers, refrigerators, food processors and other kitchen equipment employ these motors. These motors also find applications in air – conditioning fans, blower’s office machinery etc.

Single phase motors may be classified into the following basic types:

1. Single phase induction motors
2. AC. Series motor (universal motor)
3. Repulsion motors
4. Synchronous motor

## Single Phase Induction Motor

A single phase induction motor is very similar to 3 – phase squirrel cage induction motor. It has a squirrel – cage rotor identical to a 3 - phase squirrel cage motor and a single – phase winding on the stator. Unlike 3 – phase induction motor, a single phase induction motor is not self starting but requires some starting means.

Figure (1) shows 1 – phase induction motor having squirrel cage rotor and single phase distributed stator winding.

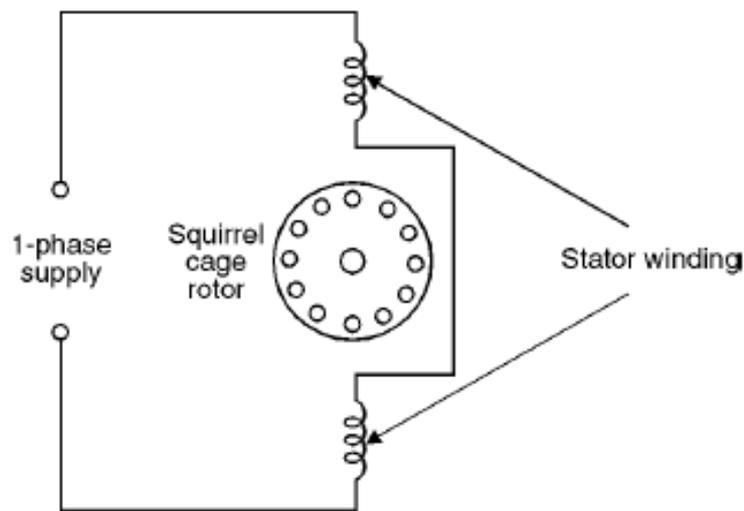


Fig. (1) Single – phase induction motor

If the stator winding is connected to single – phase a.c. supply, the stator winding produces a magnetic field that pulsates in strength in a sinusoidal manner. The field polarity reverses after

each half cycle but the field does not rotate. Consequently, the alternating flux cannot produce rotation in a stationary squirrel cage rotor. However, if the rotor is started by auxiliary means, the motor will quickly attain the final speed. The behavior of single – phase induction motor can be explained on the basis of double – field revolving theory.

### **Double – Field Revolving Theory**

The pulsating field produced in single phase AC motor is resolved into two components of half the magnitude and rotating in opposite directions at the same synchronous speed.

Let  $\Phi_m$  be the pulsating field which has two components each of magnitude  $\Phi_m/2$ . Both are rotating at the same angular speed  $\omega$  rad/sec but in opposite direction as shown in the Figure (2-a). The resultant of the two fields is  $\Phi_m \cos\theta$ . Thus the resultant field varies according to cosine of the angle  $\theta$ . The wave shape of the resultant field is shown in Figure (2-b).

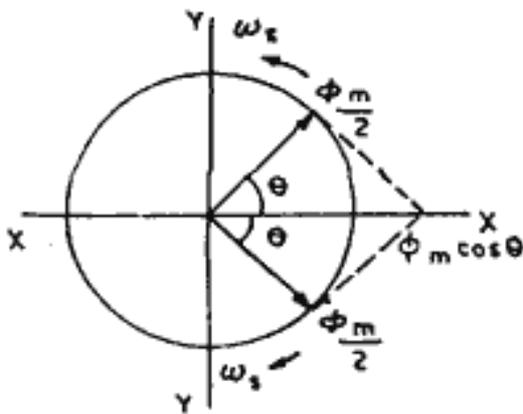


Fig. (2-a)

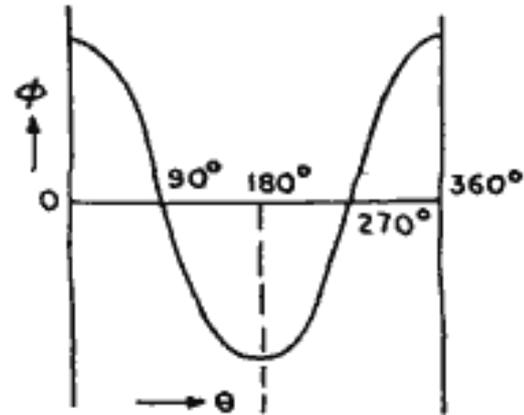


Fig. (2-b)

Thus the alternating flux produced by stator winding can be presented as the sum of two rotating fluxes  $\phi_1$  and  $\phi_2$  each equal to one half of the maximum value of alternating flux and each rotating at synchronous speed in opposite directions. Let the flux  $\phi_1$  (forward) rotate in anticlockwise direction and flux  $\phi_2$  (backward) in clockwise direction. The flux  $\phi_1$  will result in the production of torque  $T_1$  in the anticlockwise direction and flux  $\phi_2$  will result in the production of torque  $T_2$  in the clockwise direction. At standstill, these two torques are equal and opposite and the net torque developed is zero. Therefore, single – phase induction motor is not self – starting. This fact is illustrated in figure(3).

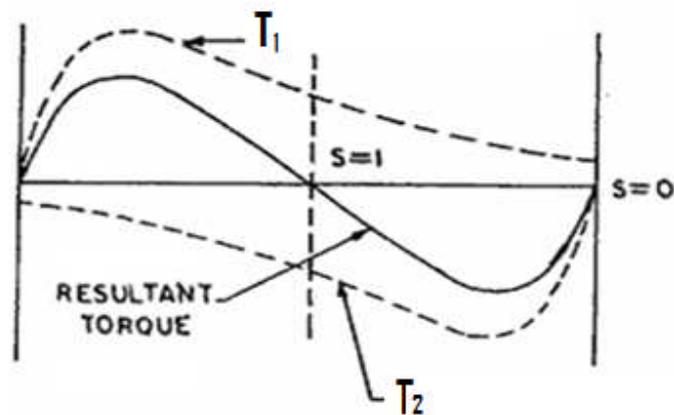


Fig. (3) Torque – slip characteristic of 1- phase induction motor

### Rotor Running

Assume that the rotor is started by spinning the rotor or by using auxiliary circuit, in say clockwise direction. The flux rotating in the clockwise direction is the forward rotating flux  $\Phi_f$  and that in the other direction is the backward rotating flux  $\Phi_b$ . The slip w.r.t. the forward flux will be

