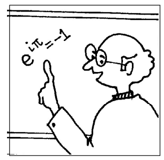
Classroom



In this section of *Resonance,* we invite readers to pose questions likely to be raised in a classroom situation. We may suggest strategies for dealing with them, or invite responses, or both. “Classroom” is equally a forum for raising broader issues and sharing personal experiences and viewpoints on matters related to teaching and learning science.

A Direct Experimental Proof of Displacement Current

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An experimental effort has been made to validate the existence of displacement current which produces magnetic field like any other form of current. The generation of emf by a loop placed parallel to the current and its square-law variation with frequency has been experimentally demonstrated. All experimental results also confirm the theory developed.

1. Introduction

The development of electromagnetic theory by James Clerk Maxwell in 1865 [1] was truly a fundamental stepping stone which laid the solid foundation for a clear understanding of *all* electromagnetic phenomena like interference, diffraction, polarization, guided waves, propagation and radiation of energy, and velocity of light. It also contained the full information of all previous existing laws of electricity and magnetism by Coulomb, Gauss, Ampere and several others [2].

Keywords
Electromagnetic theory, Maxwell's equations, displacement current.

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The Maxwell's equations in differential form are

$$\nabla \times \mathbf{H} = \sigma \mathbf{E} + \frac{\partial \mathbf{D}}{\partial t} \quad (1a)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (1b)$$

$$\nabla \cdot \mathbf{D} = \rho \quad (1c)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (1d)$$

where the symbols have their usual significance. The key and original concept of Maxwell was the introduction of the term of displacement current density, $\partial \mathbf{D}/\partial t$ (in equation (1a) above) which flows in a dielectric medium and acts, for all intents and purposes, exactly like the conduction current density, $\sigma \mathbf{E}$ flowing in a conducting medium [3]. Without this displacement current many electromagnetic phenomena like radiation and propagation of energy, guided waves, and antennas, cannot be explained at all ([4]–[8]).

The objective of the present article is to describe a *simple* experiment which will directly and convincingly show that the displacement current is as real as the conduction current.

2. Theory

The basic principle is easily understood by considering a parallel plate air dielectric capacitor as shown in *Figures* 1a and 1b. When an ac voltage $V_1 e^{j\omega t}$ is applied across the plates, there will be no conduction current in the air dielectric between the plates; however, since the electric field, $\mathbf{E} = (\mathbf{u}_z V_1/d)e^{j\omega t}$ and the displacement current vector, $\mathbf{D} = \mathbf{u}_z(\varepsilon_0 V_1/d)e^{j\omega t}$ vary sinusoidally with time, a displacement current of density,

$$\mathbf{J}_d = \frac{\partial \mathbf{D}}{\partial t} = \mathbf{u}_z \left(\frac{j2\pi f \varepsilon_0 V_1}{d}\right) e^{j\omega t}$$

is produced in the air dielectric. This, in turn, will produce a circular magnetic field, H_φ around the axis. If

