## **Physics 4051 Lab Manual**

Fall 2004

## Table of Contents

## **Table of Contents**

Table of Contents	<i>iii</i>
Preface	v
Introduction	V
Troubleshooting	V
Simple DC & AC Circuits	7
1.1. Measuring Voltages and Currents	
1.2. Thévenin's Theorem	13
1.3. Output Impedance and Input Impedance	
1.4. Oscilloscope and Function Generator	
1.5. AC Voltage Divider	
1.6. $Z_{in}$ / $Z_{out}$ Calculations	
Innut and Output Impedance Measurements and Calculations	24
A 1 INPUT IMPEDANCE GENERAL	25
A.2. INPUT IMPEDANCE: Calculation	
A.3. INPUT IMPEDANCE: Measurement	
A.4. OUTPUT IMPEDANCE GENERAL	
A.5. OUTPUT IMPEDANCE: Calculation	
A.6. OUTPUT IMPEDANCE: Measurement	
Some Circuit Theorems	
B.1. Nodal Analysis:	
B.2. Mesh or loop Analysis:	
B.3. Superposition Principle	31
B.4. Thévenin Circuit Theorem	
B.5. Norton Circuit Theorem	
Grounds and Commons	
E.1. Notation	
E.2. Floating Circuits: Battery Powered Circuits and Devices	
E.3. Grounded Circuits: Devices and Circuits Powered by the Building's Main Power	
E.4. Floating Inputs and Outputs in Power Devices	
E.5. Other Devices and Circuits Powered by the Building's Main Power	
E.6. Ground Loops	
E.7. Building Wiring Conventions	
Input and Output Impedance Concepts	
F.1. Introduction	
F.2. DU and Slowly Varying Signals	
F.J. FUWER 1 FAIISIEF	
<b>F.4. FUISES AND TERMINAUNG TEANSMISSION LINES</b>	
F.J. TEFIIIII01029: Fractical Kesistor Values	
F.O. FIODICIIIS	

## Preface

## Introduction

This manual consists of 10 individual chapters, each representing about one week of laboratory work. Although we have tried to spread the workload equally among these chapters, you will find that some chapters require more work than others. For example, chapters 2 and 4 require considerably more work than any other chapter; it is to your advantage to study the material in these chapters ahead of time.

Before doing the exercises in the manual, you should do the assigned reading in Horowitz and Hill or Diefenderfer and Holton or Polnus. Try to predict the behavior of a circuit before building it. These exercises are designed to accompany the material covered in class lectures.

### Troubleshooting

The exercises outlined in this manual require a three step process: first, building the circuit, second testing, and finally, troubleshooting or fixing the circuit. Steps one and two are always required and you will quickly discover that step three is required most of the time. You will also learn that you will spend most of your lab time on troubleshooting. Do not regard trouble shooting as a waste of time. It will help you obtain a more complete understanding of the circuit.

Here are some suggestions on how to make step three as efficient as possible.

#### **Understand the Circuit**

It is almost impossible, and to say the least -- frustrating, to fix something that you do not understand. Instead of spending an endless amount of time exchanging components and checking wires, go back to your textbook (or TA) and make sure you understand the material. Sometimes the circuit does work, yet since the student does not understand what the circuit is supposed to do he or she may spend needless time and effort modifying it.

#### **Check the Wiring**

It is a good idea for one person to build the circuit and for the lab partner to check the wiring. Some tips on wiring:

-- color code the wires (RED for positive supply voltages, GREEN for ground and BLACK for negative supply voltages).

- -- use the correct type of test leads; it greatly reduces noise and errors.
- -- check the ground wires; no ground wires should be left hanging or unattached.
- -- use as few wires as possible; beginners tend to use far too many connecting wires.

#### **Check the Components**

Sometimes components get mixed up in the bins. This is particularly true for resistors and capacitors as they are often returned to the wrong bin. Make sure you have the correct component. If it does not seem to work, try another one. After you have tried three of them, you can be certain that it must be something else in your circuit that is causing the problem. If you find faulty components, do not return them to the bins; throw them in the "dead-components" shoebox or trash.

#### **Check the Supply Voltages**

Use a scope or a voltmeter to check the supply voltages. Check these voltages at the point where they are connected to your circuit. Often you will find that for one reason or another you forgot to power your circuit.

#### **Check the Voltages inside the Circuit**

After verifying that power is indeed applied to the circuit follow the current path and measure voltages at a few easy to calculate points.

#### Typos

While we can not claim that there are no mistakes or errors in this manual, there are definitely no deliberate errors in this manual. If we find any mistakes or errors, we will post them on the blackboard in room 65. Also, the circuits in this manual have been tested and in use for the last 10 years and they do work!

#### **Rules of Thumb**

Throughout this manual you will find various "rules of thumb." They are approximations and help you remember what you should look for first when dealing with a particular component or instrument. You should completely understand and memorize these rules and you should also understand the limitations to them.

#### **Acknowledgments**

This lab manual is based on the first edition of the lab manual by Horowitz & Robinson. Many of the exercises were copied from that manual and adapted to meet our specific needs. Some of the digital exercises were developed by Prof. Zimmermann and Prof. Shupe here at the University of Minnesota. Prof. Ruddick contributed some of the exercises and also spent considerable time and effort making the manual readable. Thanks to Prof. Ganz, Prof. Rusack, Prof. Weyhmann, James Flaten, Michael Krueger and Jens Henrickson for their suggestions and proofreading and to Marty Stevens and Jon Huber for some of the drawings.

Kurt Wick Summer 1999 / 2001

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# Simple DC & AC Circuits

#### Write-up Format: Short

Reading: Appendix A of this lab manual

Horowitz&Hill: Sections 1.01 - 1.06 (pages 1 to 13) Horowitz&Hill: Sections 1.07 - 1.11 (pages 15 to 20) Horowitz&Hill: Appendix A and C

or:

Diefenderfer&Holton: Sections 1-1 - 1.13 (pages 1 to 19) Diefenderfer&Holton: Sections 6-6 - 6-7 (pages 111 to 115) Plonus: Sections 1.3 - 1.7 (pages 2 to 37)

#### **Introduction Hardware:**

This first chapter introduces you to the lab and its equipment. In the first lab session, you and your lab partner will obtain a "bread board" from your TA. For the rest of this quarter, you will "plug" various components into the breadboard to assemble and to test your circuits. Therefore, you will keep your breadboard until the end of the quarter; at the end of each lab session, store it in one of the drawers in lab and clearly mark the label on the drawer with your names.

Study the diagram below and make sure you understand how the wires are connected inside the breadboard!



• Figure 1.1. Top-view of a "breadboard" and its internal connections.

On these breadboards, the two outer strips are usually used for ground and supply voltages. Note, these outer strips are disconnected at the center of the board.

To assemble and to connect your circuits you will need wires, cables, test clips and various connectors. Have your TA show you where to find various cables and how to use them.

#### **Patch Cords and Connectors**

The lab has a large collection of color-coded **single-conductor** patch cords of various lengths. These cords are terminated with banana plugs or minigrabbers. (See below). While minigrabbers are attached to bare wires or components, banana plugs can only be used with mating connectors, called banana jacks. Most test instruments or power supplies are equipped with such banana jacks. If you want to make an electrical connection between a wire and a banana plug, you need to attach an alligator clip to the banana plug. Also observe that several banana plugs can be stacked together.



Single conductor patch cords tend not to be shielded and, therefore, are susceptible to noise. They should be used only for low frequency signals (f < 1 MHz).

The second most often used patch cords of the lab are **coaxial cables** (RG 58-U). They are black, thicker and stiffer than the single conductor patch cords. Coaxial cables contain two conductors: a center conductor and an outer conductor. The outer conductor, also known as the shield, is electrically isolated from the center conductor, which it surrounds. To shield a signal, the outer conductor is usually held at ground while the signal is fed through the center conductor.

Since coaxial cables contain two conductors, they are terminated at each end with either two connectors (two banana plugs, two minigrabbers etc.) or most commonly, a BNC connector. (BNC stands for Berkeley Nucleonics Corporation, the first company to manufacture these connectors.) The sleeve of the BNC connector is always tied to the shield of the coaxial cable while the signal passes through the center pin of the connector.



· Coaxial cable with male BNC connector

BNC connectors come in two genders, male or female. The center conductor determines the gender.

To join two or more BNC cables together, use the following connectors:



Sometimes it is necessary to join cables with BNC connectors to banana plugs. This can be done with the two adapters shown below. **Note the little extrusion shown on top of the adapters**. It identifies the banana connector connected to the shield of the BNC, which is usually its ground connection.



• Banana (Jack) to Male BNC Adapter

• Banana (Jack) to Female BNC Adapter

The BNC to banana adapter shown below is used very often to connect a BNC cable to a test instrument with banana jacks. The distance between adjacent banana jacks is standardized to allow for such adapters. For example, all our digital voltmeters can be connected this way.



• Female BNC to Banana Plug

A BNC terminator contains a shielded resistor, typically 50  $\Omega$ , that connects the shield and the center connector. It is used to reduce noise and reflections of very fast pulses.



BNC 50 ΩTerminator

#### **Introduction: Theory**

This chapter introduces two fundamental circuit analysis concepts: voltage and current dividers and input/output impedance. The concept of the voltage divider is very important because the analysis of the most complicated circuits is often accomplished by resolving the circuit into combinations of voltages or current dividers. The concept of input and output impedance is fundamental in understanding amplifiers, attenuators and transmission lines; it is explained in appendix A of this manual.

Exercise 1.5 introduces you to the oscilloscope. Since that section contains a lot of information and explanations you may want to glance over it before you come to the lab.

Finally, here is the first rule of thumb that you should remember and a note you should carefully read.

#### **Rules of Thumb about Volt- and Ammeters:**

An ideal voltmeter acts like an open circuit; it has an infinite resistance.

An ideal ammeter acts like a short circuit (i.e. an ideal wire); it has zero resistance.

**Note:** Dealing with the voltages and currents as indicated in the exercises in this book is safe for both humans and instruments. All instruments contain various safeguards to prevent them from being destroyed by you, or from destroying you if they are used improperly; so do not worry too much if you should make a mistake or if you are new to electronics. **However**, one exception is the ammeter or any device that can be set into a current measuring mode, such as a Digital Multimeter (DMM). Little or no or protection is built into such an instrument and improper use will simply destroy the instrument. Hence, always stop and think first before turning on the power in any circuit which incorporates an ammeter. Check that the current measuring device is:

1) ALWAYS hooked in SERIES with a current limiting device such as a resistor, and

2) NEVER is hooked directly ACROSS a voltage source.