$$s_f = \frac{N_s - N}{N_s}$$

Where  $N_s$  = synchronous speed

$N$  = speed of rotor in the direction of forward flux

The rotor rotates opposite to the rotation of the backward flux. Therefore, the slip w.r.t the backward flux will be

$$\begin{aligned}
 s_b &= \frac{N_s - (-N)}{N_s} = \frac{N_s + N}{N_s} = \frac{2N_s - N_s + N}{N_s} \\
 &= \frac{2N_s}{N_s} - \frac{(N_s - N)}{N_s} = 2 - s
 \end{aligned}$$

$$\therefore s_b = 2 - s$$

Thus for forward rotating flux, slip is  $s$  (less than unity) and for backward rotating flux, the slip is  $2-s$  (greater than unity) since for usual rotor resistance/reactance ratios, the torque at slips of less than unity are greater than those at slips of more than unity, the resultant torque will be in the direction of the rotation of the forward flux. Thus if the motor is once started, it will develop net torque in the direction in which it has been started and will function as a motor.

## Starting of Single Phase Induction Motors

The single phases induction motors are classified based on the method of starting method and in fact are known by the same name descriptive of the method.

### 1. Split – phase Induction Motor

The stator of a split – phase induction motor has two windings, the main winding and the auxiliary winding. These windings are displaced in space by 90 electric degrees as shown in figure (4-a).

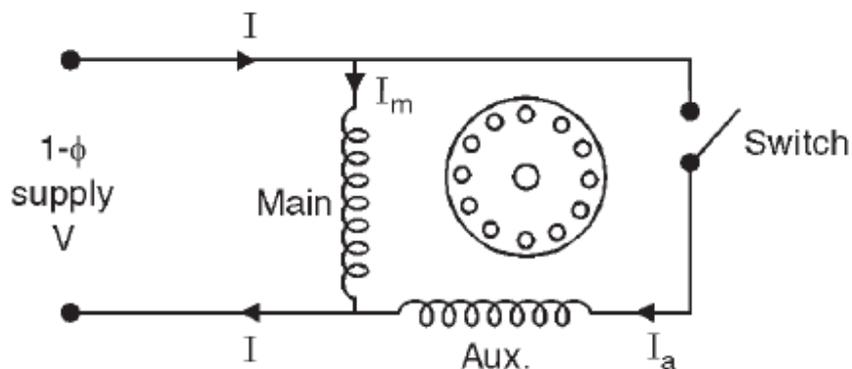


Fig.(4-a) split phase I.M.

The auxiliary winding is made of thin wire so that it has a high R/X ratio as compared to the main winding which has thick super enamel copper wire. When the two stator windings are

energized from a single – phase supply, the current  $I_m$  and  $I_a$  in the main winding and auxiliary winding lag behind the supply voltage  $V$ , and  $I_a$  leading the current  $I_m$  as shown in figure (4-b).

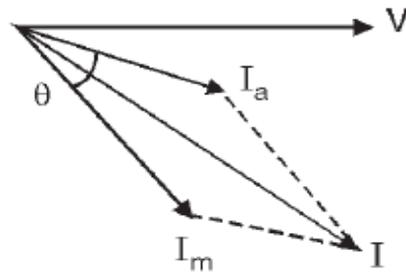
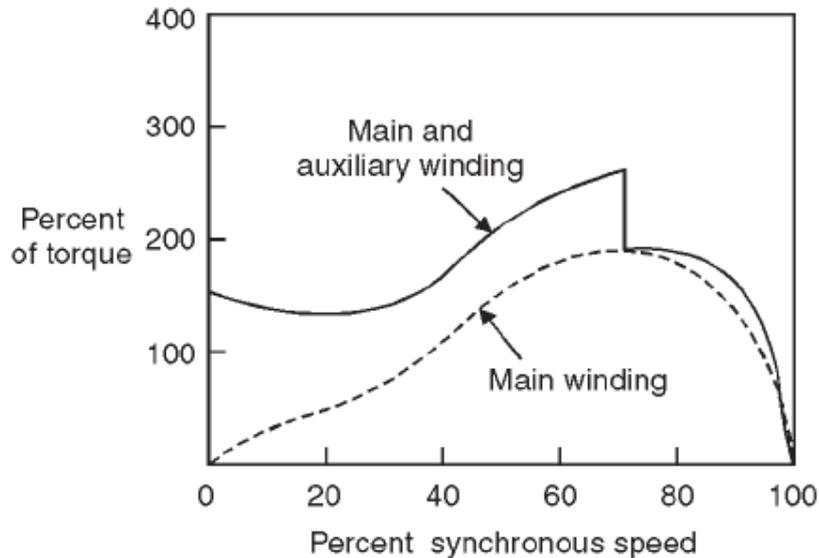


Fig.(4-b) Phasor diagram at starting

This means the current through auxiliary winding reaches maximum value first and the mmf or flux due to  $I_a$  lies along the axis of the auxiliary winding and after some time the current  $I_m$  reaches maximum value and the mmf due to  $I_m$  lies along the main winding axis. Thus the motor becomes a 2 – phase unbalanced motor. Because of these two fields a starting torque is developed and the motor becomes a self starting motor. After the motor starts, the auxiliary winding is disconnected usually by means of centrifugal switch that operates at about 75% of synchronous speed. Finally the motor runs because the main winding. Since this being single phase some level of humming noise is always associated with the motor during running. The power rating of such motors generally lies between 60- 250W.

The typical torque – speed characteristic is shown in fig (4-c).



### Characteristics

- Due to their low cost, split – phase induction motors are most popular single – phase motors in the market
- Since the starting winding is made of thin wire, the current density is high and the winding heats up quickly. If the starting period exceeds 5 seconds, the winding may burn out unless the motor is protected by built – in thermal relay. This motor is, therefore, suitable where starting periods are not frequent.

### 2. Capacitor – Start Motor

Capacitors are used to improve the starting and running performance of the single phase induction motors.

The capacitor – start motor is identical to a split – phase motor except that the starting winding has as many turns as the main winding. Moreover, a capacitor  $C$  is connected in series with the starting winding as shown in figure (5-a).