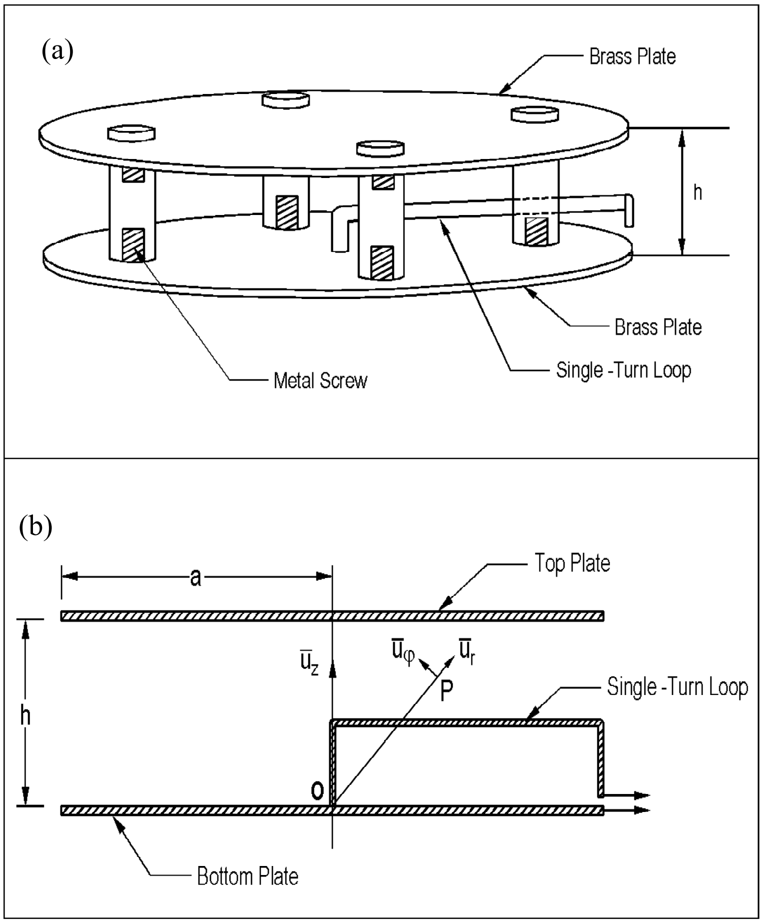

Figure 1. (a) Isometric view of the parallel plate air capacitor. (b) Schematic diagram of the capacitor showing the single-turn loop.

now a single-turn loop is held perpendicular to the magnetic field, a voltage

$$V_2 = -\frac{\partial \Psi}{\partial t} \propto V_1 f^2$$

will be induced. That is, $V_2/(V_1 f^2)$ = constant, depending only on the geometry of the experimental model (shown in *Figure* 2).

The exact solution of *Figure* 1 as a boundary value problem is a formidable task because of (i) fringing of fields near the edge of the capacitor plates, (ii) distortion of fields due to the thick conductor (dia, 2mm) forming the one-turn loop, and (iii) the propagation and radiation

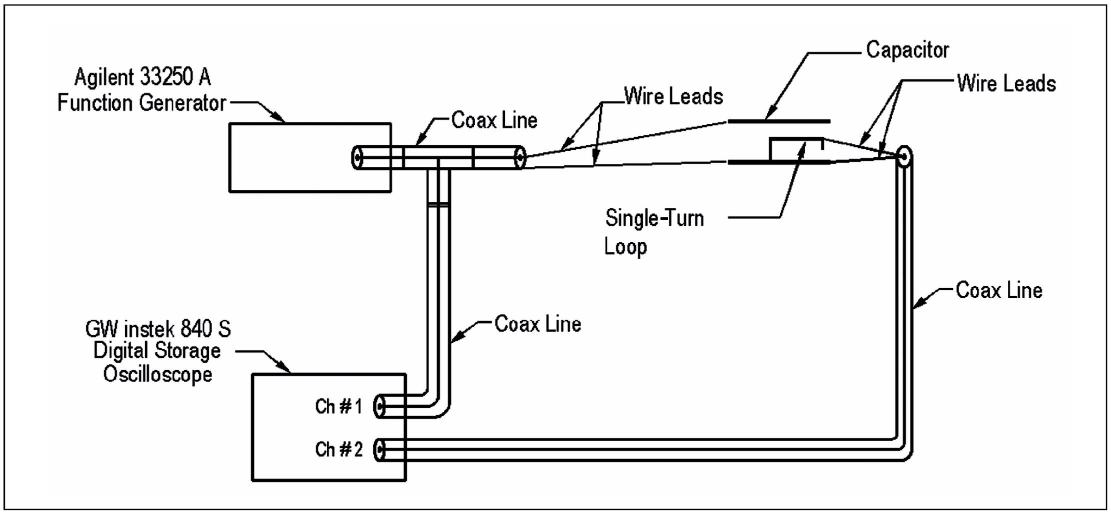

Figure 2. Schematic experimental setup for measurement of V_1 and V_2 described in the text.

effects even at the comparatively lower frequencies employed in the experiment. However, since the frequency range used in the experiment is not very high (< 25 MHz), considerable insight is obtained by employing a quasi-static approach for developing a simplified and idealistic but an appropriate theoretical analysis as given below.