## **1.1. Measuring Voltages and Currents**

Figure 1.2.a. shows a simple circuit containing a power source and a circuit element called the "device under test" or D.U.T. In this particular drawing, the power source is a battery though it also could have been a power supply, a signal generator or a transducer. The D.U.T. selected here is a resistor though it could also have been any other passive or even an active electronic component.



• Figure 1.2.a.

To measure **either** the voltage **or** the current characteristics of the D.U.T. the following configurations are used:



• Figure 1.2.b. The configuration for a voltage measurement is shown on the left while a current measurement configuration is shown in the picture on the right. The 'X' emphasizes that the circuit had been disconnected at that particular point.

In both cases, "voltmeter" and "ammeter" may refer to the same physical instrument. In the first case it is in its "voltmeter" mode, in the second case it is in its "ammeter" mode. Note that for both configurations the pervious rules of thumb are satisfied: the **voltage** is always measured with the meter across the D.U.T., i.e., **the voltmeter is in parallel with the D.U.T.**; the **current** is always measured through the device, i.e. **the ammeter is in series with the D.U.T.** 

If you already have a working circuit, measuring the voltage across a D.U.T. is usually less complicated than measuring the current because you can connect the voltmeter directly across the D.U.T. without having to rearrange the circuit. On the other hand, if you want to measure the current you need to disconnect or "break" the working circuit before or after the D.U.T. and insert the ammeter at that point.

To determine the electrical properties of a device, the current and the voltage are measured simultaneously while the power source is adjusted. The data is then plotted as an I-V plot and conventionally the voltage is plotted along the x-axis and the current along the y-axis. You will create such a plot shortly when you observe Ohm's law.



There are two possible configurations to measure both the current and the voltage of the D.U.T. **simultaneously** and they are shown below:

• Figure 1.2.c. These configurations measure both the current through the D.U.T. and the voltage across it.

Except for the rare case when the resistance of the D.U.T. is very small (on the order of the internal resistance of the ammeter (typically a few Ohms) or smaller) or very large (on the order of the internal resistance of the voltmeter (typically a few M $\Omega$ s) or larger) it does not matter which configuration you use in Figure 1.2.c. Deciding which of the two arrangements is more suitable for each of the two extreme cases stated is left to you as an exercise.

#### **Exercise:**

Measure and plot *I* vs. *V*. for a couple of resistors. First use a 33 k $\Omega$  resistor and then repeat your experiment with a 120  $\Omega$  resistor. As power source, use the adjustable DC power supply, the Agilent 3630A and connect your circuit to the COM and the +20V output. Adjust the ±20V knob on the power supply and obtain at least 5 different pairs of I-V readings per resistor using either one of the configurations in figure 1.2.c. You must use one digital meter (DVM) and one analog meter for your measurements. (You may not use two digital meters!)

#### Write-up

1.1.1. Measure and plot at least 5 different pairs of I-V readings for a 33 k $\Omega$  resistor. Plot the I-V data using standard conventions as explained earlier.

1.1.2. Repeat 1.1.1 with a 120  $\Omega$  resistor. What does its power rating of 1/8 Watt imply in terms of maximum currents and voltages? Calculate these values for the 120  $\Omega$  resistor and indicate on your plot the region where it is safe to operate the 120  $\Omega$  resistor.

- 1.1.3. Graph what the I-V relationship would look like across:
- a) an open-circuit configuration
- b) a short circuit
- c) a wire with zero resistance

(For b and c, assume that you are using a "real" voltage source with a small, non-zero output impedance.) **Do not measure it**; use your intuition or talk to your lab partner or TA. In your lab report, complete the table below:

	Current	Voltage	Resistance
a) open circuit			
b) short circuit			
c) wire			

Table 1.1.

Of the three cases (a, b, c) which two are identical? Since open and short-circuit concepts will be used continuously throughout the rest of this course be sure that you understand the above table! 1.1.4. You are to simultaneously measure *I* and *V* for a D.U.T. which has an extremely large resistance, i.e., a resistance considerably larger than the internal resistance of any voltmeter available to you. Which of the two configurations shown in figure 1.2.c is "better" and why?

## **1.2.** Thévenin's Theorem



• Figure 1.3. Voltage Divider with a voltage source. Use the Agilent 3630A power supply for the voltage source.

Thévenin's theorem is usually applied to describe a complicated circuit in terms of a much simpler circuit that has similar characteristics. In this exercise we will determine the Thévenin equivalent circuit of the voltage divider shown in figure 1.3. We then will build the Thévenin equivalent circuit and compare its characteristics with the original circuit.

Appendix B of this manual (or section 1.05 of H&H or section 1.11 of D&H) explains how to find the Thévenin equivalent circuit by calculating  $V_{TH}$  and  $R_{TH}$ . The Thévenin equivalent circuit can also be determined by measuring the appropriate currents and voltages which is what we will do in this exercise. Since we are unable to measure  $R_{TH}$  directly by connecting a meter to the circuit (because it may produce the wrong value and it also could damage the meter or the circuit) we need to measure  $V_{TH}$  and  $I_N$ ;  $R_{TH}$  can then be calculated directly from these values.

First, measure  $V_{TH}$ , or the (open-circuit) output voltage,  $V_{out}$ . Second, measure the short-circuit current,  $I_N$ , across the output. Finally, calculate  $R_{TH}$ . Before you replace the circuit with its Thévenin equivalent circuit measure  $V_{out}$  for a load: Attach a load, arbitrarily chosen to be 5.6 k $\Omega$ , and measure  $V_{out}$  again.

Now build the actual Thévenin equivalent circuit, using a variable regulated DC power supply as the voltage source for  $V_{TH}$  and check that its open-circuit voltage and short-circuit current are similar to the previously measured ones. Then attach a 5.6 k $\Omega$  load, just as you did with the original voltage divider and see if it behaves identically.

#### Write-up

1.2.1. Calculate the Thévenin equivalent circuit for the circuit in figure 1.3. (See appendix B or H&H.) Hand in a drawing of the Thévenin equivalent circuit; specify the values of the components.

1.2.2. Hand in a drawing of the Thévenin equivalent circuit (for the circuit in figure 1.3.) which you obtained from measuring  $V_{TH}$  and  $I_N$ ; again specify the values of the components. 1.2.3. Complete the table below with your measured values:

	Vout: Open Circuit	Vout: 5.6 k $\Omega$ Load	Short-circuit current
Circuit in Figure 1.3.			
Thévenin Equivalent			

Table 1.2.

1.2.4. Show (draw a schematic) how you measured the short-circuit current. Explain which assumption you made for the ammeter.

## **1.3. Output Impedance and Input Impedance**

Before you start the experiments in this section be sure to read Appendix F & A first!

A "real" battery can be thought of as an "ideal" voltage source with a series resistance; in batteries, this series or output impedance is often referred to as the "internal resistance." Measure  $V_{Source}$  and calculate  $Z_{out}$  for three of 9V batteries; use  $R_{load} = 150 \Omega$ . From your measurements, can you say what determines whether a battery is "good" or "bad"?

Measure the input impedance of your DMM (Digital Multimeter) using only the DMM itself. (If you use any additional meters you will probably obtain incorrect values!) Set your DMM to the 0-20 volts scale and adjust your DC power supply to output approximately 10 volts; measure it accurately with your DMM. Now insert a 1 M $\Omega$  resistor in series with the power supply and the DMM. Measure the voltage again. (The measurement with the 1 M $\Omega$  in place differs from the first measurement because the DMM's input impedance is affecting the measuring process.) For both cases draw a circuit diagram; indicate clearly in each which voltages you measured. (Hint: think of a "real" voltmeter as an "ideal" voltmeter (with infinite input resistance) in parallel with a very large resistor of value  $Z_{in}$ ; for simplicity, assume for the power supply  $Z_{out} = 0$ , i.e. an ideal power supply.) Compare your circuit diagrams to figure A.4. and calculate  $Z_{in}$ . Compare your value to the value the manual quotes: 11 M $\Omega$ .

By now the following rules of thumb should be "obvious":

The output impedance of an ideal voltage source is zero.

The input impedance of an ideal voltmeter is infinite.

#### Write-up

1.3.1. What is  $Z_{out}$  for three different 9 V batteries? Show how you arrived at the values. 1.3.2. What is  $Z_{in}$  for the DMM? Show how you measured this value; specifically, draw circuit diagrams and indicate clearly which voltages you measured.

1.3.3. There is a method for measuring output and input impedance which is suitable for people who abhor calculations: To measure the output impedance, first measure  $V_{open}$ . Next, attach  $R_{load}$  to the circuit and adjust  $R_{load}$  until  $V_{load} = 1/2 V_{open}$ . Now measure  $R_{load}$ . Knowing  $R_{load}$  one can determine  $Z_{out}$  without any further calculations. What is it?

## **1.4. Oscilloscope and Function Generator**

We will be using the oscilloscope ("scope"), a LEADER 1021, and the HP33120A function generator frequently. Become familiar with their operation by connecting a BNC-to-BNC cable from the function generator's OUTPUT to the scope CH1 input. Turn the function generator on and reset it by pressing the *Recall* button and the up or down arrow keys ( $\land$  or  $\lor$ ) until the display shows: *RECALL 1*. Now press the *Enter* button.



• Figure 1.6. Leader 1021 Oscilloscope Front Panel.

In order to display a signal fed into channel 1, you must "initialize" the following five scope settings:

Function	Button	# (see Fig 1.6)	Setting
Trigger Setting	MODE	28	AUTO
Trigger Setting	COUPLING	29	AC
Trigger Setting	SOURCE	30	CH1
Vertical Display Source	V-MODE	21	CH1
Vertical Display Mode	AC/GND/DC	11	AC

• Table 1.3. Initial Scope Settings

If both the scope and the function generator are turned on, adjust:

	Function	Button	# (see Fig. 1.6)	Comments
1	CH1 Vertical Position	POSITION	19	Adjust until signal is centered on the screen. (If you still don't see anything increase INTENSITY!)
2	Horizontal Sweep Speed	TIME/DIV	23	Adjust until you see a few cycles of the signal.
3	CH1 Vertical Gain	VOLTS/DIV	13	Adjust until signal fills screen.
4	Horizontal Position	POSITION	26	Adjust until you can see the beginning of the trace. (You rarely care what the end of the trace looks like!)

• Table 1.4.

This procedure works on most scopes, so become very familiar with it. If you do not understand what all these buttons do, read on; their functions are explained in the sections below.

#### **1.4.1. Scope Introduction**

"Usually" the oscilloscope is used to display one or more input voltage signal vs. time. The voltages are plotted on the vertical, i.e., the y-axis while the time is displayed along the x-axis. Though most scopes can also be used in an XY mode, where one voltage is displayed against another, for the rest, we will only consider the much more commonly used voltage vs. time mode.

