Chapter 6
GPS Relative Positioning Determination Concepts

6-1. General

Absolute positioning, as discussed earlier, will not provide the accuracies needed for most USACE control projects due to existing and induced errors. In order to eliminate these errors and obtain higher accuracies, GPS can be used in a relative positioning mode. The terms “relative” and “differential” used in this chapter and throughout this manual have similar meaning. “Relative” will be used when discussing one thing in relation to another. The term “differential” will be used when discussing the technique of positioning one thing in relation to another.

6-2. Differential (Relative) Positioning

Differential or relative positioning requires at least two receivers set up at two stations (usually one is known) to collect satellite data simultaneously in order to determine coordinate differences. This method will position the two stations relative to each other (hence the term “relative positioning”) and can provide the accuracies required for basic land surveying and hydrographic surveying.

6-3. Differential Positioning (Code Pseudo-Range Tracking)

Differential positioning using code pseudo-ranges is performed similarly to that described in Chapter 5; however, some of the major uncertainties in Equations 5-1 through 5-6 are effectively eliminated or minimized. This pseudo-range process results in absolute coordinates of the user on the earth’s surface. Errors in range are directly reflected in resultant coordinate errors. Differential positioning is not so concerned with the absolute position of the user but with the relative difference between two user positions, which are simultaneously observing the same satellites. Since errors in the satellite position (X^s, Y^s, and Z^s) and atmospheric delay estimates d are effectively the same (i.e., highly correlated) at both receiving stations, they cancel each other to a large extent.

a. For example, if the true pseudo-range distance from a “known” control point to a satellite is 100 m and the observed or measured pseudo-range distance was 92 m, then the pseudo-range error or correction is 8 m for that particular satellite. A pseudo-range correction or PRC can be generated for each satellite being observed. If a second receiver is observing at least four of the same satellites and is within a reasonable distance (300 km) it can use these PRCs to obtain a relative position to the “known” control point since the errors will be similar. Thus, the relative distance (i.e., coordinate difference) between the two stations is relatively accurate (i.e., within 0.5-5 m) regardless of the poor absolute coordinates. In effect, the GPS observed baseline vectors are no different from azimuth/distance observations. As with a total station, any type of initial coordinate reference can be input to start the survey.

b. The absolute GPS coordinates will not coincide with the user’s local project datum coordinates (Figure 6-1). Since differential survey methods are concerned only with relative coordinate differences, disparities with a global reference system used by the NAVSTAR GPS are not significant for USACE purposes. Therefore, GPS coordinate differences can be applied to any type of local project reference datum (i.e., NAD 27, NAD 83, or any local project grid reference system).

c. Code pseudo-range tracking has primary application to real-time navigation systems where accuracies at the 0.5- to 5-m level are tolerable. Given these tolerances, engineering survey applications of code pseudo-range tracking GPS are limited, with two exceptions being hydrographic survey and dredge positioning. Specifications for real-time hydrographic code tracking systems are contained in EM 1110-2-1003. See Chapter 9 for further discussion on real-time code pseudo-range tracking applications.

6-4. Differential Positioning (Carrier Phase Tracking)

Differential positioning using carrier phase tracking uses a formulation of pseudo-ranges similar to those shown in Equations 5-1 through 5-6. The process becomes somewhat more complex when the carrier signals are tracked such that range changes are measured by phase resolution. In carrier phase tracking, an ambiguity factor is added to Equation 5-1 which must be resolved in order to obtain a derived range (see Figure 5-1). Methods for resolving this ambiguity (the number of unknown integer cycles) are described in Chapter 9. Carrier phase tracking provides for a more accurate range resolution due to the short wavelength (approximately 19 cm for L1 and 24 cm for L2) and the ability of a receiver to resolve the carrier phase down to about 2 mm. This method, therefore, has primary application to engineering, topographic, and geodetic surveying, and may be employed with either static