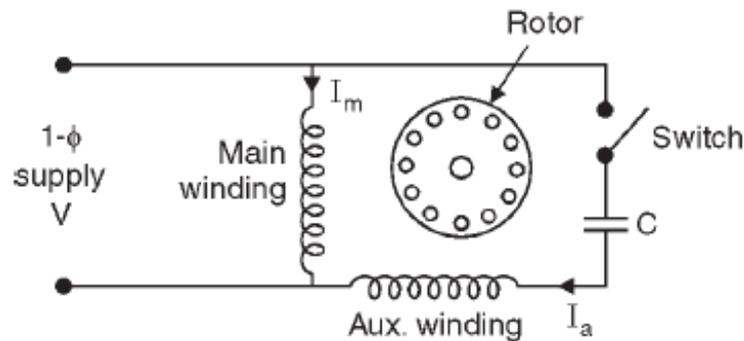
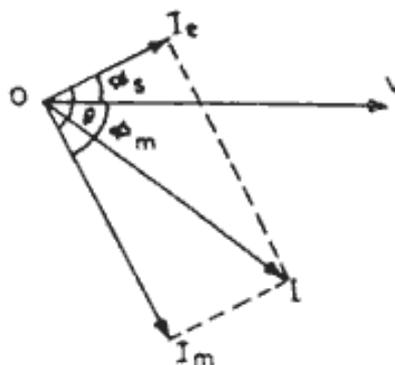
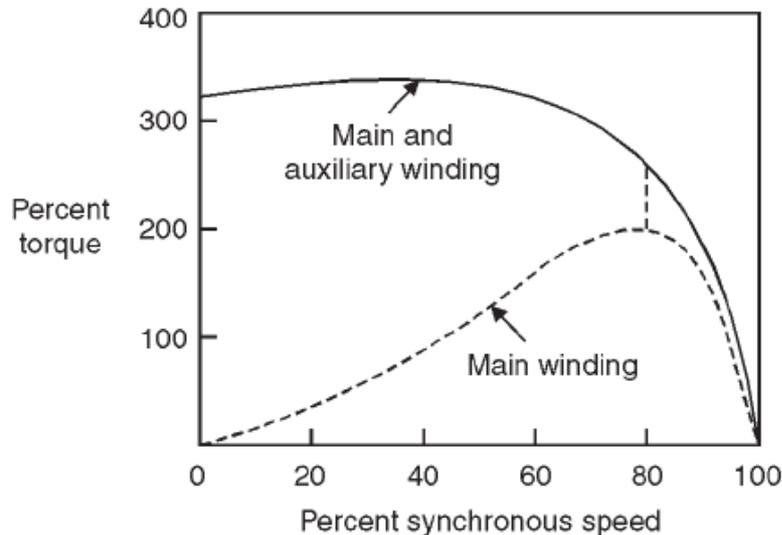


Fig.(5-a) Capacitor Start Motor

The value of capacitor is so chosen that  $I_a$  leads  $I_m$  by about  $90^\circ$  (Fig.5-b) so that the starting torque is maximum for certain values of  $I_a$  and  $I_m$ . Again, the starting winding is opened by the centrifugal switch when the motor attains about 75% of synchronous speed. The motor then operates as a single – phase induction motor and continues to accelerate till it reaches the normal speed.



The typical torque – speed characteristic is shown in fig (5-c).



### Characteristics

- Although starting characteristics of a capacitor – start motor are better than those of a split – phase motor, both machines possess the same running characteristics because the main windings are identical.
- The phase angle between the two currents is about 90° compared to about 25° in a split – phase motor. Consequently, for the same starting torque, the current in the starting winding is only about half that in a split – phase motor. Therefore, the starting winding of a capacitor start motor heats up less quickly and is well suited to applications involving either frequent or prolonged starting periods.

- Capacitor – start motors are used where high starting torque is required and where high starting period may be long e.g. to drive:

a) Compressors b) large fans c) pumps d) high inertia loads

The power rating of such motors lies between 120W and 0.75 kW.

### **3. Permanent – Split Capacitor Motor**

In this motor, as shown in fig.(6-a), the capacitor that is connected in series with the auxiliary winding is not cut out after starting and is left in the circuit all the time. This simplifies the construction and decreases the cost because the centrifugal switch is not needed. The power factor, torque pulsation, and efficiency are also improved because the motor runs as a two – phase motor. The motor will run more quietly.

The capacitor value is of the order of 20 – 50  $\mu\text{F}$  and because it operates continuously, it is an ac paper oil type. The capacitor is compromise between the best starting and running value and therefore starting torque is sacrificed. The typical torque – speed characteristic is shown in fig (6-b).

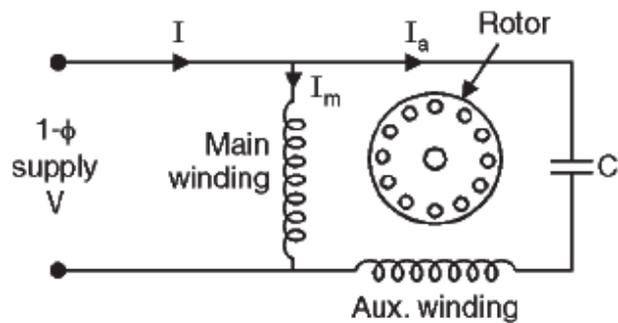


Fig.(6-a) Permanent – Split Capacitor Motor

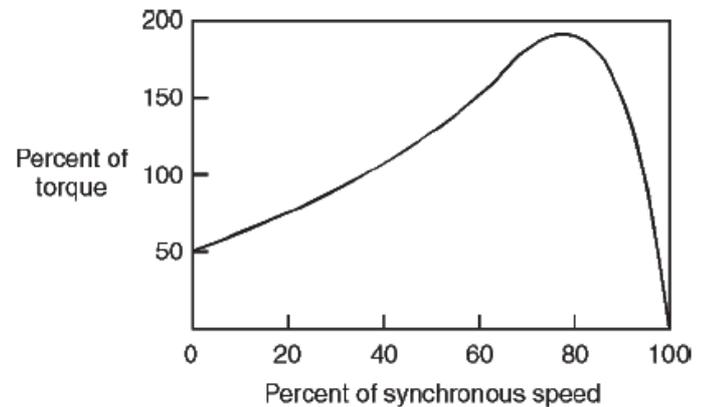


Fig.(6-b) torque – speed characteristic

### Characteristic

- These motor are used where the required starting torque is low such as air – moving equipment i.e. fans, blowers and voltage regulators and also oil burners where quite operation is particularly desirable.

### 4. Capacitor - Start Capacitor - Run

Two capacitor, one for starting and one for running, can be used, as shown in fig.(7-a).