There are 4 parts to a scope. Luckily, all the knobs are grouped in easily identifiable sections: 1) **Time Base**: The TIME/DIV setting controls the time axis (x-axis). It specifies the horizontal velocity (sweep rate) at which the electron beam is continuously swept from left to right across the display.

2) **Vertical Section**: The VOLTS/DIV setting(s) controls the vertical gain of the input voltage(s) as they are displayed along the y-axis.

3) **Trigger**: The trigger buttons specify at what point in time the electron beams begins each scan across the display.

4) **Display Adjustments**: This section of knobs is located directly below the screen. These control the visual appearance of the signal on the screen, such as the signal's focus and intensity. Since they should be self-explanatory we will not discuss them any further.

How does it work? Check for yourself and use the following scope settings:

- In the time base (TIME/DIV) select 0.2 ms / division.
- Set the (left) vertical amplification VOLTS/DIV for channel 1 to its maximum value, i.e., all the way counter clock wise, 5V / division.
- In the V-MODE chose Ch1.
- Set the TRIGGER MODE to AUTO.
- Use no input signal, i.e., temporarily disconnect the cable.

Adjust the (vertical) POSITON knob for channel 1 until you see a dot slowly wandering from left to right across the display. This dot represents the voltage at a given time and without any input it should remain constant with respect to the y-axis. What you are seeing are the electrons from the electron gun striking a tiny circular section of the fluorescent screen causing it to glow in the form of a tiny bright dot while the horizontal section is slowly sweeping the beam (horizontally) across the display. Though the screen will glow where the electrons strike it, in the absence of electrons, the glow will quickly fade and then disappear.

Now change the time base to faster sweep velocity, for example, 1 ms / division. As you change the time base, you will observe how the small dot transforms itself into a horizontal line. That event in itself is actually nothing spectacular but it really forms the basis on how the scope works.

It is WRONG to think that at higher speeds the dot spreads itself out into a horizontal line. It is still as small as before but it is now racing rapidly repeatedly across the display. The combination of the afterglow of the phosphorescence and the retina retention makes it appear like a continuous

line! (Retina retention is the physiological effect that you will see something for about 1/10 of a second after it has already disappeared.)

#### 1.4.2. Scope Trigger

We start with the scope's trigger settings because they are essential for operating a scope. To get a "clean" display of a periodic signal it is essential that each trace overlaps with the previous one, i.e. that they are all "in sync." An example of a periodic signal that is not in synch is shown below.



Figure: Showing the individual traces (snap shots) that are obtained when they are not in-sync. Though the "older" traces will ultimately fade away, if the time between each trace is very short, the overlapping traces form a messy, confusing display.

The synchronization is accomplished through the trigger. The scope starts a trace each time it receives a valid trigger event. It is created when the input signal reaches a user specified (trigger) voltage level. This level is set by the trigger **LEVEL** knob (#32).

Reconnect the HP 36120A's OUTPUT (the lower of the two BNC connectors) to the scope's CH1 and adjust the **horizontal sweep speed** (i.e. **TIME/DIV** knob) until you observe a few periods of a sine wave. With the trigger MODE set to **AUTO**, carefully watch the beginning of the trace and slowly turn the Trigger LEVEL knob. Note that when the trigger level goes past the signal's max/min values, the signal on the screen appears messy because the scope no longer receives a trigger signal. Instead, when in **AUTO** mode, if after a preset amount of time no trigger signal has been received, the scope will automatically (and arbitrarily) start a new trace, resulting in the messy display shown above. In other words, in **AUTO** mode, the scope will still display a trace even without a trigger signal.

Next move the trigger **MODE** knob, (#28) to **NORM** and try the normal triggering mode; again adjust the trigger level. In this mode, the trace is only visible when the input signal crosses the trigger level that has been set with the LEVEL knob. If the trigger level lies outside of the signal's range, no signal at all is displayed. Hence, it is probably a good idea to leave the scope in AUTO mode most of the time.



Figure: Display of a signal that is in-sync. The trigger point is indicated by the black dots. Note: though the display shows only one trace, it is actually composed of overlapping traces that are in sync. As the old trace fades away, it is "redrawn" by the new ones which are identical.

Note that whenever a specific voltage level is specified within a periodic signal, this level is reached twice within one period, once with a rising slope and once with a falling slope. Hence, to specify the intersection uniquely, the slope of the signal at the trigger level must also be specified. As you probably guessed, this is done by pulling on the trigger LEVEL button (#32). Again play with the trigger level and pull and push the button and see if it behaves the way you expect it.

#### 1.4.3. Input Channel Selection

Next consider the last two settings in table 1.3, namely the signal source and signal input coupling mode.

The signal source, or **V MODE** button (#21), selects the input channel. Setting the V MODE button to **CH1** will display the signal fed into the channel 1 (BNC connector #9). Similarly, the **CH2** setting will display the signal from channel 2 (#10). Connect an additional coaxial cable between the SYNC output of the HP function generator and the channel 2 input of the scope. Switch between channel 1 and 2 and you should see a square wave on channel 2. (You may have to adjust the vertical gain i.e., VOLTS/DIV for channel 2.)

The V MODE switch can also be used to display both channels 1 and 2 at the same time. Since most scopes have only one electron gun to draw the traces on the screen, two different methods are used to display both traces at the same time. In the **ALT** mode, the complete trace of one channel is drawn and then the trace of the other, i.e. the gun alternates drawing the two channels. See the figure below.



Figure: Scope display in the ALT mode.

At high sweep speeds, it will appear as if both channel traces are drawn at the same time but if you lower the sweep speed to 10 msec./div, you can clearly observe the alternating drawing. Try it; set V MODE to **ALT** and set the sweep speed to 10 msec/div. If you want to compare two signals at sweep speeds even lower than that, the ALT mode can become annoying.

This problem can be overcome by setting the V MODE into the **CHOP** mode. In this mode, the gun rapidly alternates between drawing small parts of each trace. At high sweep speeds, this mode may result in a trace that appears grainy or "chopped", particularly on older scopes.



Figure: In the "CHOP" mode, the display rapidly alternates multiple times during a single trace between channel 1 and 2. The "choppy" display at  $t_1$  quickly disappears once multiple traces are overlapped and displayed, as shown at  $t_2$ .

#### 1.4.4. Calibration & Measurements

Voltages or time intervals are measured by counting the number of divisions a signal takes up on the display and then multiplying this value by the corresponding knob setting. (Note: a division on the display is one square, about  $\frac{1}{2}$ " wide.) For example, if a signal is two divisions large and the vertical gain is set at 1 Volt/division, then one should be able to assume that the signal is 2 V. This answer can be wrong if the vertical gain is not in its **calibrated** mode or if you should use a **scope probe**.

Unfortunately, the VOLT/DIV value or TIME/DIV is correct only if it is in its calibrated mode, i.e. when the inner knob of the vertical gain adjustment is completely turned cw! Try it; turn the inner knob of the vertical gain (do not turn the outer knob, the VOLT/DIV knob) and turn the TIME VARIABLE knob (#24). Note, that whenever these knobs are in their uncalibrated position, a red warning light labeled UNCAL, directly adjacent to it, will turn on to indicate that your readings will be useless!

Therefore, always check that all these red warning lights are turned off! Note that these knobs are rarely ever used. Nevertheless, people love to abuse them and they are the major source of errors in faulty measurements.

One more warning: observe that the inner knob on the vertical gain knob can be pulled out. This will magnify the signal by a factor of 5. Similarly, the TIME VARIABLE knob can be pulled to produce a 10x magnification in the time scale. Also, neither of these settings will activate the red warning lights -- as they probably should. You will never need to use these knobs but you should be aware how they can drastically alter your results if you are not careful.

Your readings can also be incorrect if you measure your signal using one of the **scope probes** that you may find in the lab. (Ask your TA to show you one if you are unsure what they look like.) The advantage of a scope probe (over a "simple" cable) is that it provides  $10 M\Omega$  input impedance, as opposed to the scope's  $1 M\Omega$ . This gain in input impedance comes at cost: the signals measured are reduced by a factor of 10. (Essentially, the scope probe is a 1/10 voltage divider.) Some fancy scopes are able to sense if a scope probe is being used and then automatically adjust the gain or indicate the 10x loss on the vertical gain knob. The Leader scope used in the lab is not able to do that and, therefore, is completely unaware if you use a scope probe or a cable. So if you should ever use a scope probe, be aware of this and adjust your measured value accordingly. Also note that not all scope probes are necessarily divide by 10 circuits! (Some have adjustable gain settings.) In short, to avoid confusion, avoid scope probes.

#### 1.4.5. HP 33102A Function Generator

You will use the function generator as a signal source for most of the circuits that you will be building in this course.

The function generator has been configured to remember its settings when it is shut down. This can be annoying because the function generator has many different settings and you can never be certain which ones have been changed by a previous user.

Therefore, when you first turn it on, or if you have problems with it, reset it:

- Press the "Recall" button.
- If necessary, use the up "A" or down "V" arrow keys until the display shows: RECALL 1.
- Press the "Enter" key.

In the Recall 1 mode you should get a 1 kHz sine wave with a 200 mVpp amplitude and no offset. If this is not the case, inform your TA of the problem. (For additional troubleshooting, especially if you should measure an amplitude twice the value displayed, see also the web at: http://mxp.physics.umn.edu/f99/trouble)

The signals used for testing the circuits are usually **sine**, **square** and **triangle** waves. Connect the function generator's OUTPUT to CH1 on the scope and SYNC to CH2, as has been described in the previous sections. Push the appropriate buttons in the row labeled "Function / Modulation" and observe the corresponding signals on the scope. What happens to the SYNC output as you change from sine to square to triangle wave?

You modify the **frequency**, **amplitude** or **offset** of a signal by pressing the corresponding buttons in the bottom row labeled "MODIFY." Once you have selected an attribute that you want to modify, its current value is displayed. You can then change it by using one of three methods:

- Use the right or left arrow keys to select the digit you want to increment or decrement and then turn the round knob.
- Use the right or left arrow keys to select the digit you want to increment or decrement and then hold the up or down arrow keys.
- Push the "Enter Number" button and then enter a numerical value. (The numerical values are printed in green to the lower left of the buttons.) Complete the entry by pressing the arrow key that corresponds to the appropriate unit which is printed in green to right of it.

Which method you use is up to you.

With the scope CH1 connect to the OUTPUT and CH2 to SYNC, adjust the amplitude of a sine, square and triangle wave signal. In what way does the output signal from OUTPUT and SYNC differ when you adjust the amplitude?

Choose a waveform and change its frequency. In what way does the output signal from OUTPUT and SYNC differ when you adjust the frequency?

Summing up, **describe** how the SYNC signal is affected by the waveform, amplitude and frequency selected.

