# 4.5 Solving Systems Using Inverse Matrices

**Goals**
- Solve systems using inverse matrices.
- Use systems to solve real-life problems.

***Your Notes***

**VOCABULARY**

Matrix of variables The matrix of variables of the linear system $\begin{matrix} ax + by = e \\ cx + dy = f \end{matrix}$ is $\begin{bmatrix} x \\ y \end{bmatrix}$.

Matrix of constants The matrix of constants of the linear system $\begin{matrix} ax + by = e \\ cx + dy = f \end{matrix}$ is $\begin{bmatrix} e \\ f \end{bmatrix}$.

**Example 1** ***Writing a Matrix Equation***

Write the system of linear equations as a matrix equation.

$4x - 3y = 10$ **Equation 1**

$-3x + 2y = 7$ **Equation 2**

**Solution**

$$\overset{A}{\begin{bmatrix} 4 & -3 \\ -3 & 2 \end{bmatrix}} \overset{X}{\begin{bmatrix} x \\ y \end{bmatrix}} = \overset{B}{\begin{bmatrix} 10 \\ 7 \end{bmatrix}}$$

**Example 2** ***Solving a Linear System***

**Use matrices to solve the linear system in Example 1.**

Begin by writing the linear system in matrix form, as in Example 1. Then find the inverse of matrix $A$.

$$A^{-1} = \frac{1}{8 - 9} \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -2 & -3 \\ -3 & -4 \end{bmatrix}$$

Finally, multiply the matrix of constants by $A^{-1}$.

$$X = A^{-1}B = \begin{bmatrix} -2 & -3 \\ -3 & -4 \end{bmatrix} \begin{bmatrix} 10 \\ 7 \end{bmatrix} = \begin{bmatrix} -41 \\ -58 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

The solution of the system is (−41, −58).

*Your Notes*

**✓ *Checkpoint* Complete the following exercise.**

**1.** Write and solve the system of linear equations using matrices.

$$3x - 2y = 1$$
$$-4x + 3y = 1$$

$$\begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}; (5, 7)$$

## Example 3 *Using a Graphing Calculator*

Use a matrix equation and a graphing calculator to solve the linear system.

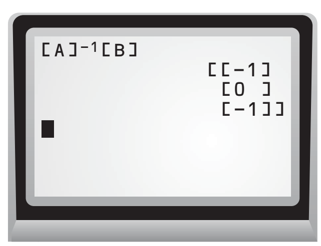
| | |
|---|---|
| $2x - y + z = -3$ | **Equation 1** |
| $-x + 2y - 2z = 3$ | **Equation 2** |
| $-3x + 2y + 2z = 1$ | **Equation 3** |

### Solution

The matrix equation that represents the system is

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -2 \\ -3 & 2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \\ 1 \end{bmatrix}.$$

Enter the coefficient matrix $A$ and the matrix of constants $B$ into a graphing calculator. Then find the solution $X = A^{-1}B$.




The solution is ( $-1$ , $0$ , $-1$ ).

*Your Notes*

**Example 4** ***Writing and Using a Linear System***

You and two friends each spend \$26 buying avocados, loaves of bread, and packages of chicken. You buy 4, 3, and 1 of each, respectively. One friend buys 2, 2, and 2 of each, while the other buys 2, 6, and 1 of each. How much does each item cost?

**Solution**

**Verbal Model**

[Total cost] = [Avocado cost] · [Number of avocados] + [Bread cost] · [Number of loaves] + [Chicken cost] · [Number of packages]

**Labels**

Avocado cost = $a$ — Number of avocados = $x$

Bread cost = $b$ — Number of loaves = $y$

Chicken cost = $c$ — Number of packages = $z$

Total cost = 26

**Algebraic Model** $26 = ax + by + cz$

Write the system of equations using the information about purchasing amounts and total cost.

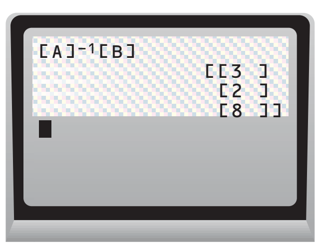
$4a + 3b + c = 26$ **Equation 1**

$2a + 2b + 2c = 26$ **Equation 2**

$2a + 6b + c = 26$ **Equation 3**

In matrix form, the system is $\begin{bmatrix} 4 & 3 & 1 \\ 2 & 2 & 2 \\ 2 & 6 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 26 \\ 26 \\ 26 \end{bmatrix}$.

Enter the coefficient matrix $A$ and the matrix of constants $B$ into a graphing calculator. Then find the solution $X = A^{-1}B$.




Avocados cost \$3 each, loaves of bread are \$2 each, and packages of chicken are \$8 each.

***Homework***