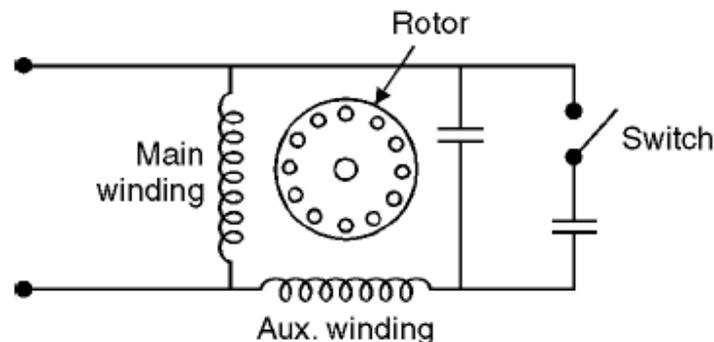


Fig. (7-a) Capacitor - Start Capacitor – Run motor

Theoretically, optimum starting and running performance can be achieved by having two capacitors. The starting capacitor is larger in value and is of the ac electrolytic type. The running capacitor permanently connected in series with the starting winding, is of smaller value and is of the paper oil type. Typical values of these capacitors for a 0.5 hp are  $C_s = 300\mu\text{F}$ ,  $C_r = 40\mu\text{F}$ . The typical torque – speed characteristic is shown in fig. (7- b).

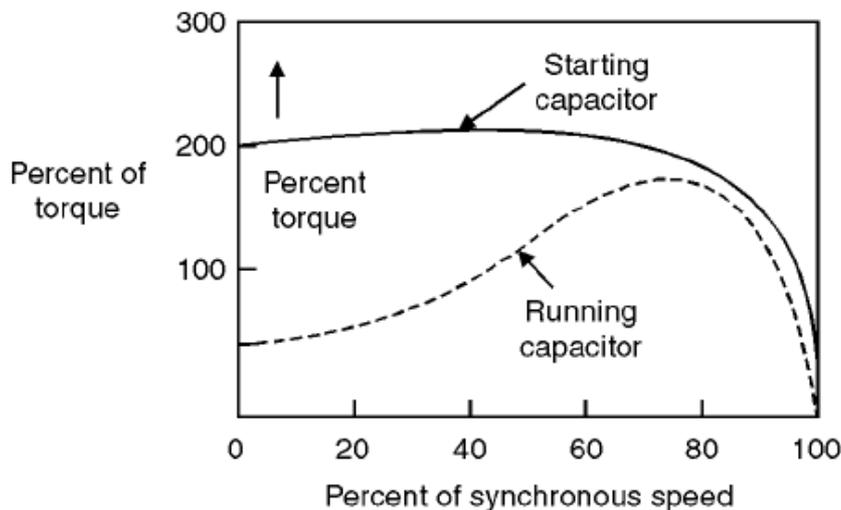


Fig.(7-b) torque – speed characteristic

### Characteristic

- Ability to start heavy loads
- Extremely quiet operation
- Higher efficiency and power factor
- Ability to develop 25 per cent overload capacity. Hence, such motors are ideally suited where load requirements are severe as in the case of compressors and conveyors ect.

## 5. Shaded Pole Induction Motor

These motors have a salient pole construction. A shaded band consisting of a short – circuited copper turn, known as a shading coil, is used on one portion of each pole, as shown in fig(8-a)

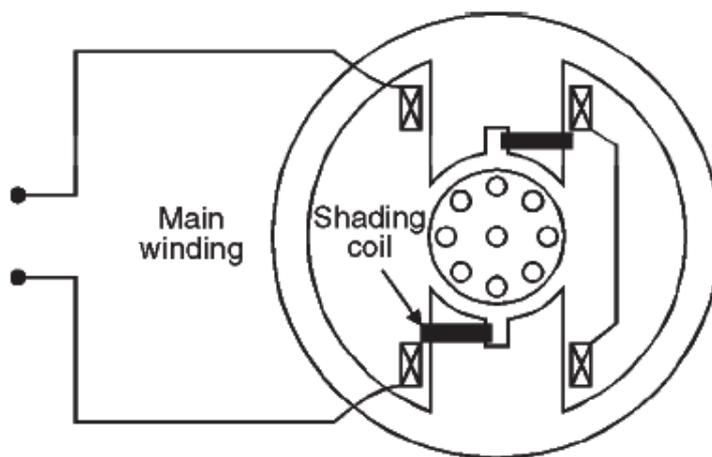


Fig (8-a) Shaded Pole Induction Motor

When alternating current flow in the field winding, an alternating flux is produced in the field core. A portion of this flux links with the shading coil, which behaves as short – circuited secondary of a transformer. A voltage is induced in the shading coil, and this voltage circulates a current in it. The induced current produces a flux called the induced flux which opposes the main core flux. The shading coil, thus, causes the flux in the shaded portion to lag behind the flux in the unshaded

portion of the pole. At the same time, the main flux and the shaded pole flux are displaced in space. This displacement is less than  $90^\circ$ . Since there is time and space displacement between the two fluxes, the conditions for setting up a rotating magnetic field are produced. Under the action of the rotating flux a starting torque is developed on the cage rotor. The direction of this rotating field (flux) is from the unshaded to the shaded portion of the pole.

The typical torque-speed characteristic is shown in fig. (8-b).

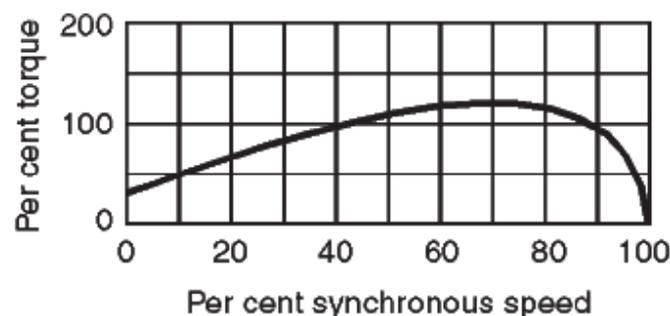


Fig.(8-b) torque – speed characteristic

### Characteristic

- The salient features of this motor are extremely simple construction and absence of centrifugal switch
- Since starting torque, efficiency and power factor are very low, these motors are only suitable for low power applications e.g. to drive: Small fans b) toys c) hair driers. The power rating of such motors is up to about 30 W.

## **Equivalent Circuit of Single – Phase Induction Motor**

When the stator of single phase induction motor is connected to single – phase supply, the stator current produces a pulsating flux. According to the double – revolving field theory, the pulsating air – gap flux in the motor at standstill can be resolved into two equal and opposite fluxes with the motor. Since the magnitude of each rotating flux is one – half of the alternating flux, it is convenient to assume that the two rotating fluxes are acting on two separate rotors. Thus, a single – phase induction motor may be considered as consisting of two motors having a common stator winding and two imaginary rotors, which rotate in opposite directions. The standstill impedance of each rotor referred to the main stator winding is  $(\frac{R_2}{2} + j \frac{X_2}{2})$ .