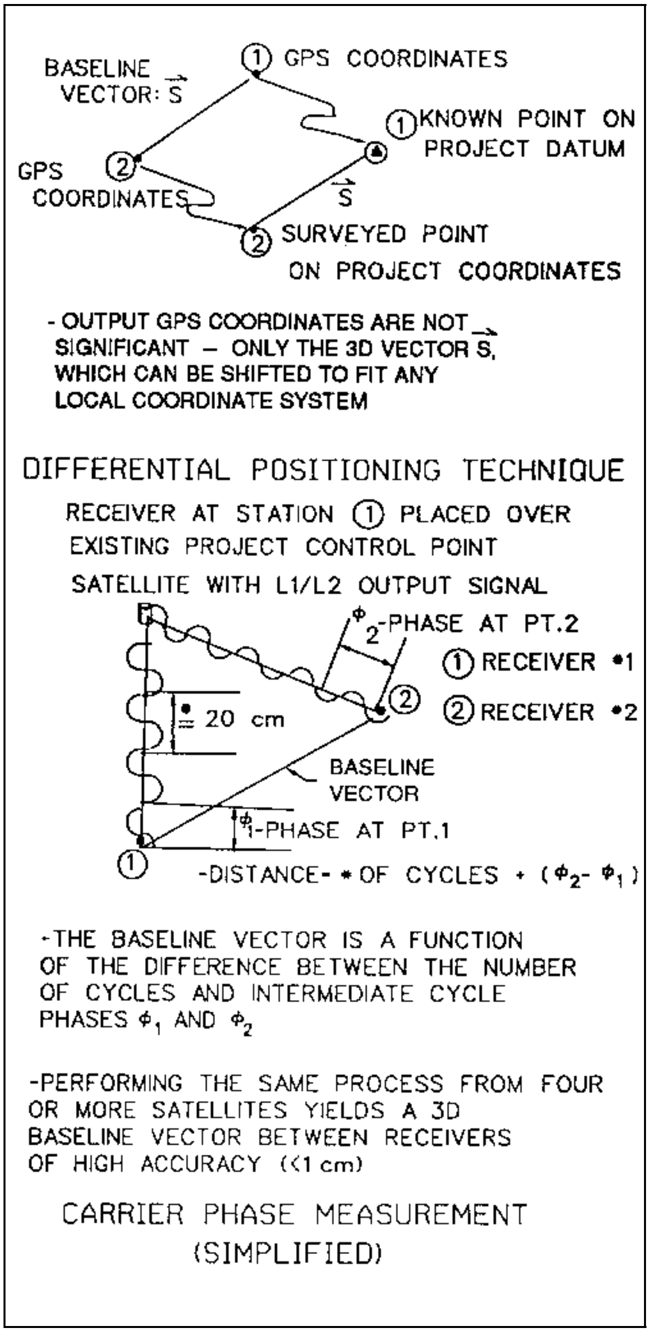

Figure 6-1. Differential positioning

or kinematic methods. There are several techniques which use the carrier phase in order to determine a station's position. These include static, rapid static, kinematic, stop and go kinematic, pseudo kinematic, and real-time kinematic (RTK) and on-the-fly (OTF) kinematic. The concepts of these techniques are explained below, but procedures can be found in Chapter 9. Table 6-1 lists these techniques, their associated accuracies, applications, and required components.

a. Static. Static surveying is the most widely used differential technique for control and geodetic surveying. It involves long observation times (1-2 hr, depending on number of visible satellites) in order to resolve the integer ambiguities between the satellite and the receiver. Accuracies in the subcentimeter range can be obtained from using the static method.

b. Rapid static. The concept of rapid static is to measure baselines and determine positions in the centimeter level with short observation times, 5-20 min. The observation time is dependent on the length of the baseline and number of visible satellites. Loss of lock, when moving from one station to the next, can also occur since each baseline is processed independent of each other.

c. Kinematic. Kinematic surveying, allows the user to rapidly and accurately measure baselines while moving from one point to the next. The data are collected and post-processed to obtain accurate positions to the centimeter level. This technique permits only partial loss of satellite lock during observation and requires a brief period of static initialization. The OTF technology, both real-time and post-processed, could eventually replace standard kinematic procedures at least for short baselines.

d. Stop and go kinematic. Stop and go kinematic involves collecting data for several minutes (1-2 min.) at each station after a period of initialization to gain the integers. This technique does not allow for loss of satellite lock during the survey. If loss of satellite lock does occur, a new period of initialization must take place. This method can be performed with two fixed or known stations in order to provide redundancy and improve accuracy.

e. Pseudo-kinematic. This technique is similar to standard kinematic procedures and static procedures combined. The differences are that there is no static initialization, longer period of time at each point (approximately 1-5 min), each point must be revisited after about an hour, and loss of satellite lock is acceptable. The positional accuracy is more than for kinematic or rapid static procedures, which makes it a less acceptable method for establishing baselines.

