Block Diagrams

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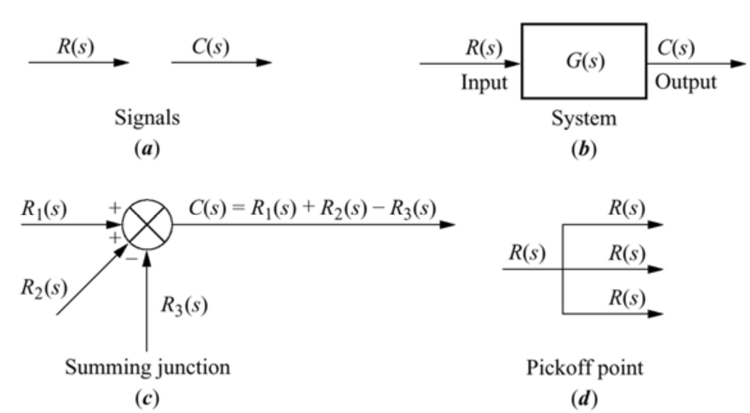
Outline

- What are block diagrams?
- Main rules: cascade, parallel, feedback.
- Interchanging: pickoff, summation.
- Combining/expanding summing junctions.
- Examples.

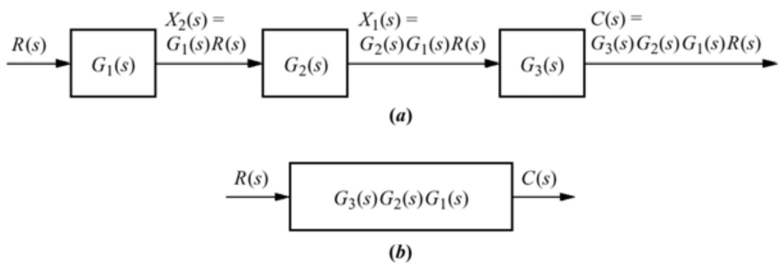
Block Diagrams

- Visual algebra: use block diagram manipulation instead of algebra.
- Block: transfer function of a subsystem.
- Line: Laplace transform of a variable.
- Simplify complex systems to obtain a single equivalent input-output transfer function.

Block Diagram Notation

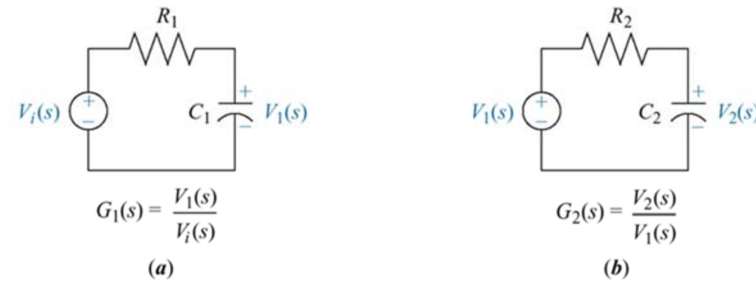


Cascade (Series) Rule



Cascade: No Loading Assumption

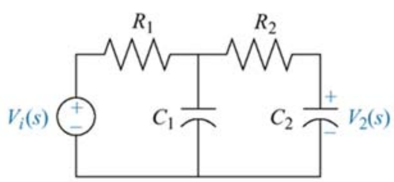
What is the transfer function of the cascade?



$$G_1(s) = \frac{\frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}} = \frac{1}{\tau_1 s + 1}, \tau_1 = R_1 C_1$$