The equivalent circuit of single – phase induction motor at standstill is shown in fig.(9).

$R_{1m}$  = resistance of stator winding

$X_{1m}$  = leakage reactance of stator winding

$X_M$  = total magnetizing reactance

$\hat{R}_2$  = resistance of rotor referred to the stator

$\hat{X}_2$  = leakage reactance of rotor referred to the stator

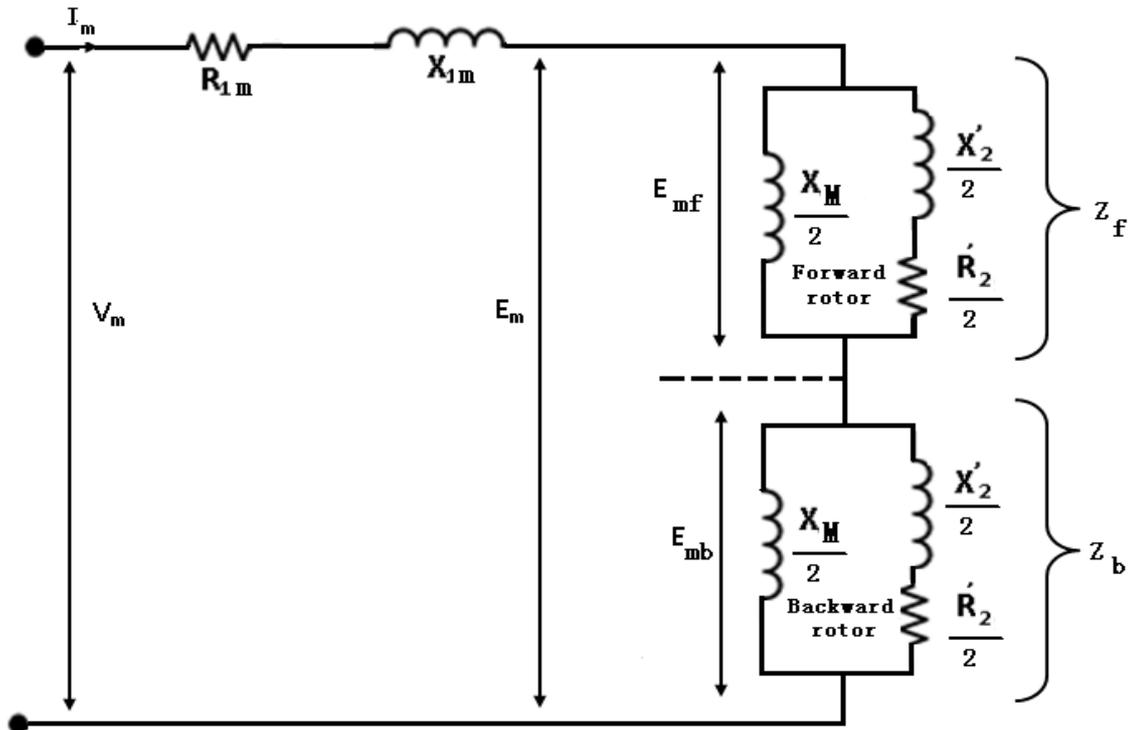


Fig.(9)

In this diagram, the portion of the equivalent circuit representing the effects of air gap flux is split into two portions. The first portion shows the effect of forward rotating flux, and the second portion shows the effect of backward rotating flux.

The forward flux induces a voltage  $E_{mf}$  in the main stator winding. The backward rotating flux induces a voltage  $E_{mb}$  in

the main stator winding. The resultant induced voltage in the main stator winding is  $E_m$ , where

$$\mathbf{E_m = E_{mf} + E_{mb}}$$

At standstill,  $\mathbf{E_{mf} = E_{mb}}$

Now suppose that the motor is started with the help of an auxiliary winding. The auxiliary winding is switched out after the motor gains its normal speed.

The effective rotor resistance of an induction motor depends on the slip of the rotor. The slip of the rotor with respect to the forward rotating flux is  $S$ . The slip of the rotor with respect to the backward rotating flux is  $(2-S)$ .

When the forward and backward slips are taken into account, the result is the equivalent circuit shown in fig.(10) which represents the motor running on the main winding alone.

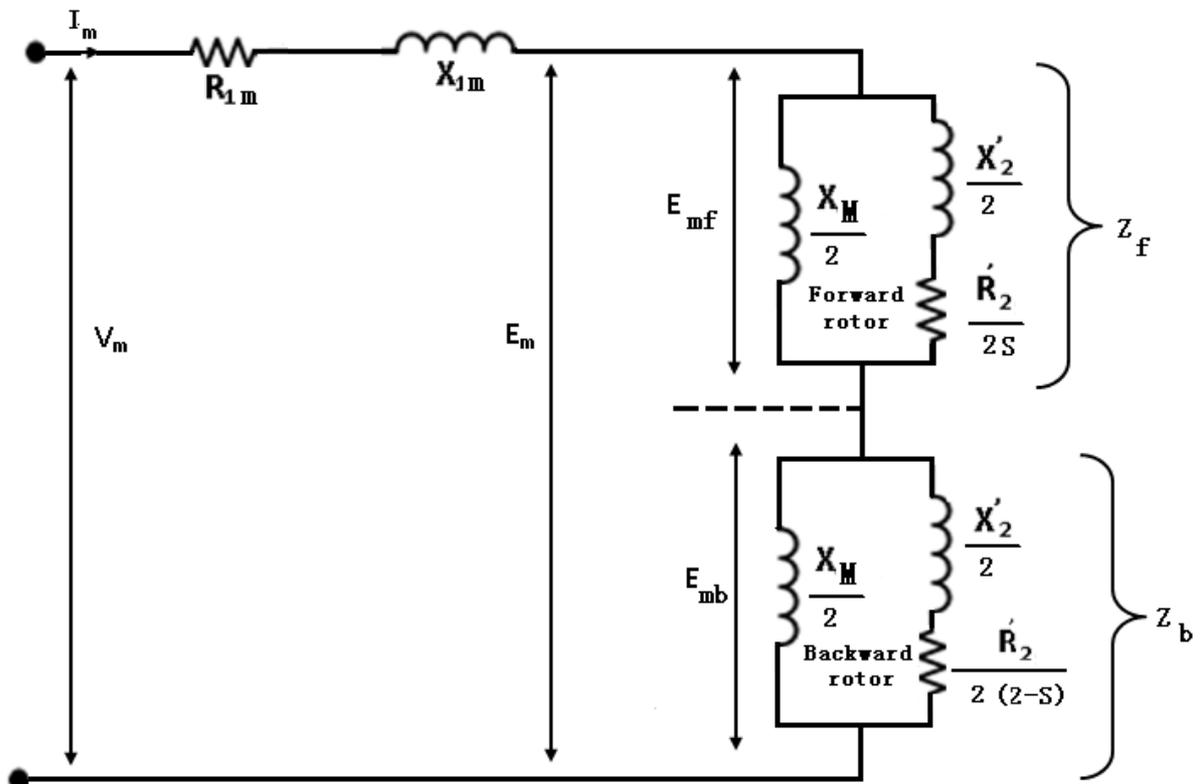


Fig.(10)