f. RTK and OTF carrier phase based positioning determination. The OTF/RTK positioning system uses

Table 6-1
Carrier Phase Tracking Techniques

Concept	Requirements	Applications	Accuracy
Static (Post-processing)	• L1 or L1/L2 GPS receiver • 386/486 computer for post-processing • 45 min to 1 hr minimum observation time[1]	• Control surveys (that require high accuracy)	• Subcentimeter level
Rapid Static (Post-processing)	• L1/L2 GPS receiver • 5-20 min observation time[1]	• Control surveys (that require medium to high accuracy)	• Subcentimeter level
Kinematic[2] (Post-processing)	• L1 GPS receiver with kinematic survey option • 386/486 computer for post-processing	• Continuous topo • Location surveys	• Centimeter level
Stop & Go Kinematic[2] (Post-processing)	• L1 GPS receiver • 386/486 computer for post-processing	• Medium accuracy control surveys	• Centimeter level
Pseudo Kinematic[2] (Post-processing)	• L1 GPS receiver • 386/486 computer for post-processing	• Medium accuracy control surveys	• Centimeter level
Real Time Kinematic/OTF Kinematic[3] (Real-time or post-processing)	For post-processing: • L1/L2 GPS receiver • 386/486 computer For real-time: • Internal or external processor (1- 386, 1- 486 computers w/dual com ports) • Min 4800 baud radio/modem data link set	• Real-time high accuracy hydro surveys • Location surveys • Medium accuracy control surveys • Photo control • Continuous topo	• Subdecimeter level

1. Dependent on satellite constellation and number of satellites in view.
2. Initialization period required and loss of satellite lock is not tolerated.
3. No static initialization necessary, integers gained while moving, and loss of satellite lock is tolerated.

GPS technology to allow the positioning to a subdecimeter in real time. This system determines the integer number of carrier wavelengths from the GPS antenna to the GPS satellite, transmitting them while in motion and without static initialization. The basic concept behind the OTF/RTK system is kinematic surveying without static initialization (integer initialization is performed while moving) and allows for loss of satellite lock. Other GPS techniques that can achieve this kind of accuracy require static initialization while the user is not moving and no loss of satellite lock while in motion.

6-5. Vertical Measurements with GPS

a. Elevation determination. **GPS is not recommended for Third-Order or higher vertical control surveys. It is recommended that it not be used as a substitute for standard differential leveling. It is, however, practical for small-scale topographic mapping or similar projects.**

b. Accuracy of GPS height differences. The height (h) component of GPS measurements is the weakest plane. This is due to the orbital geometry of the X-Y-Z position determination. Thus, GPS ellipsoidal height differences are usually less accurate than the horizontal components. Currently, GPS-derived elevation differences will not meet Third-Order standards as would be obtained using conventional spirit levels. Accordingly, GPS-derived elevations must be used with caution.

c. Topographic mapping with GPS. GPS positioning, whether operated in an absolute or differential positioning mode, can provide heights (or height differences) of surveyed points. The height h or height difference Δh obtained from GPS is in terms of height above or below the WGS 84 ellipsoid. These ellipsoid heights are not the same as orthometric heights, or elevations, which would be obtained from conventional differential/spirit leveling. This distinction between ellipsoid heights and orthometric elevations is critical to many engineering and construction projects; thus, users of GPS must exercise extreme caution in applying GPS height determinations to USACE projects which are based on conventional orthometric elevations.

(1) GPS uses WGS 84 as the optimal mathematical model best describing the shape of the true earth at sea level based on an ellipsoid of revolution. The WGS 84 ellipsoid adheres very well to the shape of the earth in terms of horizontal coordinates but differs somewhat with the established mean sea level definition of orthometric height. The difference between ellipsoidal height, as derived by GPS, and conventional leveled (orthometric) heights is required over an entire project area to adjust GPS heights to orthometric elevations. NGS has developed geoid modeling software (GEOID90, GEOID91, and GEOID93) to be used to convert ellipsoidal heights to approximate orthometric elevations. These values should be used with extreme caution.