$$G_2(s) = \frac{1}{\tau_2 s + 1}, \tau_2 = R_2 C_2$$

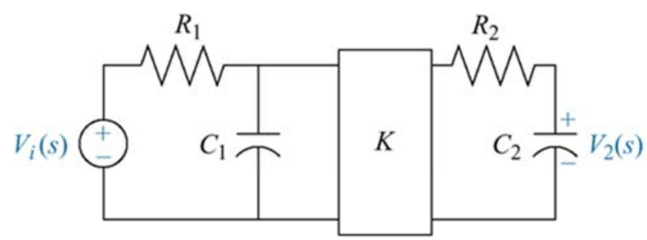
Assumption in Cascading



$$G_1(s)G_2(s) = \frac{1}{\tau_1 \tau_2 s^2 + (\tau_1 + \tau_2)s + 1}$$

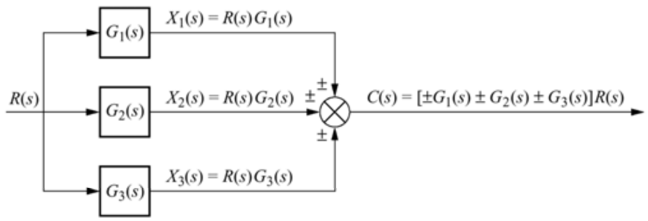
$$G_T(s) = \frac{V_2(s)}{V_i(s)} = \frac{1}{\tau_1 \tau_2 s^2 + (\tau_1 + \tau_2)s + 1 - C_2/C_1} \neq G_1(s)G_2(s)$$

Buffer Amplifier: No loading.

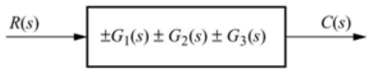


$$G_T(s) = \frac{V_2(s)}{V_i(s)} = K G_2(s) G_1(s)$$

Parallel Rule

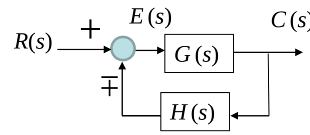


(a)



(b)

Feedback Rule: Proof

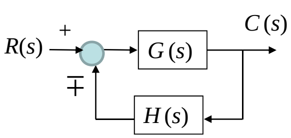


$$E(s) = R(s) \mp H(s)C(s)$$
$$C(s) = G(s)E(s)$$

$$\frac{E(s)}{R(s)} = \frac{1}{1 \pm H(s)G(s)}$$

$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}$$

Feedback Rule



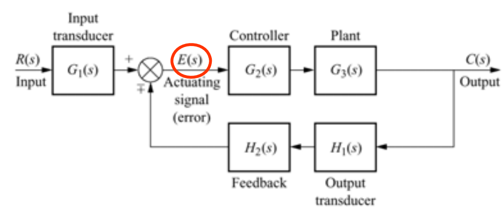
$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}$$

feed-forward gain (transfer function)

open-loop gain (transfer function)

Negative Feedback: plus sign

Feedback Control System



Closed- loop system

$$E(s) = G_1(s)[R(s) \mp H_2(s)H_1(s)C(s)]$$
$$C(s) = G_3(s)G_2(s)E(s)$$

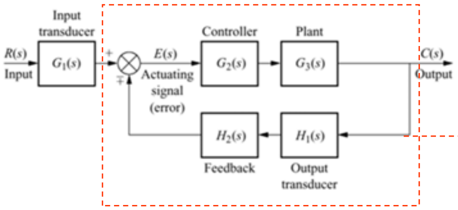
$$\frac{E(s)}{R(s)} = \frac{G_1(s)}{1 \pm H_2(s)H_1(s)G_2(s)G_3(s)}$$

feed-forward gain

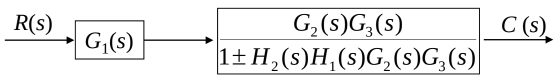
$$\frac{C(s)}{R(s)} = \frac{G_3(s)G_2(s)G_1(s)}{1 \pm H_2(s)H_1(s)G_3(s)G_2(s)}$$