The rotor impedance representing the effect of forward field referred to the stator winding  $m$  is given by an impedance

$(\frac{\hat{R}_2}{2S} + j\frac{\hat{X}_2}{2})$  in parallel with  $j\frac{X_M}{2}$ .

$$\therefore Z_f = R_f + jX_f = (\frac{\hat{R}_2}{2S} + j\frac{\hat{X}_2}{2}) \parallel (j\frac{X_M}{2})$$

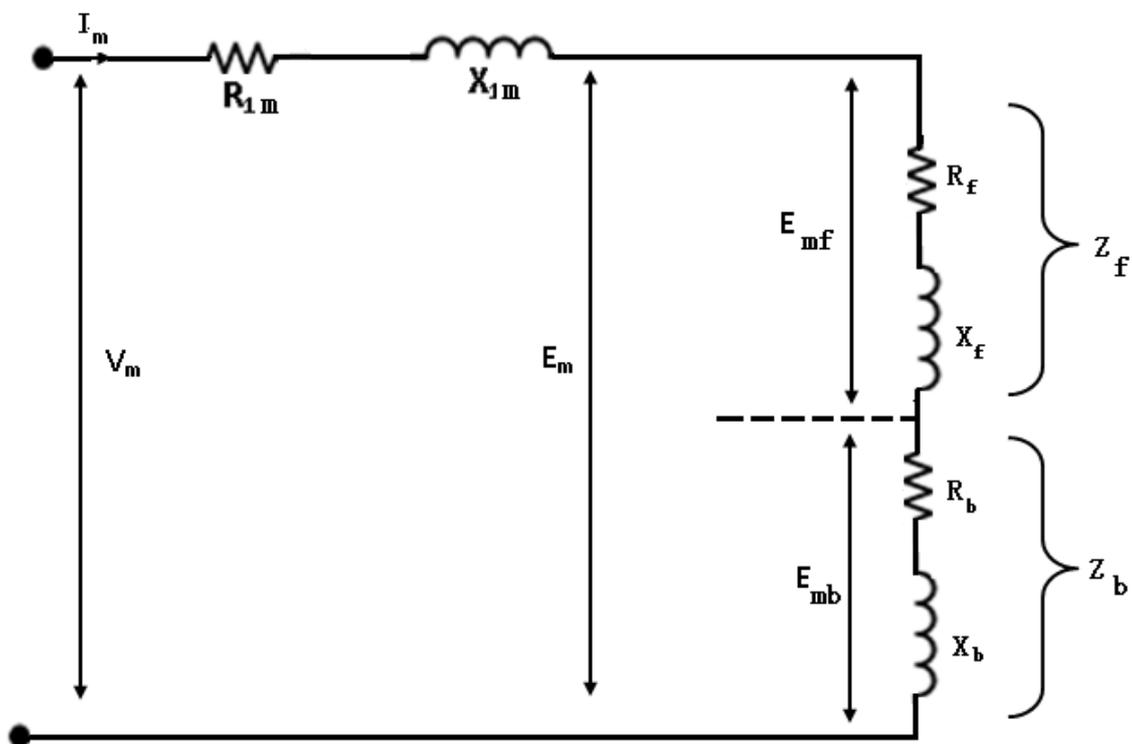
$$Z_f = \frac{(\frac{\hat{R}_2}{2S} + j\frac{\hat{X}_2}{2})(j\frac{X_M}{2})}{\frac{\hat{R}_2}{2S} + j\frac{\hat{X}_2}{2} + j\frac{X_M}{2}}$$

Similarly, the rotor impedance representing the effect of backward field referred to the stator winding m is given by an impedance  $(\frac{\hat{R}_2}{2(2-s)} + j\frac{\hat{X}_2}{2})$  in parallel with  $j\frac{X_M}{2}$ .

$$\therefore Z_b = R_b + jX_b = \left(\frac{\hat{R}_2}{2(2-s)} + j\frac{\hat{X}_2}{2}\right) \parallel \left(j\frac{X_M}{2}\right)$$

$$Z_b = \frac{\left(\frac{\hat{R}_2}{2(2-s)} + j\frac{\hat{X}_2}{2}\right)\left(j\frac{X_M}{2}\right)}{\frac{\hat{R}_2}{2(2-s)} + j\frac{\hat{X}_2}{2} + j\frac{X_M}{2}}$$

The simplified equivalent circuit of single – phase induction motor with only main winding energized is shown in fig.(11).



The current in the stator winding is

$$I_m = \frac{V_m}{Z_{1m} + Z_f + Z_b}$$

The torque of the backward field is in opposite direction to that of the forward field, and therefore the total air – gap power in a single phase induction motor is

$$P_g = P_{gf} - P_{gb}$$

Where  $P_{gf}$  = air – gap power for forward field

$$P_{gf} = I_m^2 R_f$$

Where  $P_{gb}$  = air – gap power for backward field

$$P_{gb} = I_m^2 R_b$$

∴

$$P_g = I_m^2 R_f - I_m^2 R_b = I_m^2 (R_f - R_b)$$

The torque produced by the forward field

$$T_f = \frac{1}{\omega_s} P_{gf} = \frac{P_{gf}}{2\pi n_s}$$

The torque produced by the backward field

$$T_b = \frac{1}{\omega_s} P_{gb} = \frac{P_{gb}}{2\pi n_s}$$

The resultant electromagnetic or induced torque  $T_{int}$  is the difference between the torque  $T_f$  and  $T_b$  :

$$T_{int} = T_f - T_b$$

As in the case of the 3 - phase I.M., the induced torque is equal to the air gap power divided by synchronous angular velocity.

$$T_{int} = \frac{P_g}{\omega_s} = \frac{1}{\omega_s} (P_{gf} - P_{gb}) = \frac{I_m^2}{\omega_s} (R_f - R_b)$$