**Note:** Unless specified otherwise, whenever we talk about the "output signal" from the function generator, we refer to "OUTPUT" output. Therefore, always connect your scope and circuits to the "OUTPUT." The SYNC output is rarely used.

#### **1.4.6. Signal Input Coupling**

Finally you get ready to measure some "real" signals. A sinusoidal signal can be expressed by:

$$V(t) = A + B\sin\left(\frac{2\pi t}{T}\right)$$
(1.1.)

"A" is usually referred to as the DC offset voltage, "B" as the amplitude and "T" is the period. **Draw** the above function and clearly indicate in your picture what A, B and T are.

If you followed the instructions, you had your channel 1 input coupling (the lever marked **AC/GND/DC**, or #11) set to AC. In this mode any DC offset is ignored and equation (1.1.) reduces to:

$$V(t) = B\sin\left(\frac{2\pi t}{T}\right)$$

Hence, if all you want to measure is the amplitude and frequency of a signal this mode is fine. Measure (with the scope) and **report** the largest sine wave **amplitude** that can be generated by the function generator OUTPUT at 1 kHz and 15 MHz. (Check that the DC OFFSET is off, i.e., set to zero.) For the two frequencies, what is the percentage error between the frequency set on the function generator and the frequency measured from the scope?

To measure a DC offset is a two-step process. First, you need to know where ground potential is situated on your screen (i.e. where V = 0 is) since its position can be arbitrarily adjusted by the Vertical Position knob (#19). Set the input coupling lever (AC/GND/DC switch, #11) to GND (ground); this shorts out the input (i.e. physically disconnects the input signal) and turns equation (1.1.) into:

$$V(t) = 0$$

(1.3.)

(1.2)

Set your trigger MODE to AUTO and adjust your Vertical Position until you see the corresponding signal on the screen. Position it vertically wherever you consider it convenient and remember where that location is. Explain to your lab partner why you cannot get a steady display with trigger MODE set to NORM. (Try it.)

Secondly, switch the input coupling lever to DC. Adjust the function generator to produce a 0.5 V amplitude, 1 kHz sine wave. Now, adjust the function generator DC OFFSET from one extreme position to the other. **Report** the maximum positive and negative DC offset voltages measured.

Remember, we said that with the input coupling set to AC, any DC offset voltages are ignored by the scope. Check that it is true by setting the input coupling to AC and again change the function generator's DC offset voltage.

#### Write-up

- 1.4.1. Describe how the SYNC signal is affected by the waveform, amplitude and frequency.
- 1.4.2. Draw the function (1.1.) and clearly indicate in your picture what A, B and T are.

1.4.3. Measure and report the largest sine wave amplitude that can be generated by the HP function generator's OUTPUT at 1 kHz and 15 MHz.

1.4.4. Report the maximum positive and negative DC offset voltages measured for a 0.5 V amplitude, 1 kHz sine wave.

## **1.5. AC Voltage Divider**

First spend a minute thinking about this question: How would the analysis of a resistive voltage divider be affected by an input voltage that changes with time (for example, a sinusoidal input signal)? Now hook up the voltage divider from lab exercise 1.2.; replace the 15 V voltage source with the function generator and see what it does to a 1 kHz sine wave (use function generator and scope), comparing input and output signals.

#### Write-up

1.5.1. Draw pictures of  $V_{in}$  vs. *t* and  $V_{out}$  vs. *t* as observed on the scope. Draw both pictures on the same graph.

1.5.2. Explain in detail (use equation 1.1.) why it must act that way...

## **1.6.** Z<sub>in</sub> / Z<sub>out</sub> Calculations

Find the answer to any three of the five problems, F.6.1 to F.6.5., in Appendix F.6. at the end of this manual.

#### Write-up

1.6.1. Hand in the solutions to three of the five problems in Appendix F.6.



## Input and Output Impedance Measurements and Calculations

When you design or analyze all but the most trivial circuits it is useful to break them up into a number of simpler circuits that can be more easily understood. Because the voltage divider (or the current divider) is very basic, we would like to treat any complex circuit like an assembly of voltage dividers. Therefore, if we know the "equivalent" resistance of parts of a circuit then we can predict how each part affects the other. Furthermore, we can also predict how the entire circuit would affect a source or a load attached to it.

When we simplify a circuit and quote its resistance, or as in the AC case, its impedance, we need to know exactly where in a given circuit this value refers to. Though the impedance could be measured anywhere, usually we are only interested in the impedance at the input and output of a circuit.



• Figure A.1. Not all circuits have an input and an output. For example, a battery has only an output terminal.

Sometimes it is obvious which part of the circuit is the input terminal and which one the output. If this is not the case it is important that the points are clearly identified; generally the circuits are drawn with the left side as the input and the right side the output.



To predict how a circuit affects a source, we replace the circuit with a resistor (and maybe a capacitor) that corresponds to the measured or calculated input impedance of a circuit. The same

holds true for how a load affects a circuit except that in this case we replace the actual circuit with a resistor equal to the value of the output impedance.

While the input and output impedance represent a real physical characteristic of a circuit they usually do not describe every electronic property of a given circuit. Therefore, we cannot expect that replacing an entire circuit with its equivalent input or output impedance will reproduce every physical property of a circuit. It should be understood that replacing the circuit with its equivalent input and output impedance is merely a very powerful tool for circuit analysis. This concept is used in modeling "real" devices as "ideal" ones and for maximum power transfer applications (i.e. impedance matching).

Because this method is very important you will be asked to calculate or to measure the input and output impedances many times during this course. In order to be able to calculate the impedances it is assumed that you have enough information about the circuit and that you know how to calculate it; this is often nontrivial especially if active elements such as transistors and opamps are part of the circuit. Measuring the impedances can be tricky if the impedances are very large or very small.

What follows are various recipes and conventions for finding the input and output impedances and we will use these during the entire course. Nevertheless, you should understand that while these conventions are the most common ones they are not the only ones that people use. Therefore, when you are quoting your results you should always indicate how you arrived at them. You should always specify which impedance you measured (input or output); in addition, if there are any other terminals in the circuit indicate if they were kept open- or short-circuited.

## A.1. INPUT IMPEDANCE GENERAL

The **input impedance** of a circuit is the impedance seen by a signal source driving a circuit. It is generally an indication of how much a circuit affects a signal source. Note: if the circuit has an output terminal, then all our calculations and measurements are done with NO load attached, i.e. the input impedance is always measured with the output open-circuited.

## A.2. INPUT IMPEDANCE: Calculation



• Figure A.3.

1) Identify the input and output terminals of the circuit.

2) If the circuit has an output terminal, there should be no load attached to it.

3) Short-circuit all independent voltage sources and open-circuit all independent current sources in the circuit.

4) Calculate R<sub>Thévenin</sub> looking into the input terminal; this is the input impedance of the circuit.

### A.3. INPUT IMPEDANCE: Measurement

To determine the input impedance,  $Z_{in}$ , treat the circuit in figure A.4. as a voltage divider. The current in the circuit is:

$$I = \frac{V_R}{R} = \frac{V_C}{Z_{in}}$$
(A.1.)

Solving for  $Z_{in}$ , we find:

$$Z_{in} = R \frac{V_C}{V_R} \tag{A.2.}$$

To measure the input impedance,  $Z_{in}$ :

1) Remove any load attached to output of the circuit.

2) Insert a series resistor R between a signal source and the circuit under test and measure the voltage drop across the source,  $V_{\rm S}$ .

3) Measure the voltage drop across the circuit under test,  $V_{C}$ .

4) Calculate  $V_R$ :  $V_R = V_S - V_C$ 

Finally use equation A.2 to calculate the input impedance.



(You may wonder why the extra steps 3 and 4 were introduced to calculate  $V_R$  instead of directly measuring it. The reason is subtle: some measurement instruments require that a voltage is always measured with respect to the same point in the circuit. Using the method above measures both voltages with respect to point *A* often referred to as "ground.")

## A.4. OUTPUT IMPEDANCE GENERAL

The **output impedance** of a circuit is the impedance seen by a load attached to the output of the circuit as it looks back into the circuit. If the circuit has an input terminal then the output impedance is calculated and measured with an ideal voltage source attached to the input terminal. Since an ideal voltage source has zero output impedance (i.e. its impedance is that of a short circuit) we can say that we measure the output impedance with the input short-circuited.

## A.5. OUTPUT IMPEDANCE: Calculation



1) Identify the input and output terminal of the circuit.

2) If the circuit has an input terminal attach an ideal voltage source to it.

 Short-circuit all independent voltage sources and open-circuit all independent current sources in the circuit.

4) Calculate  $R_{Thévenin}$  looking into the output terminal; this is the output impedance of the circuit. (Remember that for finding  $R_{Thévenin}$  ideal voltage sources are replaced with short circuits.)

## A.6. OUTPUT IMPEDANCE: Measurement

Notice the circuit in figure A.6. is just a voltage divider with:

$$V_{Load} = V_{Open} \frac{R_{Load}}{R_{Load} + Z_{Out}}$$
 (A.3.)

where  $V_{Open}$  is the voltage measured at the output with no load attached. Solving A.3. for  $Z_{Out}$  results in:

$$Z_{Out} = R_{Load} \frac{V_{Open} - V_{Load}}{V_{Load}} \quad (A.4.)$$

To measure the output impedance:

1) Attach a signal source to the input of the circuit.

2) Measure the open-circuit output voltage  $V_{Open}$  without any load attached.

3) Attach a "reasonable" load resistor  $R_{Load}$  across the output; measure the output voltage with the load attached, i.e. measure  $V_{Load}$ . Now use equation A.4. and calculate  $Z_{Out}$ .



• Figure A.6. Thévenin Circuit with Load

Here are a couple of exercises to test your skills. Calculate the input and output impedances (actually, in this case resistance would be appropriate) for the following circuits:



#### A.7. Answers:

Circuit 1:  $Z_{in} = 10 \text{ k}, Z_{out} = 0$ Circuit 2:  $Z_{in} = R1, Z_{out} = R2$ Circuit 3:  $Z_{in} = 2R, Z_{out} = R/2$ Circuit 4:  $Z_{in} = R, Z_{out} = R$ 



## Some Circuit Theorems

The currents and voltages in simple circuits are most easily determined by repeated application of the voltage-divider principle. Nevertheless, for more complex circuits, two basic methods exist for the mathematical analysis. They are nodal and mesh or loop analysis and they are applicable in most circumstances. Sometimes, the mathematics involved in solving for the individual voltages and currents can become tedious; in such cases, applying the superposition theorem and Thévenin or Norton theorem may simplify the analysis.

## **B.1. Nodal Analysis:**

A node is defined as a point where three or more circuit elements are joined together. In nodal analysis the voltage at each node is calculated by summing all the currents,  $i_k$ , flowing into and out of each node. From charge conservation, it follows that for each node:  $\sum i_k = 0$ . This simply means that the current flowing into a node must equal the current flowing out of that node.