open-loop gain

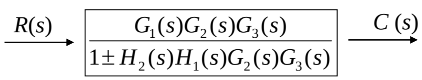
Simplify Block Diagram



$$\frac{G_2(s)G_3(s)}{1 \pm H_2(s)H_1(s)G_2(s)G_3(s)}$$

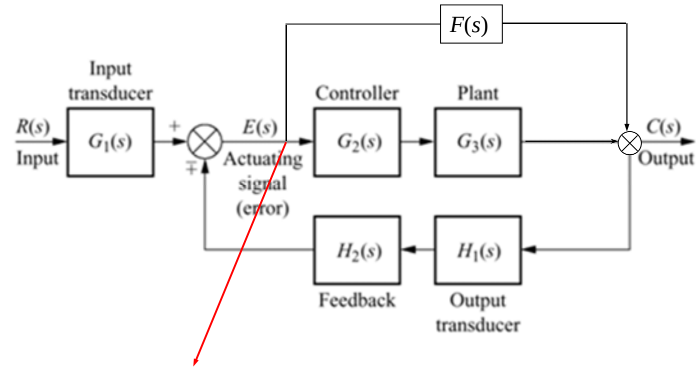


$$R(s) \rightarrow G_1(s) \rightarrow \frac{G_2(s)G_3(s)}{1 \pm H_2(s)H_1(s)G_2(s)G_3(s)} \rightarrow C(s)$$



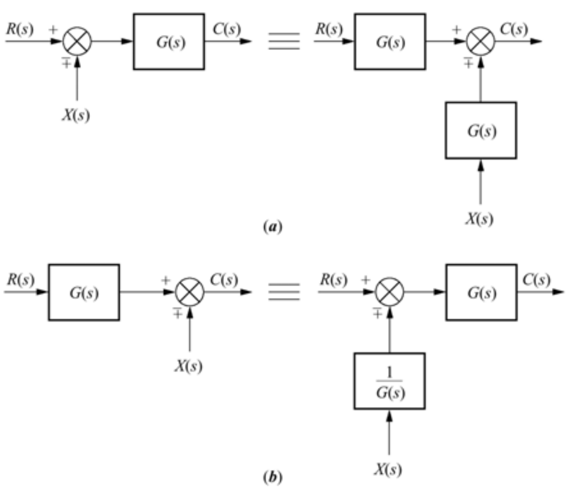
$$R(s) \rightarrow \frac{G_1(s)G_2(s)G_3(s)}{1 \pm H_2(s)H_1(s)G_2(s)G_3(s)} \rightarrow C(s)$$

Effect of Pickoff

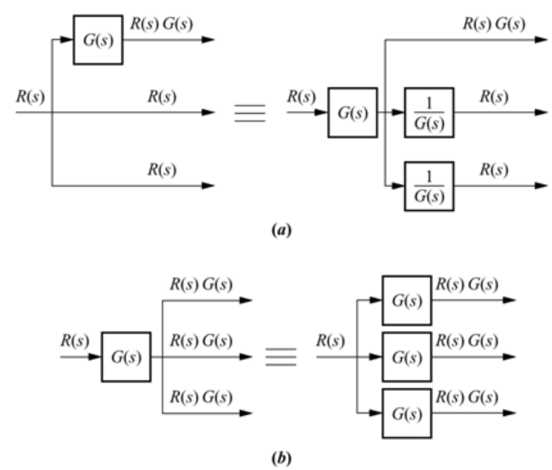


CANNOT use the feedback formula since $E(s)$ is needed.

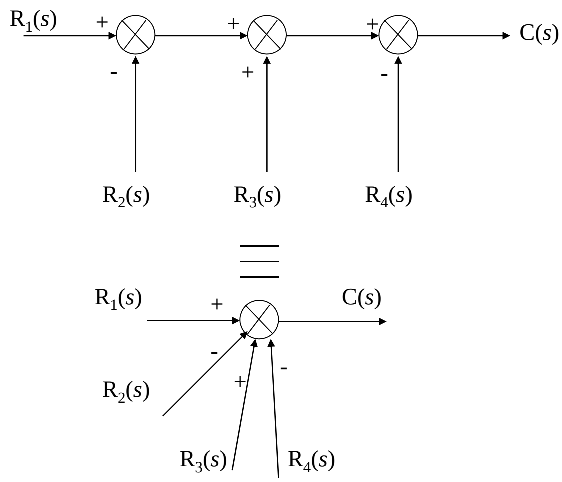
Interchange order of Blocks and Summing Junction