Referring to *Figure* 1b, for an applied voltage V across the capacitor plates,

$$V = V_1 e^{j\omega t},$$

the electric field $\mathbf{E}_z$ and the displacement $\mathbf{D}_z$ in the air dielectric are

$$\mathbf{E}_z = \frac{V}{d} = \left(\frac{V_1}{d}\right) e^{j\omega t}$$

$$\mathbf{D}_z = \varepsilon_0 \mathbf{E}_z = \left(\frac{\varepsilon_0 V_1}{d}\right) e^{j\omega t}$$

Thus, the displacement current density is

$$\begin{aligned} \mathbf{J}_d &= \partial \mathbf{D}/\partial t \\ &= \mathbf{u}_z \left(\frac{j\omega\varepsilon_0 V_1}{d}\right) e^{j\omega t}. \end{aligned} \qquad (2)$$

Since the current is along the z-direction, the magnetic field $\mathbf{H}$ at a point $\mathrm{P}(r, \phi, z)$ will be circular along the ϕ-

direction and may be obtained from Ampere's law as

$$\begin{aligned} \oint \mathbf{H} \cdot d\mathbf{l} &= I \\ \text{or } 2\pi r H_\varphi &= J_d \pi r^2 \\ \text{or } H_\varphi &= \frac{J_d\, r}{2}, \quad r \leq a. \end{aligned}$$

The magnetic flux Ψ linked with the single-turn radial loop of height h and length a is, from (2)

$$\begin{aligned} \Psi &= \int_0^a \mu_0 H_\varphi h \mathrm{d}r \\ &= \left(\frac{j\omega \mu_0 \varepsilon_0 a^2 h V_1}{4d} \right) e^{j\omega t} \end{aligned} \tag{3}$$

The emf V_2 induced in the single-turn loop is, from (3)

$$\begin{aligned} V_2 &= \frac{-\partial \Psi}{\partial t} \\ &= \left(\frac{\pi^2 h a^2}{c^2 d} \right) V_1 f^2, \end{aligned} \tag{4}$$

where $c = 1/\sqrt{\mu_0 \varepsilon_0}$ is the velocity of light.

Equation (4) shows that

$$\frac{V_2}{V_1 f^2} = \frac{\pi^2 h a^2}{c^2 d} = \text{constant}, \tag{5}$$

dependent solely on the geometry of the capacitor and the loop.

For the capacitor made for the present experiment, $h = d/2$, $a = 0.102$ m, and hence

$$\frac{V_2}{V_1 f^2} = \left(\frac{\pi^2 a^2}{2c^2} \right) = k = 55 \times 10^{-20}$$

from (4). If V_2 is in millivolt, V_1 in volt, f in MHz, then

$$\frac{(V_2)_{m\mathrm{V}} \times 10^{-3}}{(V_1)_{\mathrm{V}} \times (f_{\mathrm{MHz}})^2 \times 10^{12}} = k$$

$$\text{i.e.,} \quad \frac{(V_2)_{m\mathrm{V}}}{(V_1)_{\mathrm{V}} \times (f_{\mathrm{MHz}})^2} = k \times 10^{15} = 55 \times 10^{-5}$$

under idealized conditions.

3. Experiment

The experimental arrangement is shown schematically in *Figure* 2. As the frequency f of the signal source (Agilent 33250A, 80 MHz Function/Arbitrary Waveform Generator) is varied, the input voltage V_1 and the output voltage V_2 at each frequency are measured by the voltage recorder's two channels (GW instek 840 s, Digital Storage Oscilloscope). In spite of precautions taken so that the voltage level V_2 is higher than the noise level it was not always possible by the measuring instrument at some frequencies. The experimental results are given in *Table* 1.

