RLC Circuits

Centre College

Physics 230 Lab 8

1 Preliminaries

- Objective
  To study the electrical characteristics of an alternating current circuit containing a resistor, inductor, and capacitor.

- Equipment
  Oscilloscope, function generator, probes, leads, wire clips, resistor, capacitor, inductor, RLC meter.

2 Theory

To study the effect of a sinusoidal source on a series combination of a resistor, capacitor, and an inductor, it is convenient to apply a current source, as shown in Figure 1.

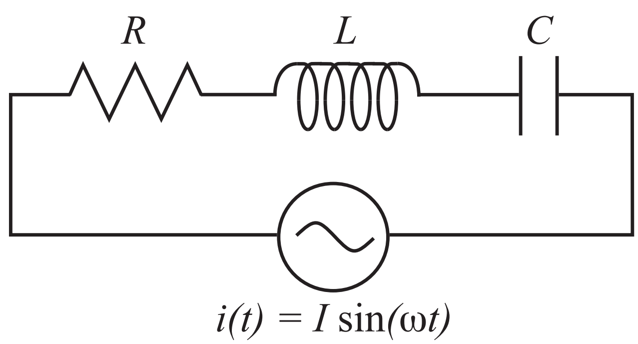


Figure 1: A series RLC circuit with a current source.

In class you learned when a sinusoidal voltage (of frequency f) is applied to a capacitor or an inductor, the current leads or lags the applied voltage. It is reasonable, therefore, to assume that the voltage, $v(t)$, across the RLC circuit of Figure 1 is out of phase with the current, $i(t)$:

$$v(t) = E_m \sin(\omega t + \phi),$$

where E_m is the maximum EMF across the RLC circuit and ϕ is the phase angle between $v(t)$ and $i(t)$, and $\omega = 2\pi f$.

Our objective is to find E_m and ϕ in terms of I, R, L, C, and f. The equation for E_m will be Ohm's law for the RLC circuit, and ϕ will provide us with information about how the voltage, $v(t)$ is related to $i(t)$.

Phasor Solution

The phasor description provides a convenient method for finding E_m and ϕ. We will begin by noting that the current elements are in series so at any instant the current, $i(t) = I\sin(\omega t)$, must be the same at all points in the circuit. The phasor diagrams for R, L, and C are shown in Figure 2. Since the current, I, is the same in each circuit element, it is placed along the x-axis in each graph. You should understand that this is true only at a particular instant, since the phasors rotate counterclockwise with angular velocity $\omega = 2\pi f$. However, the I phasor of each graph will always point in the same direction since $i(t)$ must be the same for each series element.

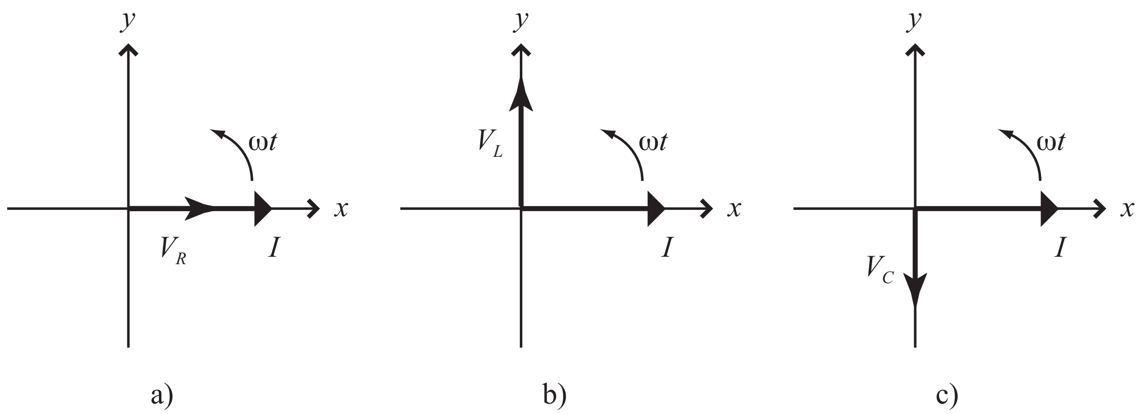


Figure 2: Phasor diagrams for the current and voltage across the a) resistor b) inductor, and c) capacitor.