The total copper loss is the sum of rotor copper loss due to the forward field and the rotor copper loss due to the backward field.

$$P_{cr} = P_{crf} + P_{crb}$$

And rotor copper loss in a 3 – phase induction motor

$$P_{cr} = \text{slip} * \text{air gap power}$$

$$P_{cr} = sP_{gf} + (2 - s)P_{gb}$$

The power converted from electrical to mechanical form in a single phase induction motor is given by

$$P_{mech} = P_{conv} = \omega T_{ind}$$

$$\begin{aligned} P_{mech} &= (1 - s)\omega_s T_{ind} \\ &= (1 - s)P_g = (1 - s)(P_{gf} - P_{gb}) \end{aligned}$$

Or

$$P_{mech} = I_m^2 (R_f - R_b) (1 - s)$$

Shaft output power

$$P_{out} = P_{mech} - \text{core loss} - \text{mechanical losses} - \text{stray losses}$$

$$P_{out} = P_{mech} - P_{rot}$$

Where

$$P_{rot} = \text{rotational losses}$$

**Example**

A 230 V, 50 Hz, 4 – pole single phase induction motor has the following equivalent circuit impedances:

$$R_{1m} = 2.2\Omega, \quad \hat{R}_2 = 4.5\Omega$$

$$X_{1m} = 3.1\Omega, \quad \hat{X}_2 = 2.6\Omega, \quad X_M = 80\Omega.$$

Friction, windage and core loss = 40 W

For a slip of 0.03pu, calculation (a) input current, (b) power factor, (c) developed power, (d) output power, (e) efficiency.

**Solution.** Form the given data

$$\frac{\hat{R}_2}{2s} = \frac{4.5}{2 \times 0.03} = 75\Omega$$

$$\frac{\hat{R}_2}{2(2-s)} = \frac{4.5}{2(2-0.03)} = 1.142\Omega$$

$$\frac{1}{2}\hat{X}_2 = \frac{1}{2} \times 2.6 = 1.3\Omega$$

$$\frac{1}{2}X_M = \frac{1}{2} \times 80 = 40\Omega$$

For the forward field circuit

$$\begin{aligned}
 Z_f &= R_f + jX_f = \frac{\left(\frac{\hat{R}_2}{2s} + j\frac{\hat{X}_2}{2}\right) \left(j\frac{X_M}{2}\right)}{\frac{\hat{R}_2}{2s} + j\frac{\hat{X}_2}{2} + j\frac{X_M}{2}} \\
 &= \frac{(75 + j1.3)(j40)}{75 + j1.3 + j40} = \frac{(75.011\angle 0.993^\circ)(40\angle 90^\circ)}{85.619\angle 28.84^\circ} \\
 &= 35.04\angle 62.15^\circ \Omega = 16.37 + j30.98\Omega
 \end{aligned}$$

For the backward field

$$\begin{aligned}
 Z_b &= R_b + jX_b = \frac{\left(\frac{\hat{R}_2}{2(2-s)} + j\frac{\hat{X}_2}{2}\right) \left(j\frac{X_M}{2}\right)}{\frac{\hat{R}_2}{2(2-s)} + j\frac{\hat{X}_2}{2} + j\frac{X_M}{2}} \\
 &= \frac{(1.142 + j1.3)(j40)}{1.142 + j1.3 + j40} = \frac{(1.73\angle 48.7^\circ)(40\angle 90^\circ)}{41.316\angle 88.4^\circ} \\
 &= 1.675\angle 50.3^\circ = 1.07 + j1.29\Omega \\
 Z_{1m} &= R_{1m} + jX_{1m} = 2.2 + j3.1
 \end{aligned}$$

The total series impedance

$$\begin{aligned}
 Z_e &= Z_{1m} + Z_f + Z_b \\
 &= 2.2 + j3.1 + 16.37 + j30.98 + 1.07 + j1.29
 \end{aligned}$$

$$= 19.64 + j35.37 = 40.457 \angle 60.96^\circ \Omega$$

(a) Input current

$$I_m = \frac{V_m}{Z_e} = \frac{230 \angle 0^\circ}{40.457 \angle 60.96^\circ} = 5.685 \angle -60.96^\circ \text{ A.}$$

(b) Power factor =  $\cos(-60.95^\circ) = 0.4856$  *lagging*.

(c) Developed power

$$P_{conv} = P_d = I_m^2 (R_f - R_b)(1 - s)$$

$$= (5.685)^2 (16.37 - 1.07)(1 - 0.03) = 479.65 \text{ W}$$

(d) Output power =  $P_d - P_{rot} = 479.65 - 40 = 439.65 \text{ W}$

$$\text{Input power} = VI_m \cos \phi = 230 \times 5.685 \times 0.4856 = 634.9 \text{ W}$$

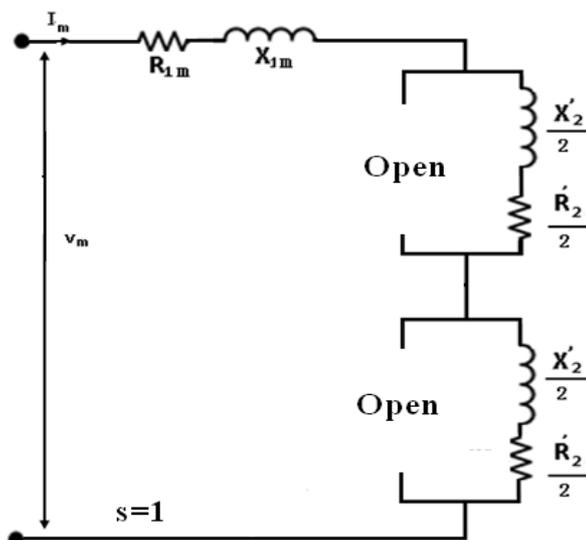
(e) Efficiency =  $\frac{\text{output}}{\text{input}} = \frac{439.65}{634.9} = 0.692 \text{ pu.}$

## Determination of Equivalent Circuit Parameters

The parameter of the equivalent circuit of single – phase induction motor can be determined from the blocked – rotor and no – load tests. These tests are performed with auxiliary winding kept open, except for the capacitor – run motor.

### Blocked – rotor test

In this test the rotor is at rest (blocked). A low voltage is applied to the stator so that rated current flows in the main winding. The voltage ( $V_{scr}$ ), current ( $I_{scr}$ ) and power input ( $P_{scr}$ ) are measured. With the rotor blocked,  $s = 1$  the impedance  $\frac{X_M}{2}$  in the equivalent circuit is so large compared with  $(\frac{\hat{R}_2}{2} + j\frac{\hat{X}_2}{2})$  that it may be neglected from the equivalent circuit. Therefore the equivalent circuit at  $s=1$  is shown in fig.(12).