Here are the individual steps for performing a nodal analysis:

- 1) Identify and label each node in a circuit as  $v_1$ ,  $v_2$  etc.
- 2) Label the current that flows between each node, for example,  $i_1$ ,  $i_2$ , etc.
- 3) Choose an arbitrary direction for the currents and indicate that in your circuit.
- 4) Choose a node and write down all currents that flow into and out of that node. Remember that the sum of currents flowing into the node must equal the sum of currents flowing out!
- **5)** Apply Ohm's law and replace each current in step 4 with the corresponding  $\Delta V/R$ ; keep your sign convention consistent with your current direction!
- 6) Repeat step 4 and 5 for all other nodes and currents.
- **7)** Solve for the individual  $v_{i}$ .

Here is an example:



First, we identify and label all nodes and indicate the currents in the circuit. The direction of the currents are chosen arbitrarily. Nevertheless, once a current direction has been chosen for a particular node, the direction must be maintained for the entire loop! For example, by selecting  $i_1$  to flow out of  $v_1$  we imply that it must flow into  $v_2$ .



• Figure B.2. Circuit with nodes and currents.

The circuit in figure B.2 has only two nodes. Picking node  $v_1$  and applying step 4, we get:  $-i_1 + i_2 + i_3 = 0$  (B.1.)

Note, that we used a positive sign for currents flowing into a node and a negative sign for currents flowing out of a node.

Working through step 5, applying Ohm's law, we end up with:

$i_1 = (v_1 - V_b - v_2)/R_2$	(B.2.)
$i_2 = (v_2 + V_a - v_1)/R_3$	(B.3.)
$i_3 = (v_2 - v_1)/R_1$	(B.4.)

Again observe the sign convention: for a current flowing from node *A* to node *B*,  $\Delta V$  is:  $V_A - V_B$ . Furthermore, current flowing through a voltage source from the negative to the positive terminal, causes a positive voltage drop; current flowing through a voltage source in the opposite direction is considered a negative voltage drop.

Finally, we substitute equations B.2., B.3 and B.4. into B.1. At this point we could solve for either  $v_1$  or  $v_2$  though they are dependent on each other. If we arbitrarily ground  $v_2$ , i.e.  $v_2 = 0$  then we find that:

for 
$$v2 = 0$$
,  $v1 = \frac{V_a R_1 R_2 + V_b R_1 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$  (B.5.)

## **B.2. Mesh or loop Analysis:**

A mesh is defined as a loop of a circuit that does not contain any other loops within it. In mesh analysis Kirchhoff's law is applied which states that the voltage changes in a complete circuit loop must add up to zero, i.e.  $\sum v_i = 0$ .

The individual steps for performing a mesh analysis are:

- 1) Find each mesh and assign a "mesh" current to it.
- 2) Calculate the voltage drop or increase at each point in the mesh and recall that  $\sum v_i = 0$ .
- **3)** Apply Ohm's law to every resistive term of step 2; if a component is shared with another mesh, then the current through that component is the difference between the two mesh currents.
- 4) Repeat step 2 and 3 for all other meshes.
- **5)** Solve for the individual  $i_k$ .

As an example we use the same circuit as in the nodal analysis, figure B.1. First, we indicate the mesh currents  $i_1$  and  $i_2$  as flowing in a clockwise direction.



• Figure B.3. Notice how the mesh currents are drawn in clockwise fashion to increase symmetry.

The voltage drops going around mesh 1 and 2 are:

$$V_{a} - V_{R_{3}} - V_{R_{1}} = 0$$

$$-V_{R_{2}} - V_{b} - V_{R_{1}} = 0$$
(B.6.)

Again note the conventions. If the mesh current goes through a voltage source from the negative to the positive terminal, then we indicate it as a positive voltage source. If the current flows the other direction, we label it as a negative source. All voltage drops across resistors are negative.

Next we apply Ohm's law to equations B.6.:

$$V_{a} - i_{1}R_{3} - (i_{1} - i_{2})R_{1} = 0$$
  
-  $i_{2}R_{2} - V_{b} - (i_{2}i_{1})R_{1} = 0$  (B.7.)

Finally, we are able to solve for i1 and i2:

$$i_{1} = \frac{R_{1}V_{a} + R_{2}V_{a} - R_{1}V_{b}}{R_{1}R_{2} + R_{1}R_{3} + R_{2}R_{3}}, \quad i_{2} = \frac{R_{1}V_{a} - R_{1}V_{b} - R_{3}V_{b}}{R_{1}R_{2} + R_{1}R_{3} + R_{2}R_{3}}$$
(B.8.)

Assuming that  $v_2 = 0$  and solving for  $v_1 = (i_1 - i_2) R_1$  results again in equation B.5.

Situations do at times arise when it is difficult to apply the nodal or the mesh analysis. For example, the circuit below, which is very similar to the one above, can not be analyzed using the nodal analysis method. (At least not to the extent that it is covered here.)



• Figure B.4. Circuit to be analyzed

If a situation arises where the nodal analysis does not work, then generally a mesh analysis will work and vice versa.

## **B.3. Superposition Principle**

The superposition principle states: In any linear resistive circuit containing several sources, the voltage across any resistor or source may be calculated by adding algebraically all the individual voltages caused by each independent source acting alone, with all other independent voltage sources replaced by short circuits and all other independent current sources replaced by open circuits.

Let's apply the superposition principle and determine  $v_1$  in the circuit in figure B.1. (For simplicity, we assume that  $v_2 = 0$ .) Since there are two independent voltage sources,  $V_a$  and  $V_b$ , we need to apply the superposition principle twice. First, we find the voltage at  $v_1$  due to  $V_a$ . This is done by shorting  $V_b$ .



• Figure B.5. First application of the superposition principle to the circuit in figure B.1.

We can see that  $R_1$  in parallel with  $R_2 (R_1 || R_2)$  forms a voltage divider with  $R_3$ . This allows us to write immediately:

$$\mathbf{v}_{1}\Big|_{V_{b}=0} = V_{a} \frac{R_{1} \|R_{2}}{R_{3} + R_{1} \|R_{2}} = V_{a} \frac{R_{1}R_{2}}{R_{3}(R_{1} + R_{2}) + R_{1}R_{2}}$$
(B.9.)

Next, we find the voltage at  $v_1$  due to  $V_b$ ; therefore, we shorten  $V_a$ :



• Figure B.6. Second application of the superposition principle to the circuit in figure B.1.

Again applying the equation for a voltage divider allows us to write down immediately:

$$V_1\Big|_{V_a=0} = V_b \frac{R_1 \|R_3}{R_2 + R_1 \|R_3} = V_b \frac{R_1 R_3}{R_2 (R_1 + R_3) + R_1 R_3}$$
(B.10.)

Finally, from the superposition principle,  $v_1$  is:

$$v_{1} = v_{1}|_{v_{a}=0} + v_{1}|_{v_{b}=0}$$
(B.11.)  
$$v_{1} = \frac{V_{a}R_{1}R_{2}}{R_{3}(R_{1}+R_{2}) + R_{4}R_{2}} + \frac{V_{b}R_{1}R_{3}}{R_{2}(R_{1}+R_{3}) + R_{4}R_{3}}$$
(B.12.)

Simplifying this yields again equation B.5.

Below is a circuit example that was used in a final exam in this course. Applying the superposition principle you should see without any further ado that  $V_{out}$  is -1 V.



• Figure B.7. Sample circuit.

## **B.4. Thévenin Circuit Theorem**

Thévenin's theorem is used to represent a circuit by a voltage source  $V_{Th}$  and a series resistor  $R_{Th}$  (or impedance  $Z_{Th}$ ). As in the previous appendix on input and output impedance, the Thévenin equivalent circuit can either be determined by measurement or by calculation. The method selected depends on the particular situation though both methods yield the same results. As you will see, determining the input impedance of a circuit is identical to finding its Thévenin resistance.

To calculate the Thévenin equivalent of a circuit:

- Identify and label two points (A and B) in the circuit across which you will determine the Thévenin equivalent circuit.
- **2)** To find  $V_{Th}$ , calculate the voltage across these two points.
- **3)** To determine  $R_{Th}$ , first reduce all independent sources in the network to zero by short-circuiting all voltage sources and by open-circuiting all current sources; now calculate the resistance seen across points *A* and B; this is  $R_{Th}$ .

To determine the Thévenin equivalent circuit by <u>measurement</u>, first measure the voltage that appears across the two points *A* and B; this  $V_{Th}$ .  $R_{Th}$  is identical to  $Z_{in}$  and can be determined by the method explained in the previous appendix.

For example, the Thévenin equivalent of the circuit in figure B.7. is:



#### **B.5. Norton Circuit Theorem**

Norton's theorem is the dual of Thévenin's theorem. It is used to represent a circuit by a current source,  $I_N$ , and a parallel impedance  $R_N$  or  $Z_N$ .

The Norton equivalent circuit can be determined by first finding the Thévenin equivalent circuit. The Norton equivalent circuit is then be found from:  $I_N = V_{Th} / R_{Th}$  and  $R_N = R_{Th}$ .

The Norton equivalent circuit can also be obtained by either measuring (or calculating) the short-circuit current between the two points of interest in the circuit; this short-circuit current corresponds to  $I_N$ . The Norton impedance,  $R_N$ , is found in the same manner as  $R_{Th}$ . (Read the previous section.)

For example, the Norton equivalent circuit of figure B.7. is:





## Grounds and Commons

#### Introduction

The following paragraphs are intended to clear up some issues regarding the concepts of "common" and "ground."

## E.1. Notation

Some of the confusion about the concepts discussed here may come from sloppy notation. For example, Ohm's law is usually written as V = IR. While this is (mostly) correct we know that "V" always refers to energy, i.e., a potential difference. Therefore, more properly, Ohm's law should be written as  $\Delta V = IR$ .

By convention, the voltage difference,  $\Delta V$ , between two points in a circuit, *A* and *B*, is defined as:  $\Delta V = V_{AB} = V_A - V_B.$ 

Using this notation, Ohm's law reduces to its familiar form of  $V_A = V = I R$  only when  $V_B = 0$ , a situation that is by no means always given!

To illustrate the dangers of carelessly applying *V*, instead of  $\Delta V$ , a simple voltage divider circuit is shown in the picture below. It is a slightly modified version of a problem used on a recent final exam. Students were asked to calculate  $V_{XY}$ .



• Figure E.1. Voltage divider powered by two batteries.

Students remembered and applied the equation for a voltage divider, derived from Ohm's law:  $V_{out} = V_{in} R_1 / (R_1 + R_2)$ 

They (correctly) assumed that  $V_{in}$  corresponds to  $V_{AB}$  = 10V and obtained  $V_{out} = V_{XY} = 9V$ . But is this correct?