When these graphs are combined into one, we may add these voltages vectorally. Figure 3 shows how the magnitude of this resultant voltage is computed. The phase angle is also readily determined from this phasor diagram.

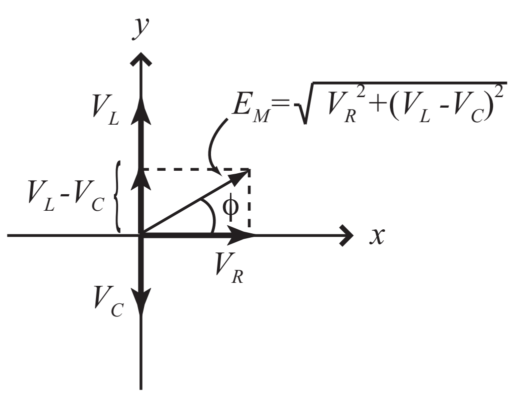


Figure 3: Resultant voltage and phase angle.

The maximum EMF is the length of the phasor constructed from the resultant voltage. From Figure 3:

$$E_m = \sqrt{V_R^2 + (V_L - V_C)^2}.$$

But,

$$V_R = IR, \quad V_L = IX_L, \quad V_C = IX_C,$$

so

$$E_m = I\sqrt{R^2 + (X_L - X_C)^2}.$$

Or,

$$E_m = IZ, \tag{1}$$

where

$$Z = \sqrt{R^2 + (X_L - X_C)^2}.$$

Equation 1 is Ohm's Law for an AC circuit. The term Z corresponds to the resistance of a DC circuit and is called the *impedance*. The units of impedance are ohms.

The phase angle may also be found from Figure 3:

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{IX_L - IX_C}{IR},$$

$$\phi = \arctan\left(\frac{X_L - X_C}{R}\right). \tag{2}$$

These equations reveal that we need only consider the reactances (X_L and X_C) and the resistance (R) to determine ϕ. If the correct phase relation is maintained from the phasor diagram, a graph of X_L, X_C, and R is sufficient to find ϕ and Z. This is illustrated in Figure 4.

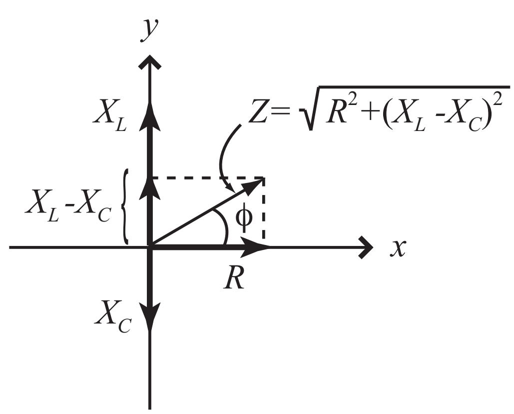


Figure 4: Impedance and phase angle determined from resistance and reactances.

Recall that the phase angle, ϕ, tells us by how much the voltage leads the current:

$$i(t) = I\sin(\omega t),$$

$$v(t) = E_m\sin(\omega t + \phi) = IZ\sin(\omega t + \phi).$$

If we relate the sign of ϕ to Figure 4,

$$\begin{array}{llll} \text{when} & X_L > X_C, & \phi > 0, & \text{and } v(t) \text{ leads } i(t), \\ & X_L > X_C, & \phi > 0, & \text{and } v(t) \text{ lags } i(t), \\ & X_L > X_C, & \phi > 0, & \text{and } v(t) \text{ and } i(t) \text{ are in phase.} \end{array}$$

When the last condition above is satisfied ($X_L = X_C$) the circuit is said to be in *resonance*. At anytime, the average power dissipated in the circuit is

$$\begin{aligned} P_{AV} &= \frac{I}{\sqrt{2}}\frac{V}{\sqrt{2}}\cos\phi, \\ &= I_{RMS}V_{RMS}\cos\phi, \end{aligned}$$

where

$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}.$$

At resonance, $X_L = X_C$ and $\cos(\phi) = 1$. Further, because $\omega = 2\pi f$, this condition gives

$$f_{res} = \frac{1}{2\pi\sqrt{LC}}. \tag{3}$$

At resonance, maximum power is dissipated in the circuit.

3 Procedure

1. Connect the resistor, inductor, and capacitor in series across the signal generator output. Connect the positive output of the signal generator to the resistor and the negative output (ground) to the inductor, as shown in the figure below.