If now a graphical plot is made of the ratio $V_2/\left(V_1 f^2\right)$ vs frequency, the ratio should remain constant, as required by equation (5). The experimental results (shown in *Figure* 3a) confirm the theory.

Similarly, from (5) it is also seen that $\log(V_2/V_1) = 2\log f$+constant. A graphical plot of the above on a log–log graph paper should have a slope of 2. The experimental results (*Figure* 3b) show the slope to be 1.87 (error = 6.5%). Further, equation (5) gives that V_2/V_1 = constant $\times f^2$. A graph of this should be a straight line passing through the origin as confirmed from the experimental results (*Figure* 3c).

4. Conclusions

As mentioned earlier in section 2, the value of the geometrical constant $\pi^2 a^2/2c^2$ in (5) has been derived assuming idealized conditions. In the case of the actual experimental model (*Figure* 1b), however, the induced voltage V_2 (as may be obtained by the solution of the boundary value problem pertaining to the model) will

f MHz	f^2 $(\text{MHz})^2$	V_1 Volts P-P	V_2 Millivolts P-P	$\frac{V_2}{V_1} \times 10^2 = B$	$\frac{V_2}{V_1 f^2} \times 10^3 = A$
2.0	4	20.8	7.00	33.65	84.13
3.0	9	21.0	12.80	60.95	67.72
4.0	16	20.8	20.00	96.15	60.09
5.0	25	20.6	30.40	147.57	59.03
6.0	36	20.4	44.00	215.69	59.91
7.0	49	20.0	64.00	320.00	65.31
8.0	64	19.2	72.00	375.00	58.59
9.0	81	18.8	96.00	510.64	63.04
10.0	100	17.4	116.00	666.67	66.67
11.0	121	16.8	112.00	666.67	55.10
12.0	144	15.6	108.00	692.31	48.08
13.0	169	15.0	120.00	800.00	47.34
14.0	196	14.0	144.00	1028.57	52.48
15.0	225	13.4	154.00	1149.25	51.08
16.0	256	12.8	180.00	1406.25	54.93
17.0	289	12.0	204.00	1700.00	58.82
18.0	324	11.8	212.00	1796.61	55.45
19.0	361	11.0	196.00	1781.81	49.36
20.0	400	10.6	216.00	2037.74	50.94
21.0	441	10.2	240.00	2352.94	53.35
22.0	484	9.8	240.00	2448.98	50.60
23.0	529	9.4	280.00	2978.72	56.31
24.0	576	8.8	232.00	2636.36	45.77

Table 1. Experimental results of measurement; see Figures 3a to 3c.

be very much higher because of the following factors:

- fringing of electric field around the circumference of the capacitor's circular plates,

- considerable distortion of the magnetic field due to

(i) small eccentricity of the placement of the single-turn loop between the capacitor plates and

(ii) large diameter $\sim$2 mm of the loop wire resulting in a much larger magnetic flux linkage with the loop and