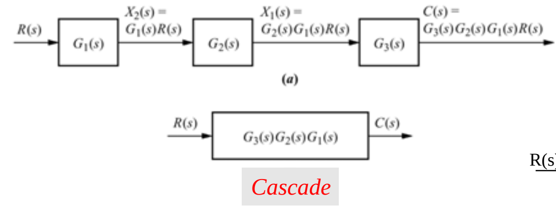
Interchange Order of Blocks and Branching

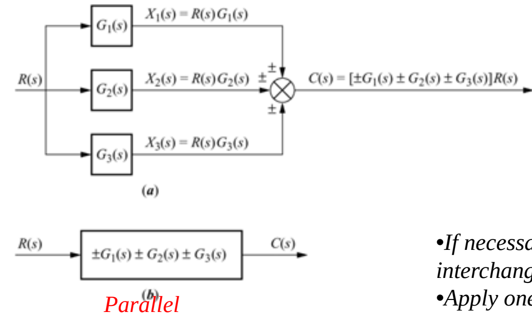


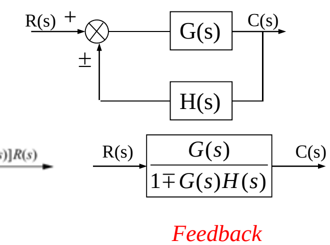
Combining/Expanding Summing Junctions



Problem Solution: 3 Basic Rules

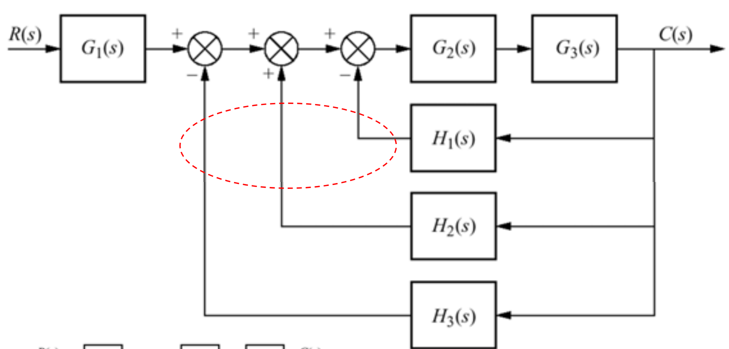


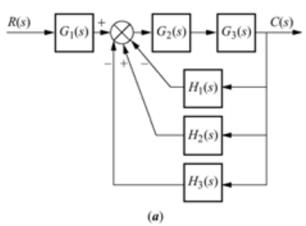




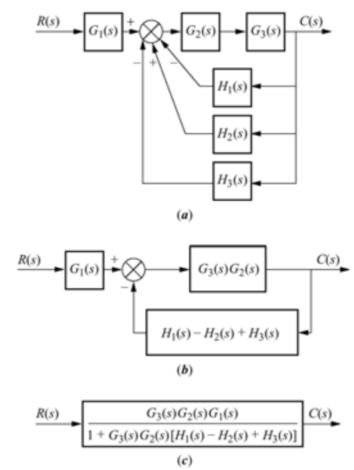
- *If necessary, interchange.*
- *Apply one of the 3 rules.*