Unfortunately, the right answer is  $V_{XY} = 4V$ . So what went wrong? The voltage divider equation quoted above was derived from Ohm's law with the assumption that  $V_B = 0$ . In the situation shown above,  $V_B$  is 5 Volts below  $V_Y$  and, therefore, this offset must be included in the derivation of the voltage divider equation.

If  $V_B$  is not 0 you will get a different expression for the voltage divider.

To sum up what we has been written so far: when analyzing circuits one should be very careful to understand with respect to what (reference) voltage one is analyzing the circuit. Since the math gets easier when the reference voltage is 0 V, circuit designers prefer to designate an arbitrary reference point in the circuit called a "common" and then arbitrarily assume that it always is at 0 V.

#### E.2. Floating Circuits: Battery Powered Circuits and Devices

The simplest example of a "floating" circuit (or a floating device) is a circuit (or device) powered only by batteries and not connected to the building's wiring. Though the voltages within such a circuit are well defined with respect to other points within the circuit, the voltage with respect to the (lab) environment or the building's wiring is unknown or unimportant; it may even change over time, i.e., it is "floating."

Contrary to what one might think, this is a very desirable property because, if required, it is very simple to turn a floating circuit into one that has a fixed voltage with respect to the other instruments or the building's wiring. (Going the other way can be extremely difficult.) Floating circuits also are less affected by electrical noise, especially if that noise comes from devices connected to the wiring in the lab.

An example of a floating circuit was shown in Figure E.1. When analyzing it we paid special attention to specify the reference voltage to which a voltage was calculated or measured. For example, for the circuit in Figure E.1.,  $V_X$  really has no meaning,  $V_{XY}$  does.

On the other hand, we can pick and identify an arbitrary point in the circuit, i.e., a common reference point or a "**common**," and define that it is at 0 V. In the previous example, point Y was (arbitrarily) chosen as our reference point. (See Figure E.2. below and note the symbol.) Now writing  $V_X$  is no longer ambiguous since  $V_{XY} = V_X - V_Y = V_X - 0 = V_X$ . Of course, selecting a different common may result in voltages that are off by a constant.



• Figure E.2. Voltage divider with a "common" reference point.

Setting the common point at Y allows us now to derive the answer to the final exam question in the previous section. Using the fact that in any voltage divider the current remains constant,  $I_1 = I_2$ , and applying (the full version of) Ohm's law to calculate  $I_1$  and  $I_2$  yields:  $(V_A - V_X) / R_1 = (V_X - V_B) / R_2$ . Noting that  $V_A = +5V$ ,  $V_B = -5V$  and solving for  $V_X$  gives us  $V_{XY}$ .

Since the terms "common" and "ground" (more on "grounds" in the next section) are often incorrectly used it is important to point out one important, but subtle, fact. Remember that we assumed that the battery operated circuit is floating, i.e., since it is not connected to the building's wiring it has no fixed or predetermined voltage relationship to it. By the same reasoning, the "common" also is floating and has no fixed relation ship to the building's wiring.

Can a circuit have more than one common? Sure, but each common is at the same voltage, i.e., by convention also at 0 V. If you recall that (ideal) wires have no voltage drop across them, then it follows that all points connected to the common with wires must therefore also be at 0 V.

Applying these ideas shows that the two circuits shown below are identical. In the right drawing, the wires between the two common points have been omitted because by convention all common points are at 0 V. Whether one prefers the first diagram or the second diagram is really a matter of taste. (If you do not like the second representation, you can always draw the connecting wires between the common points back in.) Generally, the method pictured on the right is preferred because it allows grouping of circuit elements into "easily" recognizable circuit blocks, such as filters, voltage dividers, amplifiers etc., though it has the following danger: when building such circuits, the (required) connecting wires between the commons are often overlooked! Be aware of this mistake and try to avoid it!



• Figure E.3. Floating circuits with common

To sum up, floating battery operated circuits and devices:

a) can have a common point at any arbitrary point;

b) though a common point is assumed to be 0V, with respect to the lab it is floating and, therefore, at some arbitrary potential to it.

## **E.3. Grounded Circuits: Devices and Circuits Powered by the Building's** Main Power

Things do get a bit more complicated, and sometimes murky, when using devices such as power supplies, oscilloscopes and function generators. These devices are all plugged into the building's main power. For them to work safely they must be built so that their casing is physically connected to a common point in the building which itself is connected to "**ground**," i.e., short for "common ground".

This ground is not some arbitrary (virtual) reference point but an actual physical point in a building's power circuit. It is either a long stake driven into the ground or a clamp tied to the main water inlet of a house. In either case, it is always connected to the house wiring with a very heavy, thick copper wire able to withstand 100's of amps, and it would be dangerous to think one could (or should) override or ignore it! Furthermore, this ground point is at 0 Volts with respect to all other devices that are connected to the power in that building. Unlike a floating circuit where the common could be offset to any arbitrary voltage, this common ground always remains, or should remain, at 0 V.



• Figure E.4. Floating circuit

For the first example consider measuring the voltage  $V_{AX}$  in the (floating) circuit above using an oscilloscope. Before doing so carefully examine the connectors to the scope inputs: the outer conductor of each coaxial connector is directly mounted to the casing of the scope which, as mentioned previously, is itself connected to the building ground. In other words, the outside, by convention the black connector, of a coax cable is always connected to ground when it is hooked up to the oscilloscope. Once you connect one of the ground connectors to the floating circuit shown above, a ground point in your circuit has been established, and no other ground point can exist or be at a different potential!



• Figure E.5. An oscilloscope has been connected to the floating circuit from the previous picture. The grounds drawn are redundant but are shown to emphasize that both the case and the outside of the input connectors are grounded.

Now consider what happens if we try to measure  $V_{AX}$  with the scope's channel 1 and  $V_{XB}$  with channel 2 as shown in the picture below.



• Figure A4: Setup with the scope simultaneously (trying) to measure VAX and VXB.

The circuit shown above has now two common ground points, X and B. This causes both points X and B to be 0 volts. To visualize this effect, remember that we can replace the connection between ground points with wires; an equivalent drawing of previous circuit is shown below.



• Figure E.6. Same setup up as in the previous figure except that the effect of the hooking up the grounds to different points in the circuit is shown explicitly, i.e., the effect of shorting out the 9k resistor.

This circuit is no longer identical to the original one because the 9k resistor has been shorted out by the ground wires, forcing  $V_{XB}$  to remain at 0 V always and making  $V_{AX}$  5V. So, back to our original question, how do we then measure  $V_{AX}$  and  $V_{XB}$  simultaneously using the scope?

The short answer is that it cannot be done directly. If you want to observe both signals simultaneously, you could

a) either make point X the common ground point and observe  $V_{AX}$  and  $V_{BX}$  and note that  $V_{XB} = -V_{BX}$ , (see the figure below) or



b) observe  $V_{AB}$  and  $V_{XB}$  and then calculate  $V_{AX} = V_{AB} - V_{XB}$ .

• Figure E.7. This setup allows observing V<sub>AX</sub> and V<sub>BX</sub> simultaneously. Note that point X is grounded and that the ground leads from channel 2 should be left unconnected since point X is already grounded through the leads from channel 1. Connecting the ground leads from channel 2 to point X can actually cause problems because it establishes "ground loops." (See the next section.)

You should now understand why it is so crucial that there is <u>only one point</u> in your circuit to which the (black) ground leads must be connected to.

### E.4. Floating Inputs and Outputs in Power Devices

The device in the previous section, the oscilloscope, had grounded inputs. Though the casings of all devices and instruments powered by the building's electrical power are (hopefully) always grounded, its outputs or inputs are not necessary. Unfortunately, aside from examining the device as we did in the previous section there is no simple and easy method for determining if a device's outputs or inputs are grounded short of consulting the manufacturer's specifications or testing it. To illustrate this, here is an example using the lab's HP3630A power supply.

The last picture in the previous explained how to measure  $V_{AX}$  and  $V_{BX}$  using the battery-powered circuit. Could we still use the same setup if we replaced the floating batteries with a power supply? The answer depends on how the power supply was constructed. Is one of its outputs grounded or were both they left floating?



• Figure E.8. The adjustable power supply on the left has one of its outputs internally grounded to the case while the outputs on the power supply to the right were left floating. (Note, the 110 AC input line to the power supply has been omitted from the drawing for simplicity. The arrow across the "battery" symbol indicates that it is an adjustable voltage source.)

It turns out that the lab power supply, the HP3630A has floating outputs. The COM output is not connected to the ground. Therefore, the power supply outputs behave like a battery-operated floating circuit. A more detailed schematic of all the outputs of the HP3630A is shown below.



• Figure E.9. Agilent E3630A power supply outputs. (Note the symbol used for ground and that the casing (and one output) is still connected to the building ground for safety reasons.)

When powering the lab circuits, which output should you use, COM or ground?

- If you use only one of the adjustable voltage sources from the E3630A power supply then COM must always be connected to your circuit to complete the current path.
- On the other hand, if two or more voltage sources are used in series, for example the +20V and the -20V, then the circuit is complete without connecting COM. Whether and how you want to connect COM in such a case depends on if you want to leave COM floating or if you want to tie it to ground.
- Connecting the ground output to your circuit is optional; it is mainly used to reduce noise or for safety reasons, for example, if the voltages applied are very large, 50 V or more.

A final comment on notation: As we have seen now, it is possible for a circuit to have both a ground and a common with the common point being offset from ground. This results in confusion interpreting voltages such as  $V_X$ , i.e., was  $V_X$  measured with respect to ground or the common? Since  $V_X$  only makes sense if the reference voltage is 0, it must be concluded that under such circumstances the voltage is with respect to ground, and not the common. To make sure though, it's best to specify that!



To test how well you understand the previous concepts, calculate for the four circuits shown below the voltages  $V_{AX}$ . Assume that you use an oscilloscope to measure  $V_{AX}$  and that its ground lead is connected to point X.

• Figure E.10. "N.C." stands for "Not Connected."

The voltages observed on the scope would be: a) +1V, b) undetermined, i.e. incomplete circuit, c) 10V d 0V.

## E.5. Other Devices and Circuits Powered by the Building's Main Power

If you're a bit dazzled by all this it is not surprising. First, we said that devices plugged into the building's main power usually have grounded outputs (and inputs), and then we showed that the lab power supply has floating outputs. So what can be said about all the other devices that are plugged into the building's power supply? The short answer is: not much. The safe answer is that unless you know everything about the device, most likely one of its outputs (or inputs) is grounded and, therefore, deal with them accordingly.