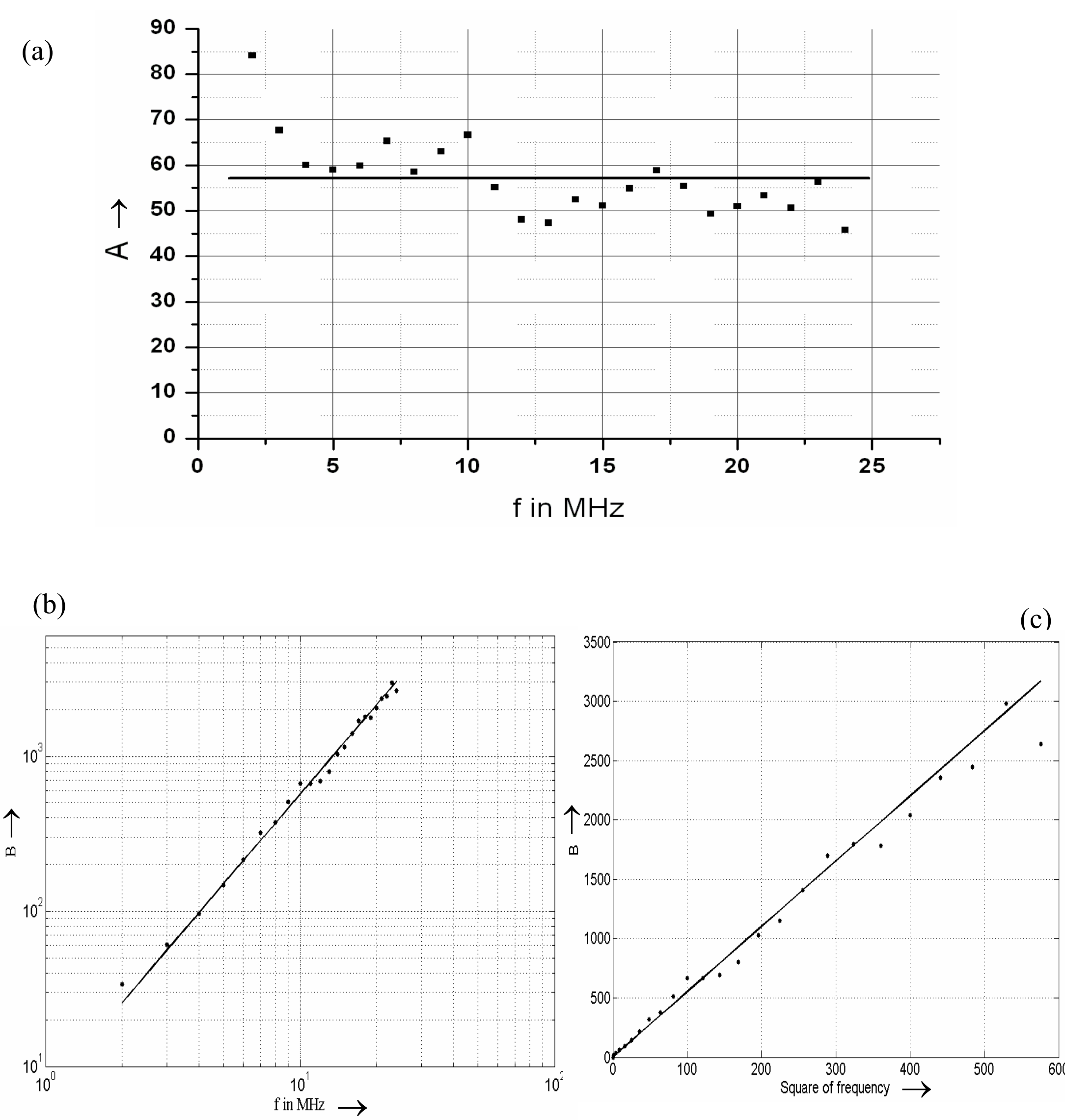

Figure 3. (a) Graphical plot of $(V_2/V_1f^2) \times 10^3 = A$ versus frequency f showing the former to be constant as given in equation (5). (b) A plot of $(V_2/V_1) \times 10^2 = B$ versus f in a log–log graph paper shows the variation to be linear with a slope of 1.87 instead of 2 as given in equation (5). (c) Graphical plot of $(V_2/V_1) \times 10^2 = B$ versus f^2 showing the straight line nature of variation passing through the origin as given in equation (5).

(iii) effect of the metallic screws holding the capacitor plates, and

(iv) induced emf picked up by the leads to measure V_2.

However, since the intrinsic purpose of this experiment is to provide a direct experimental proof of the existence of the displacement current, it is enough to show that $(V_2/V_1 f^2)$ is a constant (not necessarily the one given by equation (5)) with frequency as in *Figure* 3a; also, the other results of *Figures* 3b and 3c agree very closely with the theory as given by equation (5). Thus, all the experimental results confirm the theoretical consideration. It is, therefore, concluded that a direct experimental proof of the displacement current has been firmly established.

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Suggested Reading

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