Example 5.1

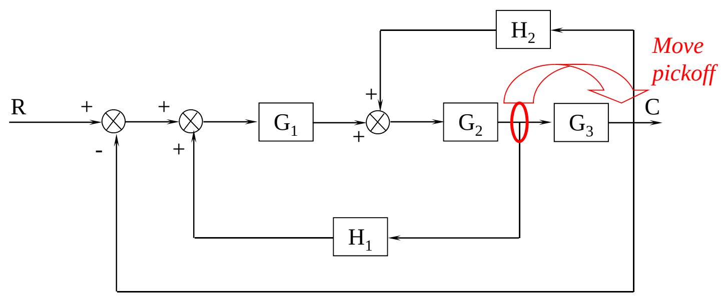




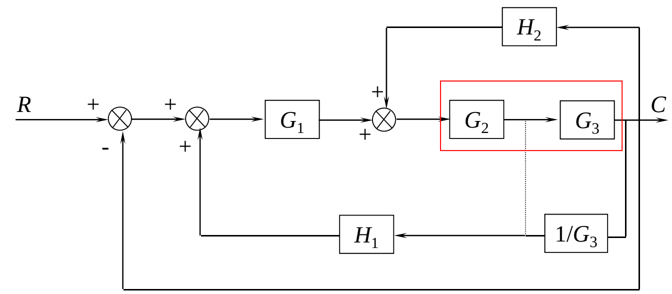
Simplify



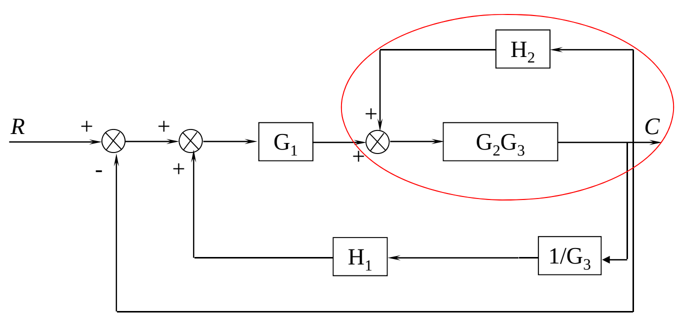
Example



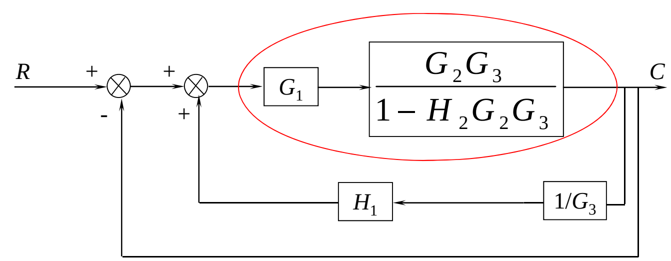
Move Pickoff Point



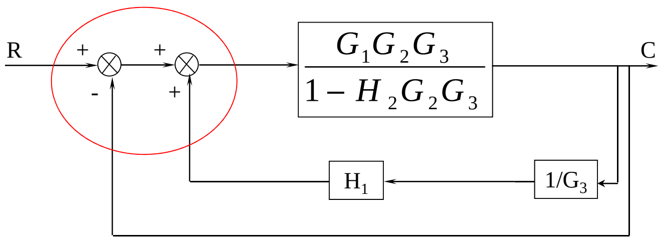
Cascade Rule



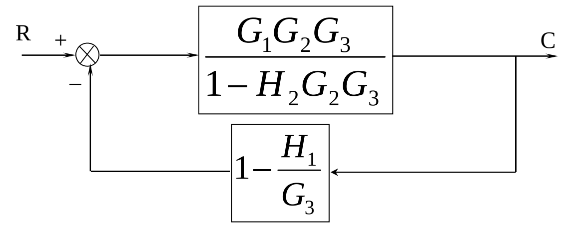
Feedback Rule



Cascade Rule



Parallel Rule

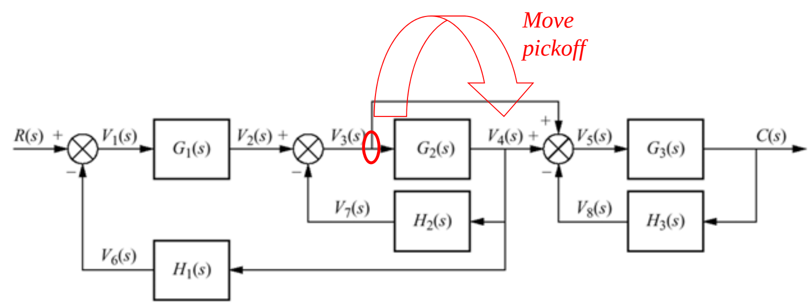


Feedback Rule

$$T(s)=\frac{C(s)}{R(s)}$$

$$=\frac{\dfrac{G_1G_2G_3}{1-H_2G_2G_3}}{1+\left(\dfrac{G_1G_2G_3}{1-H_2G_2G_3}\right)\left(1-\dfrac{H_1}{G_3}\right)}$$