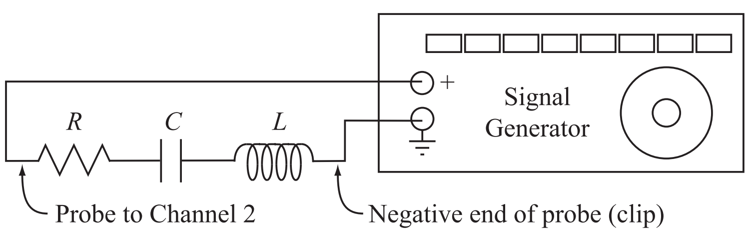


Figure 5: Finding the voltage across the entire circuit.

Use the Channel 2 probe from the oscilloscope to measure the voltage across all elements of the circuit. To do this, connect the probe lead to the left side of the resistor as shown in 5. The negative end of the probe (the clip) will always be connected to the ground side of the circuit (this is not shown in subsequent figures).

Set the frequency of the sine wave to 25.0 kHz. Use the signal on Channel 2 of the oscilloscope to be certain of this frequency (measure the period or record the frequency from the lower right side of the oscilloscope display). Adjust the signal generator to an amplitude of 4.0 Volts. (Remember, to obtain a stationary signal on the scope, you will have to trigger on Channel 2.)

Adjust the voltage range and sweep rate of the scope so that much of the screen is used by approximately two cycles of the signal. Carefully draw what you observe on the grid below, and record the scope settings (don't forget units).

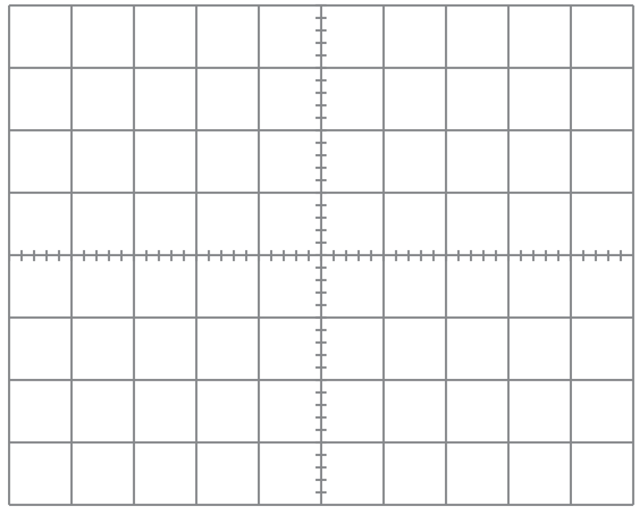
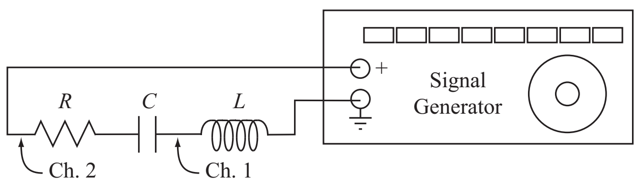


Figure 6: Voltage across the entire circuit, $v(t)$.

Volts/Div: __________

Sec/Div: __________

2. Now connect a probe to Channel 1 and measure the voltage across the inductor.



You may need to adjust the vertical scale to obtain the peak-to-peak voltage across L. Draw the signals of both the voltage across the entire circuit (Ch. 2) and the voltage across the inductor (Ch. 1) on the grid below. Measure the peak-to-peak voltage across the inductor carefully using the cursors and use it to find the voltage amplitude.

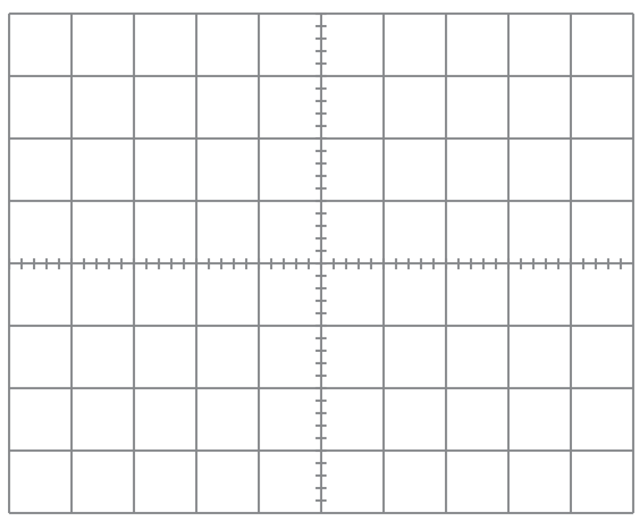


Figure 7: Voltage across the entire circuit, and across the inductor.