(2) Static or kinematic GPS survey techniques can be used effectively on a regional basis for the densification of low-accuracy vertical control for topographic mapping purposes. Existing benchmark data (orthometric heights) and corresponding GPS-derived ellipsoidal values for at least three stations in a small project area can be used in tandem in a minimally constrained adjustment program to reasonably model the geoid. More than three correlated stations are required for larger areas to ensure proper modeling of the geoidal undulations in the area. The model from the benchmark data and corresponding GPS data can then be used to derive the unknown orthometric heights of the remaining stations occupied during the GPS observation period.

(3) Procedures for constraining GPS observations to existing vertical control are detailed in Section 11 of Leick and Lambert (1990). Step-by-step vertical control planning, observation, and adjustment procedures employed by the NGS are described in some of the publications listed in Appendix A (see Zilkoski 1990a, 1990b; Zilkoski and Hothem 1989). These procedures are recommended should a USACE field activity utilize GPS to densify low-order vertical control relative to the orthometric datum.

6-6. Differential Error Sources

The error sources encountered in the position determination using differential GPS positioning techniques are the same as those outlined in Chapter 5. In addition to these error sources, the user must ensure that the receiver maintains lock on at least three satellites for 2D positioning and four satellites for 3D positioning. When loss of lock occurs, a cycle slip (a discontinuity of an integer number of cycles in the measured carrier beat phase as recorded by the receiver) may occur. In GPS absolute surveying, if lock is not maintained, positional results will not be formulated. In GPS static surveying, if lock is not maintained, positional results may be degraded, resulting in incorrect formulations. Sometimes, in GPS static surveying, if the observation period is long enough, post-processing software may be able to average out loss of lock and cycle slips over the duration of the observation period and formulate positional results that are adequate; if this is not the case, reoccupation of the stations may be required. In all differential surveying techniques, if loss of lock does occur on some of the satellites, data processing can continue easily if a minimum of four satellites have been tracked. Generally, the more satellites tracked by the receiver, the more insensitive the receiver is to loss of lock. In general, cycle slips can be repaired.

6-7. Differential GPS Accuracies

There are two levels of accuracies obtainable from GPS using differential techniques. The first level is based on pseudo-range formulations, while the other is based on carrier beat phase formulations.

a. *Pseudo-range accuracies*. Pseudo-range formulations can be developed from either the C/A-code or the more precise P-code. Pseudo-range accuracies are generally accepted to be 1 percent of the period between successive code epochs. Use of the P-code where successive epochs are 0.1 µs apart produces results that are around 1 percent of 0.1 µs or 1 ns. Multiplying this value by the speed of light gives a theoretical resultant range measurement of around 30 cm. If using pseudo-range formulations with the C/A-code, one can expect results 10 times less precise or a range measurement precision of around 3 m. Point positioning accuracy for a differential pseudo-range formulated solution is generally found to be in the range of 0.5-10 m. These accuracies are largely dependent on the type of GPS receiver being used.

b. Carrier beat phase formulations. Carrier beat phase formulations can be based on either the L1 or L2, or both carrier signals. Accuracies achievable using carrier beat phase measurement are generally accepted to be 1 percent of the wavelength. Using the L1 frequency where the wavelength is around 19 cm, one can expect a theoretical resultant range measurement that is 1 percent of 19 cm, or about 2 mm. The L2 carrier can only be used with receivers which employ a cross correlation, squaring, or some other technique to get around the effects of A/S.

(1) The final positional accuracy of a point determined using differential GPS survey techniques is directly related to the geometric strength of the configuration of satellites observed during the survey session. GPS errors resulting from satellite configuration geometry can be expressed in terms of DOP. Positional accuracy for a differential carrier beat phase formulated solution is generally found to be in the range of 1-10 mm.

(2) In addition to GDOP, PDOP, HDOP, and VDOP, the quality of the baselines produced by GPS differential techniques (static or kinematic) through carrier phase recovery can be defined by a quantity called relative DOP (RDOP). Multiplying the uncertainty of a double difference measurement by RDOP yields the relative position error for that solution. Values of RDOP are measured in meters of error in relative position per error of one cycle in the phase measurement (m/cycle). Knowledge of an RDOP or a value equivalent to it is extremely important to the confidence one assigns to a baseline recovery. Key to understanding RDOP is to remember that it represents position recovery over a whole session of time and is not representative of a position recovery at an instant in time. When carrier phase recovery techniques are used, RDOP values around 0.1 m/cycle are considered acceptable.