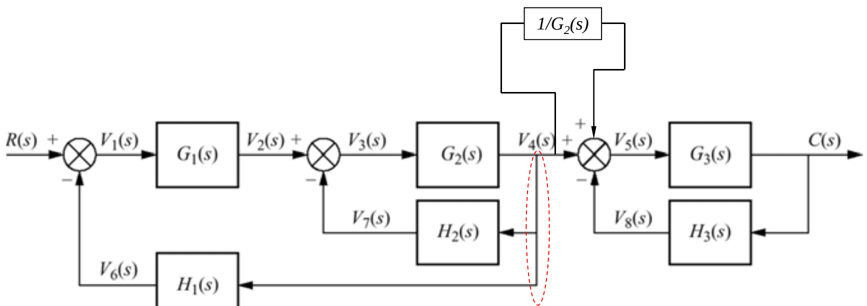
Example



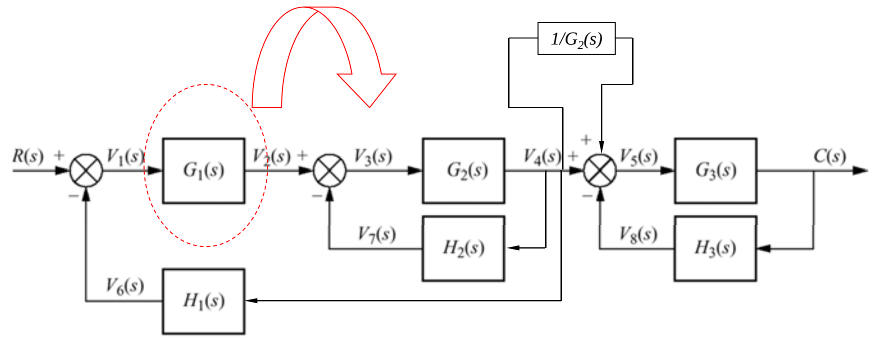
$$V_4(s)=G_2(s)V_3(s)$$

$$V_3(s)=\frac{1}{G_2(s)}V_4(s)$$

Two Feedback Loops



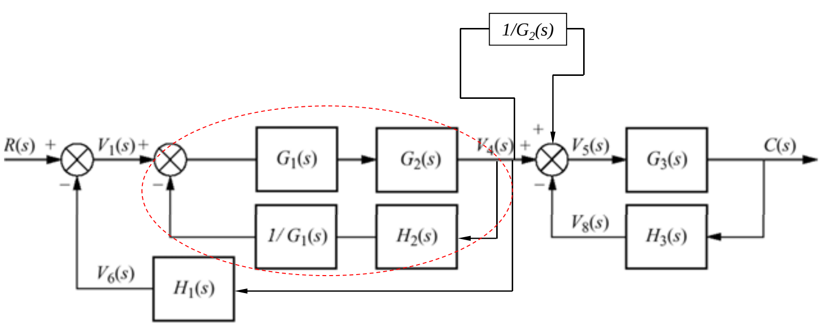
Book: Move Block



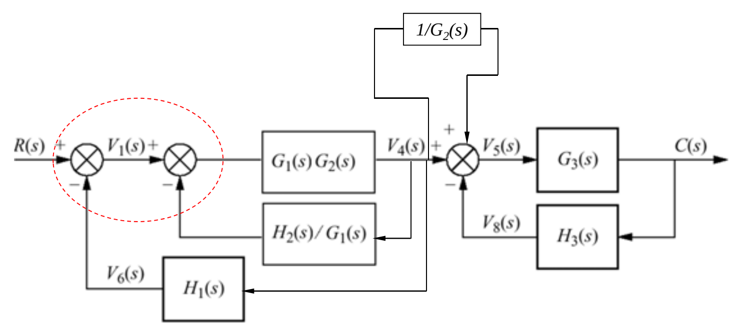
Easier: Use feedback rule