Below are some rules that should be taken with a grain of salt:

- Power supplies: Outputs of "cheap" power supplies are usually not floating; expensive ones almost always are but only over some manufacturer's specified range!
- AC signal sources and function generators: Since it is easy to float the output from an AC source, a large capacitor or a transformer will do the job, you may assume that most such devices use floating outputs.

 Voltmeters Powered by the Building's Main Power: They should have floating inputs if they really want to measure accurate voltages but the question is over which range they can be floated, especially when measuring DC voltages. So if you're not sure, use a handheld meter.

### E.6. Ground Loops

It was stated that all the (black) ground leads should be connected to a single point in your circuit. Unfortunately, if there is more than one (ground) path to this common ground point, ground loops exist. As far as circuit design goes, though ground loops are not "illegal," they are more of a nuisance because they typically manifest themselves through oscillations and noise in your circuit.

For example, in the picture below a function generator has been connected to a device under test (DUT) and to an oscilloscope. Ch 2 is used to observe the signal before it goes through the DUT and CH 1 shows the signal after the DUT. This is a typical setup and easy to implement using a BNC Tee.



• Figure E.11. Example of a ground loop.

Notice that the oscilloscope's ground is connected to point X through two different paths, A and B. Current could flow from the scope through A into X and back through B, causing oscillations. Disconnecting the ground clips from either cable A or B (but not both) would be the proper thing to do.

## E.7. Building Wiring Conventions

Finally, here are some closing comments regarding the building's wiring system and conventions.

The circuits treated so far were powered by two wires; usually one held at ground and the other providing the signal path. If that is the case why do then most (safe) power cords use three wires?

The answer to that question is related to the very small, but finite resistance of real wires which we have so far ignored. We were able to ignore this effect because the currents in the circuits in the lab exercises are on the order of milliamps. Therefore, a resistance of a few Ohms, which is typical, results in a negligible voltage drop. In instruments, appliances and especially in accidents, large currents up to 100 amps and more can flow before a fuse or a circuit breaker will interrupt the current flow. Under such circumstances a "small" resistance of 1 Ohm can quickly lead to a large voltage difference of 100 Volt or more between one end of the wire and the other. The assumption we have made so far, namely that all wires connected to a common point are at the same

potential, is no longer true in such a situation! Touching a device carrying large currents while coming in contact with one carrying little current could be lethal!



• Figure E.12. Main power with an electrical outlet (female connector) and wire color-coding conventions.

Therefore, a third wire, called "earth," or "ground," has been added to most power cords. One end is connected to the building's ground; the other makes contact with the casing of the instrument. Under normal circumstances no or very little current will flow through the ground wire. Therefore, even with a small resistance in the wire, the casing will remain very close to the ground potential. Meanwhile any current used by the device is supplied through the "hot" wire and returned through the "neutral" wire. Hot and neutral can carry large currents forcing to the actual device inside the casing to a voltage different from ground. Since the entire device is shielded by the case this should pose no danger to the user. are there for!

# Appendix Input and Output Impedance Concepts

## F.1. Introduction

#### F.1.1. Overview

The input and output impedance  $Z_{in}$  and  $Z_{out}$ , describes how a device is affected when it is connected to another device. Specifically, the devices discussed here are voltage sources such as power supplies, detectors and transducers connected to a load, for example, a scope, a voltmeter or an amplifier.

The input / output impedance is viewed under three different conditions. First, its effect on slowly varying, including DC signals, or steady state signals is considered and "ideal" input / output conditions are derived. Next, the optimal power transfer is explained. Finally, it ends by explaining what result the input / output impedance has on very fast or transient signals in a transmission line.

#### F.1.2. Mechanical Analogy

Input and output impedance is not limited to electronic circuits. It can exist in mechanical devices. Therefore, it may be helpful to start with a mechanical example.

As mentioned earlier,  $Z_{out}$  (and  $Z_{in}$ ) play important roles in power transfer when one device, for now called the "transmitter," is connected to a "load." See Figure 1 below.



Figure 1: Mechanical" transmitter" with a load.

The transmitter consists of a (infinitely powerful) motor or oscillator bolted to the floor and connected to a spring with stiffness  $k_{out}$ . The load consists of a spring with stiffness  $k_{load}$  attached to

a rigid wall. Our main interest is to observe the amplitude of the horizontal displacement, dx, between the two devices.

It should be clear that the relative stiffness of the two springs has a great impact on the displacement amplitude. For example, if the load spring is significantly stiffer than the  $k_{out}$  spring then the observed amplitude will vanish entirely. (If you are not convinced then consider the extreme case with the load spring being a rigid bar attached to the wall and the  $k_{out}$  spring a very weak, easily compressible spring.)

How does this simple mechanical analogy relate to electrical circuits? The motor represent a voltage (or current) source and the stiffness of the spring attached to it is inversely proportional to the device's output impedance, i.e.,  $Z_{out} \propto 1/k_{out}$ . (The stiffer the spring, the "lower" its impedance.) The displacement, dx, is the voltage detected at the load.

Studying how the load and the output impedance affects the signal allows to predict how much of a signal we are able to transmit to another device, what its optimal power match would be and how much of it might reflect back into the transmitter.

## F.2. DC and Slowly Varying Signals

#### F.2.1. Introduction

At first we will examine how the output and input impedance affects the amplitude of a signal transmitted for slowly varying signals, f < 1 MHz. (Note: At this point we are not interested in what input and output impedance will give us the best power transfer; this is a separate issue, and one which provides a different answer and which will be addressed in the next chapter.) Instead we approach this topic from an experimental viewpoint where the signal source is usually some sort of detector whose strength (amplitude) we want to measure accurately at the load without having to worry about the attenuation introduced either by  $Z_{out}$  or any load attached to the transmitter. Additionally, we might also ask ourselves, what makes an "ideal" transmitter?

#### F.2.2. Output Impedance: Z<sub>out</sub>

#### F.2.2.1. "Ideal" Transmitters

To answer the question what constitutes and "ideal" transmitter we once more use the mechanical analogy from the previous section. We ask ourselves, what type of spring constant, i.e.,  $k_{out}$ , would be "ideal" for such an "ideal" transmitter? By "ideal" we mean a transmitter whose output signal is the least affected by any other device attached to it and where the entire amplitude of the driver is directly applied to a load and not "wasted" by  $k_{out}$ .

Clearly, an infinitely stiff spring attached to the motor, or if you prefer a metal bar, satisfies this condition. As long as the motor is powerful enough, the displacement of any device attached to it will be identical to that of the motor. In electrical terms, this corresponds to  $Z_{out}$  being ideally zero or "small." If this condition is fulfilled, then any device attached to the voltage source will get the full voltage drop across its inputs.

At this point you may wonder what catch is and why we would spend any effort discussing voltage sources with finite or large output impedances or why anyone would use a non-ideal source. There are two situations when it is important to consider the (finite) output impedance: when dealing with "real" voltage sources (as opposed to "ideal" ones) and when building a voltage source with passive components, such as resistors.

#### F.2.2.2. Real Voltage Sources

Unfortunately, ideal voltage sources do not exist though a good (= expensive) power supply will come close to it. The limitation of most "real" sources is that they are not able to supply an infinite amount of power even over a short time to maintain a fixed voltage. A battery is a good illustration of this limitation.

Though the exact behavior of each real voltage source is complex, for mathematical purposes, one can approximate its behavior by thinking of the real device as composed of an "ideal" device (with infinite power) and with a finite output resistance. See the figure below.



Figure 2: 9 Volt battery and its circuit representation

If the device is attached to a "simple" resistor, as shown below, you can see how  $Z_{out}$  and  $R_{Load}$  form a voltage divider. All the power dissipated by  $Z_{out}$  is essentially "wasted" in the battery and never reaches the load.



*Figure 3: 9 Volt battery with load* 

#### Zout Example 1: Batteries

Most car batteries are nominally 12 VDC. Though stringing 8 AA batteries in series will also produce 12 VDC there is no way you will be able to substitute your heavy (and expensive) car battery with the (cheap) AA batteries if you want your car to start even in a balmy Minnesota summer day. Why is this the case since both configurations provide 12 VDC?

Certainly, to judge from its size and weight alone, a car battery can store more (chemical) energy and, therefore, provide power for an extended time. Additionally, we also have to consider the output impedance of the two types of batteries.

Some additional Information to analyze this problem: Typically, the starter on a car requires up to 100 Amps. In other words, at 12 VDC, you may think of the load resistance as  $V_{Load}/I_{Load} = 0.12 \Omega$ .

Lets apply our previous knowledge and assume that both types of batteries have identical (infinitely powerful) voltage sources, at least for a short time, (both at 12 VDC) but substantially different output impedances. Though we don't know what the output impedance of the car battery is, we know it must be substantially less than  $0.12 \Omega$ . Assuming that we want to receive at least 90% of the 100 Amps required, then  $Z_{out}$  must be  $0.012 \Omega$  or less.

What is the  $Z_{out}$  for the AA batteries? As you will measure in one of the lab exercises, it is on the order of a few ohms. Even if it were as low as 10  $\Omega$ , the batteries could at best only supply 0.12 Amp, far short of the required 100 Amps.

#### F.2.2.3. Real Voltage Sources: Transducers and Detector

Another category of "real" voltage sources that play an important role in physics experiments are transducers and detectors. They too can be modeled on voltage or current sources whose output changes with respect to physical conditions such as pressure, temperature, light, etc. Unfortunately, most of these sources not only provide small signals but their output impedance is extremely large! Usually we can not alter the actual physical process that produces these signal so we have to learn to live with these high output impedances and keep this very fact in mind, especially when dealing with input impedance!

#### F.2.2.4. Voltage Sources Created with Passive Elements

It has already been stated that a (good) power supply approaches the behavior of ideal voltage sources. Unfortunately, electronic components require various different supply voltage levels; +15, +12, +5, -12, -15 VDC are some of the more typical ones. When using a large number of electronic components, it is therefore unavoidable that many different supply voltages will be required. Instead of employing a large collection of power supplies, each set to a specific supply voltage, most circuits are usually driven at most by only two power supplies: one providing the largest positive and, if required, another the most negative voltage. Any voltage level in between can then be obtained using a voltage divider built from a pair of resistors, (passive components.) (More fancy solutions do exist but are too complicated to discuss here.) As you will see in the following example, considering  $Z_{out}$  for this type of voltage source will be very important.

#### Zout Example 2: Resistive Voltage Divider

Assume that you are designing a circuit that needs two different supply voltages (in addition to ground) to power its devices, let's say +20 VDC, +10 VDC and that you want to use only one, though very good, power supply which for all practical purposes has a  $Z_{out}$  = 0.

Since it is very difficult to step up (increase) a DC voltage, your only choice is to set the power supply to +20 VDC and then to use a (resistive) voltage divider to obtain the required +10 VDC. Since  $Z_{out} = 0$ , we will ignore the output impedance of the actual voltage source and assume that  $V_2$  always remains at +20 VDC.