Volts/Div (Ch. 1): __________

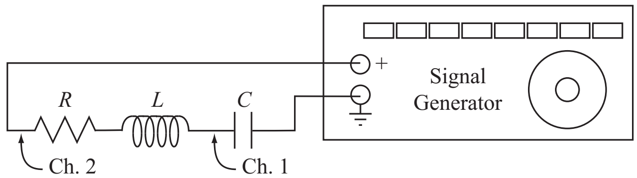
P-P Voltage (Ch. 1): __________

Voltage Amp (Ch. 1): __________

Volts/Div (Ch. 2): __________

Sec/Div: __________

3. To record the voltage across the capacitor, C, you will have to rearrange the circuit so one end of the capacitor is connected to the negative side of the signal generator. (This is because the oscilloscope probe must be placed across the circuit element you are measuring with ground on one side of the element.)



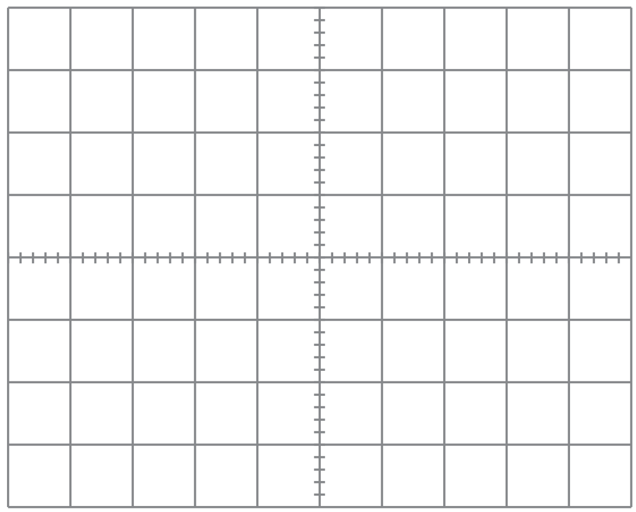


Figure 8: Voltage across the entire circuit, and across the capacitor.

Volts/Div (Ch. 1): ____________

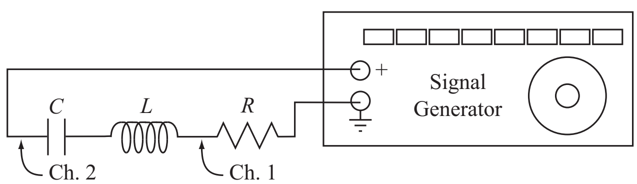
P-P Voltage (Ch. 1): ____________

Voltage Amp (Ch. 1): ____________

Volts/Div (Ch. 2): ____________

Sec/Div: ____________

4. Repeat the procedure to measure the voltage across the resistor, R.



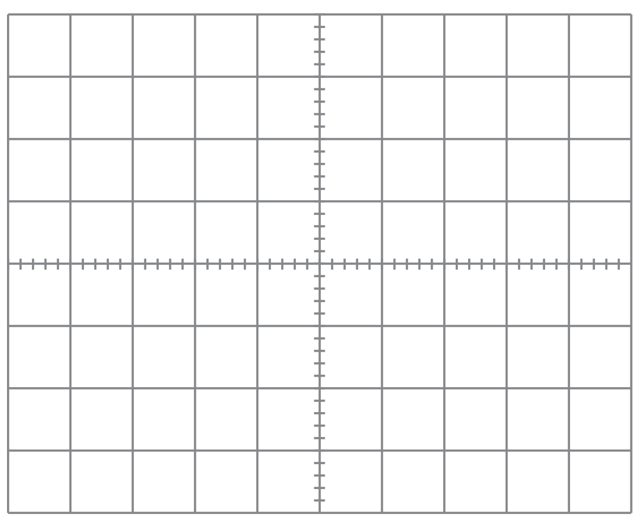


Figure 9: Voltage across the entire circuit, and across the capacitor.