$$\frac{V_4(s)}{V_2(s)} = \frac{G_2(s)}{1 + G_2(s)H_2(s)}$$

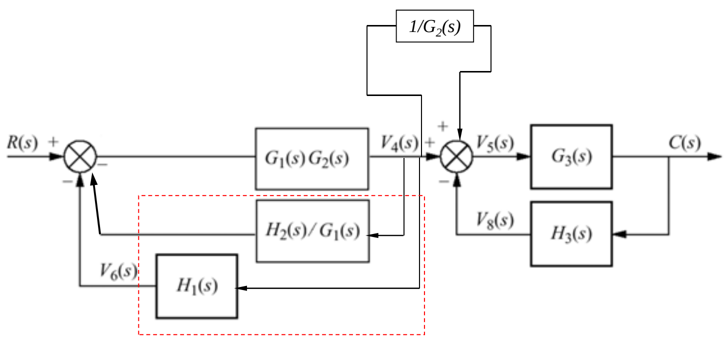
Feedback Rule



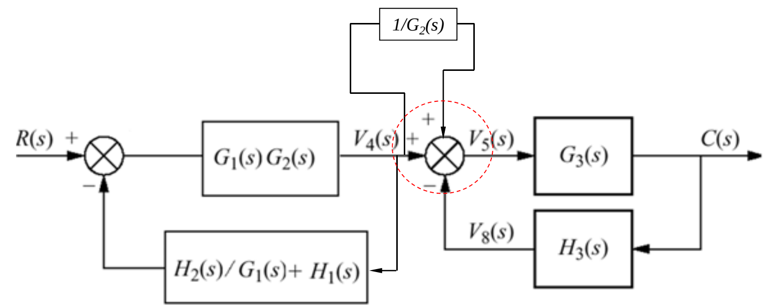
Combine Summing Junctions



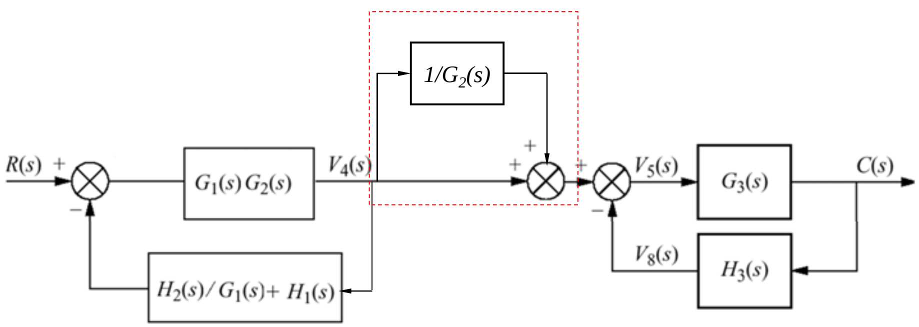
Parallel Rule



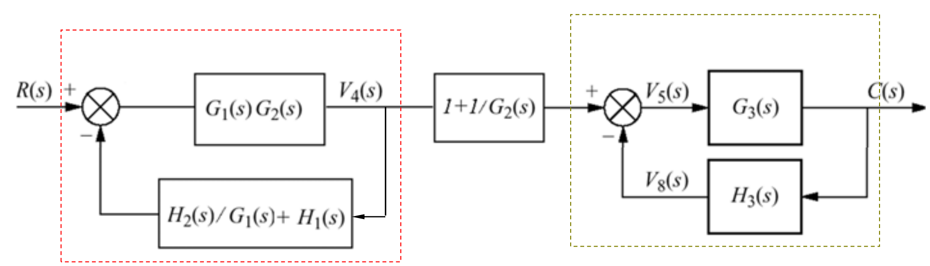
Can we use parallel rule?