**Fig.(12) simplified equivalent circuit of single phase I.M. with locked rotor**

$$Z_e = \frac{V_{scr}}{I_{scr}}$$

From fig.(12), the equivalent series resistance  $R_e$  of the motor is

$$R_e = R_{1m} + \frac{R'_2}{2} + \frac{R'_2}{2} = R_{1m} + R'_2 = \frac{P_{scr}}{I_{scr}^2}$$

Since the resistance of the main stator winding  $R_{1m}$  is already measured, the effective rotor resistance at line frequency is given by

$$R'_2 = R_e - R_{1m} = \frac{P_{scr}}{I_{scr}^2} - R_{1m}$$

From fig.(12), the equivalent reactance  $X_e$  is given by

$$X_e = X_{1m} + \frac{X'_2}{2} + \frac{X'_2}{2} = X_{1m} + X'_2$$

Since the leakage reactance  $X_{1m}$  and  $X'_2$  cannot be separated out we make a simplifying assumption that  $X_{1m} = X'_2$ .

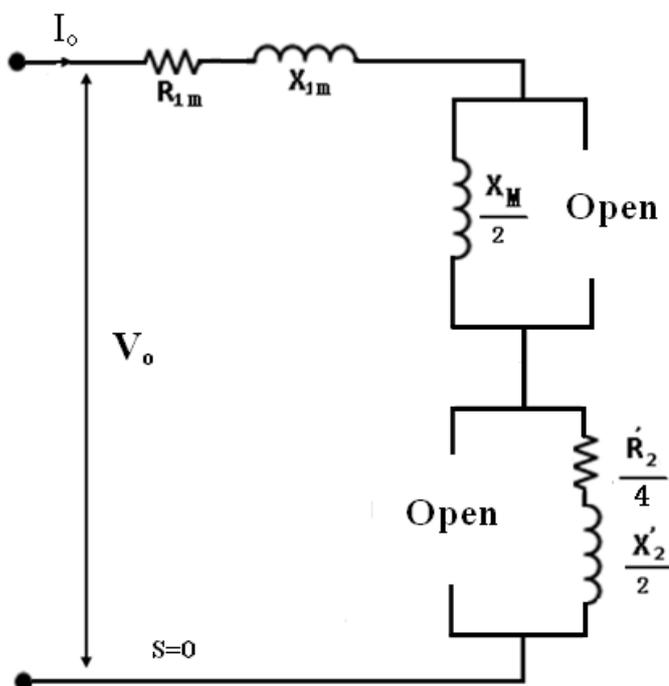
$$\therefore X_{1m} = X'_2 = \frac{1}{2} X_e = \frac{1}{2} \sqrt{Z_e^2 - R_e^2}$$

Thus, from blocked - rotor test, the parameters  $R'_2, X_{1m}, X'_2$  can be found if  $R_{1m}$  is known.

### No - load test

The motor is run without load at rated voltage and rated frequency. The voltage ( $V_o$ ), current ( $I_o$ ) and input power ( $P_o$ ) are measured. At no load, the slip  $s$  is very small close to zero and  $\frac{\hat{R}_2}{2s}$  is very large as compared to  $\frac{X_M}{2}$ .

The resistance  $\frac{\hat{R}_2}{2(2-s)} \cong \frac{\hat{R}_2}{4}$  associated with the backward rotating field is so small as compared to  $\frac{X_M}{2}$ , that the backward magnetizing current is negligible. Therefore, under no load conditions, the equivalent circuit becomes as shown in fig.(13).



**Fig.(13) simplified equivalent circuit of single phase I.M. at no load**

From the fig (13), the equivalent reactance at no load is given by

$$X_o = X_{1m} + \frac{X_M}{2} + \frac{X_2}{2}$$

Since  $X_{1m}$  and  $X_2$  are already known from the blocked rotor test, the magnetizing reactance  $X_M$  can be calculated from above equation.

And

$$X_o = Z_o \sin \phi_o = Z_o \sqrt{1 - \cos^2 \phi_o}$$

$$\cos \phi_o = \frac{P_o}{V_o I_o}$$

$$Z_o = \frac{V_o}{I_o}$$

**Example**

A 220 V, single – phase induction motor gave the following test results:

Blocked – rotor test : 120V, 9.6A, 460W

No – load test : 220V, 4.6A, 125W

The stator winding resistance is  $1.5\Omega$ , and during the blocked – rotor test, the starting winding is open. Determine the equivalent circuit parameters. Also, find the core, friction and windage losses.

**Solution**

Blocked – rotor test

$V_{scr}=120\text{V}$ ,  $I_{scr} = 9.6\text{A}$  ,  $P_{scr}=460\text{W}$

$$Z_e = \frac{V_{scr}}{I_{scr}} = \frac{120}{9.6} = 12.5\Omega$$

$$R_e = \frac{P_{scr}}{I_{scr}^2} = \frac{460}{(9.6)^2} = 4.99\Omega$$

$$X_e = \sqrt{Z_e^2 - R_e^2} = \sqrt{(12.5)^2 - (4.99)^2} = 11.46\Omega$$

$$X_{1m} = \acute{X}_2 = \frac{1}{2} X_e = \frac{1}{2} * 11.46 = 5.73\Omega$$

$$R_{1m} = 1.5\Omega$$

$$R_e = R_{1m} + \acute{R}_2$$

$$\acute{R}_2 = R_e - R_{1m} = 4.99 - 1.5 = 3.49\Omega$$

No – load test:  $V_o=220V$ ,  $I_o = 4.6A$  ,  $P_o=125W$

$$\cos \phi_o = \frac{P_o}{V_o I_o} = \frac{125}{220 * 4.6} = 0.1235$$

$$\therefore \sin \phi_o = 0.9923$$

$$Z_o = \frac{V_o}{I_o} = \frac{220}{4.6} = 47.83\Omega$$

$$\therefore X_o = Z_o \sin \phi_o = 47.83 * 0.9923 = 47.46\Omega$$

Core, friction and windage losses

=power input to motor at no load – no load copper loss

$$\begin{aligned} &= P_o - I_o^2 \left( R_{1m} + \frac{\acute{R}_2}{4} \right) \\ &= 125 - (4.6)^2 \left( 1.5 + \frac{3.49}{4} \right) = 74.8W \end{aligned}$$