Volts/Div (Ch. 1): ____________

P-P Voltage (Ch. 1): ____________

Voltage Amp (Ch. 1): ____________

Volts/Div (Ch. 2): ____________

Sec/Div: ____________

5. Since the voltage across the resistor and the current are always in phase (see Figure 2a), the phase angle, $\phi = \omega \Delta t$, can be obtained by measuring the time between successive peaks of the voltage across the resistor and the voltage across the entire circuit. Use the cursors to carefully measure Δt and use the result to find the phase angle, ϕ. Think carefully about whether this phase angle should be positive or negative.

6. With the Ch. 1 probe across the resistor as in Step 4, adjust the signal generator frequency until you observe resonance (that is, until $v(t)$ and $v_R(t)$ are in phase). Record this resonant frequency, f_{res} (from the oscilloscope).

7. Using an RLC meter, measure the R, L, and C as appropriate for each circuit element.

$R =$ ____________ $L =$ ____________ $C =$ ____________

4 Analysis

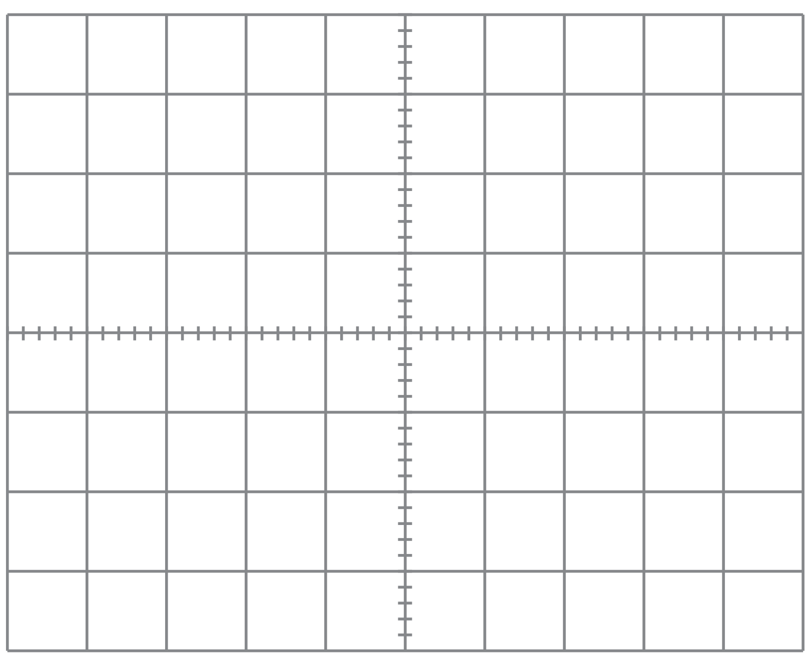
1. Replot the voltage across the entire circuit along with all the voltages across each component *on the same scale* below. **Label** the various signals, and indicate whether they lead, lag, or are in-phase with the *current*.

Volts/Div:

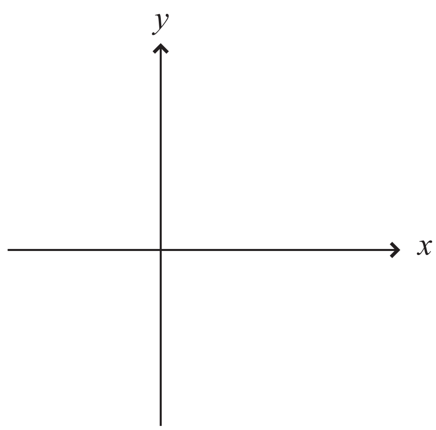
____

Sec/Div:

____



2. Create a figure similar to Figure 3, where the vectors V_L, V_C, and V_R are given by the amplitude of the various voltage signals you recorded. Draw the resultant vector, E_m, and the phase angle, ϕ. Does the sign of ϕ agree with what you found above?



3. Compute the sum of the voltage amplitudes across R, L, and C. Compare this with the voltage amplitude across the entire circuit. Why are they different?

4. With the phase angle you found in the procedure, and the measured values of R and C, use Eq. 2 to compute the inductance of the inductor coil. (Derive an equation for L and show your work clearly.)

5. Use the resonant frequency to compute the inductance of the inductor coil. Compare this value with your measured value of L using the RLC meter and the value you obtained by measuring the phase angle.