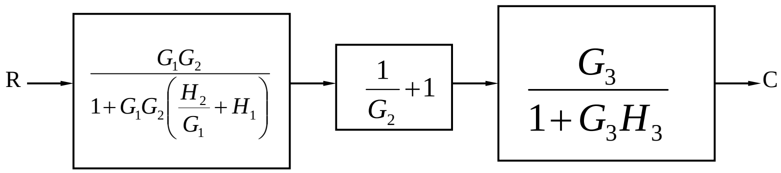
Expand Summation



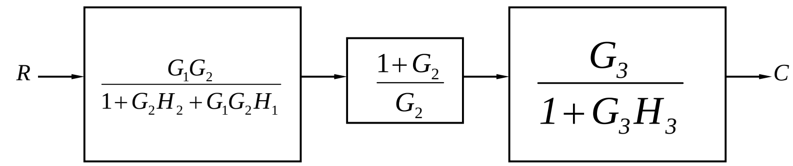
Parallel Rule



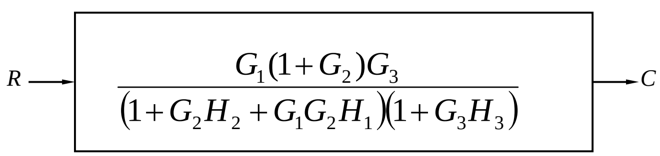
Feedback Rule



Simplify



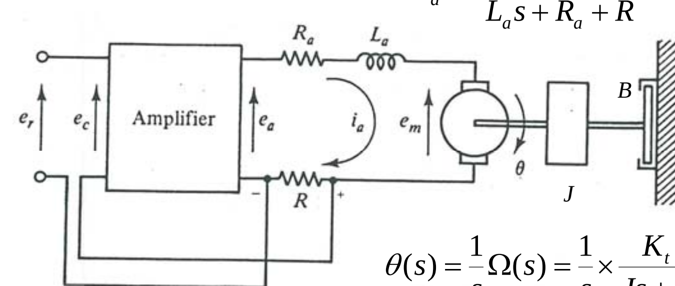
Cascade Rule



Motor with Feedback

$$E_m(s) = K_e s\theta(s)$$

$$I_a = \frac{E_a(s)-E_m(s)}{L_a s + R_a + R}$$

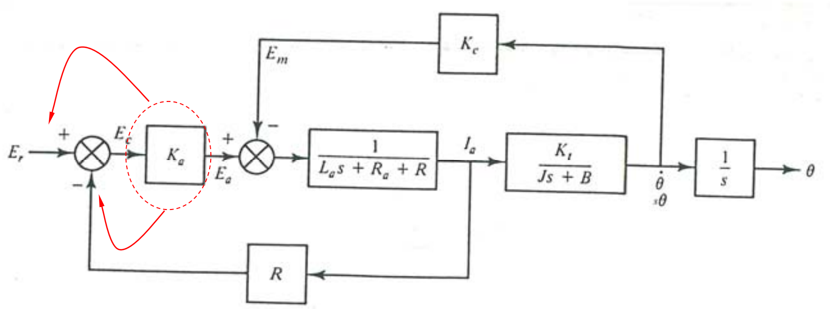


$$\theta(s) = \frac{1}{s}\Omega(s) = \frac{1}{s}\times\frac{K_t}{Js+B}I_a$$

$$E_a(s) = K_a E_c(s)$$

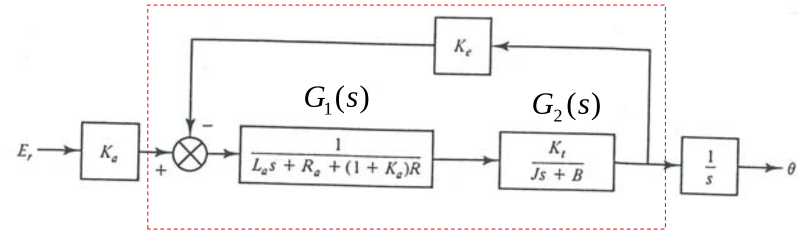
$$E_c(s) = E_r(s) - RI_a(s)$$

Block Diagram



$$G_1(s) = \frac{1/(L_a s + R_a + R)}{1 + K_a R/(L_a s + R_a + R)} = \frac{1}{L_a s + R_a + R + K_a R}$$

Feedback Loop



$$\frac{\theta(s)}{E_r(s)} = \frac{K_a G_1(s) G_2(s)}{1 + K_e G_1(s) G_2(s)} \frac{1}{s}$$