Figure 4: Voltage source created from resistive components

Close inspection will reveal that, without any load,  $V_1 = +10$  VDC for  $R_1 = R_2$ . Applying the voltage divider equation, it appears that the actual values of  $R_1$  and  $R_2$  are unimportant as long the two resistors are identical.

Therefore, let's first study this circuit by selecting large resistors, arbitrarily chosen at  $R_1 = R_2 = 100$  k $\Omega$ . It should come as no surprise that when you connect a "good" voltmeter to this circuit you should read  $V_1 = 10.0$  VDC.

This exercise, as it has been stated, is still incomplete. Since we are building a primitive "power supply" we also want to connect some load to it, or what's the point? Arbitrarily, for illustration, we use a "medium" load, 1 k $\Omega$  resistor which would draw 10 mA of current at the expected 10 VDC. (In reality we would like to specify the maximum current drawn from this voltage divider.) Measuring  $V_1$  again with the 1 k $\Omega$  load shows that it has now dropped to about 0.2 V, far from the 10 V required. (Please confirm these numbers for yourself!)

What happened? Our simple initial calculation neglected to take the  $Z_{out}$  of  $V_1$  into consideration. (Note: if  $Z_{out}$  of  $V_s$  were not 0 we would have to take that into consideration too!) For now, let's state that the output impedance of  $V_1$ ,  $Z_{out1}$ , is  $R_1$  and  $R_2$  in parallel, i.e.,  $R_1R_2/(R_1+R_2)$  with  $V_{S1} = V_1$ . (For a more detailed analysis see the section following this one.) In our case,  $Z_{out1} = R_1/2 = 50 \text{ k}\Omega$ , a rather large value. In other words,  $V_1$  is the result of the voltage divider formed by  $Z_{out1}$  and  $R_{Load}$ ,  $V_1 = V_s Z_{out1}/(Z_{out1} + R_{Load}) = ~ 0.2V$ . By choosing large values for  $R_1$  and  $R_2$  we created a voltage divider that is easily affected by  $R_{Load}$ . So how do we fix this problem?



Figure 5: The voltage source created from resistive components and its equivalent circuit

Since we started with  $R_1$  and  $R_2$  being large, we chose in our second approach small values, say 100  $\Omega$ . Applying the load resistor used previously we find that  $V_1$  with a load = 9.5 V. That's not

great but it is within 5% of the requested 10.0V, i.e., within the accuracy of typical electronic components.

Continuing along the same path of reasoning, one might wonder what prevents us from selecting even smaller values for  $R_1$  and  $R_2$ . Doing so would decrease  $Z_{out1}$  even more and make the voltage divider even "stiffer". Before we proceeding, let's quickly calculate the power being dissipated, i.e., wasted, by  $R_1$  and  $R_2$  when no load is connected. (The current flowing through the voltage divider resistors without any load attached is often referred to as the **quiescent** current, i.e., the current that flows when all is "quiet.") Using two100  $\Omega$  resistors corresponds to "wasting". 25W in each resistor, or a total 0.5 W. On the other hand, if we were to decrease  $R_1$  and  $R_2$  to 10  $\Omega$ s, we get a far stiffer voltage divider;  $V_1$  with the 1 k $\Omega$  load would be 9.95V, but the power being wasted without any load attached amounts to 10 W in each resistor! At that point, even if the power supply can handle that, you are starting to create a small electric heater instead of an electric circuit. (If those figures do not mean anything, consider how hot a 40 W light bulb gets as it burns twice the amount of power as our 10  $\Omega$  circuit.)

<u>Conclusion</u>: generally, we would like to have the smallest possible output resistance. Nevertheless, when designing circuits, especially when using only resistive elements we have no choice but find a compromise between how low we want the  $Z_{out}$  to make and how much power we are willing to waste. Once you learn about active components, transistors op-amps and ICs in general, you will learn of different ways to create a low output impedance without having to waste quite as much power though as a general rule, you will always be faced with this dilemma.

A final comment before concluding this section: if you read the analysis carefully you may have wondered why we do not use an entirely different approach. Since we already (arbitrarily) selected a 1 k $\Omega$  load, we could solve the voltage divider equations more carefully for  $R_1$ ,  $R_2$  and  $R_{Load}$  so that  $V_1$  is exactly 10 V. As you can easily prove to yourself, in such a case  $R_2 = R_{Load} R_1/(R_{Load} + R_1)$  would fulfill this condition. We did not use this approach because  $V_1$  without any load attached could greatly deviate from its "nominal" value. For example, if we keep  $R_2 = 100$  k, and  $R_1 = 909$   $\Omega$ , then only with a 1 k $\Omega$ ,  $V_1 = 10$ V but without a load it would be at 18.3 V.

Since loads attached to a circuit usually vary, the choice between this approach and the former one is whether we want  $V_1$  to exceed or to fall short of the nominal value. Since devices are more easily damaged if the voltage is exceeded then falls short of it, we stay with our original approach. Also, once you have managed the concept of output impedance, you will get a pretty good feeling what type of loads you can "safely" attach to a device. Just keep the simple fact in mind that when your load is identical to the output impedance, the "nominal" voltage will drop by half!

#### Zout Calculations for the Previous Example

When replacing circuit 2a with the equivalent one, circuit 2b, how do we determine  $V_{S1}$  and  $Z_{out1}$ ? A (correct) approach would be to find circuit 2a's Thevenin equivalent but for completeness, we determine it here by first principle.

What do we mean by the circuits "being equivalent"? Essentially two conditions must hold: 1)  $V_1$  for circuit 2a must be identical to  $V_1$  for circuit 2b when no load is attached. 2)  $V_1$  for circuit 2a must be identical to  $V_1$  for circuit 2b when identical loads are attached to each circuit. Note, condition 2 must hold for any arbitrary load!

Let's start with condition 1, the open circuit voltage:  $V_{1open}$  for circuit 2a is the product of the voltage divider formed by  $R_1$  and  $R_2$ , i.e.,  $V_{1open} = V_{s2} R_1/(R_1 + R_2)$ .

 $V_{1open}$  for circuit 2b is simply  $V_{s1}$ . From this:  $V_{s1} = V_{s2} R_1 / (R_1 + R_2)$  Next we consider condition 2, the voltage with a load, *R*<sub>Load</sub>, attached:

For circuit 2a:  $V_{1Load} = V_{s2} R_1 / |R_{Load} / (R_1 / |R_{Load} + R_2)$  where ' $R_1 / |R_{Load}$ ' means  $R_1$  in parallel with  $R_{Load}$ . For circuit 2b:  $V_{1Load} = V_{s1} R_{Load} / (R_{Load} + Z_{out1})$ 

Combining these last two equations leads to:  $V_{s2} R_1 / R_{Load} / (R_1 / R_{Load} + R_2) = V_{s1} R_{Load} / (R_{Load} + Z_{out1})$ 

Finally combining this equation with the last one from condition 1 and solving for  $Z_{out1}$  yields after some tedious algebra:

 $Z_{out1} = R_1 R_2 / (R_1 + R_1)$ 

#### F.2.3. Input Impedance: Zin

#### F.2.3.1. Introduction

 $Z_{in}$  is the "inverse" of the  $Z_{out}$  concept. If our main concern is to observe the largest possible signal amplitude then we would like the input impedance to be as large as possible. Going back to our mechanical analogy, it represents the stiffness of the spring that is connected to some existing system. You may think of this spring as the "recorder" observing the motion of a system we are monitoring. Clearly, if the monitored system is very weak, (it already exhibits a large output impedance) or if it exhibits very minute oscillations, hooking it to a stiff spring might entirely alter or even destroy the behavior we want to observe. For this reason, we want to connect it to a spring that extremely floppy so that the original system is perturbed as little as possible. Again, we are not interested in the optimal power transfer from the output device to the input but what we want is that the original signal from the source reaches the receiver with no attenuation.

#### F.2.3.2. Transducers, Detectors and Followers

As already mentioned, transducers are notorious for their large output impedance. From an engineering viewpoint, it is often impossible to reduce the output impedance significantly by changing the physical characteristics of the device without sacrificing sensitivity. Therefore, such devices are usually directly connected to an amplifier. Contrary to what one might think, these amplifiers provide very little or no voltage gain. (An amplifier with a voltage gain of 1 is called a "follower.") Instead their purpose is to provide an **impedance change**. They output the original input while providing a very low  $Z_{out}$ . For now we skip on how this is accomplished but generally it involves active components such as transistors or op-amps. From an energy conservation viewpoint, one can see that such a device will only work if it receives some (additional) external power.



Figure 6: Follower circuit: The external power inputs that are required to power  $V_s$  are not shown.

We have now specified the ideal output impedance of a follower but what should its input impedance be? Since  $V_{out} = V_{in}$ , we want  $V_{in}$  to be as large as possible. Similarly, we don't want to perturb, load down, or attenuate  $V_{in}$ . Therefore,  $Z_{in}$  should be as large as possible.

#### F.2.3.2.1. Example

The circuit below is deceivingly simple consisting only of an (ideal) voltage source and a resistor.



Figure 7:

Without any calculations, you should see that  $V_{out}$  (without any load) attached should be 10 V. Nevertheless, when you actually measure it with a decent digital DVM you will observe a value that is far below the 10 V; depending on the actual DVM used in the lab, it will indicate 5 volts or less. So what's going on?

When using a (good) measuring device, it is tempting to assume that its input impedance is infinite so that it does not perturb the output signal. Digital voltmeters and oscilloscopes have finite input impedances in the 1 to 10 M $\Omega$  range. Most of the times, we get away ignoring the effects that such a large input impedance has on the measurements, which is exactly why the desired input impedance was chosen to be large. Nevertheless, there are situations, this example being one of them, where we have to consider the effect the input impedance has on the measurements. This is certainly the case when the device's output impedance is comparable to the input impedance, as in this example. Assuming that the DVM's input impedance is 10 M $\Omega$ , then the measured signal will be half of  $V_s$ .



Figure 8: Same circuit as figure 7 but with a Digital Volt Meter (DVM) attached.

Another word of caution: the input impedance for a measurement device is usually not constant and can vary when the input range (sensitivity) of the device is changed! This may explain (sometimes) why you obtain different readings when switching between different range scales!

## F.3. Power Transfer

In the previous sections, our intention was to observe the largest possible voltage signal and we concluded that under ideal conditions our "transmitter" should have a  $Z_{out}$  close to 0. Though we approached the problem only in terms of voltage, it still would hold if were to calculate the power transmitted to the "receiver" or load. In such a case, see the circuit below:  $P_{Load} = I_{Load} V_{Load} = I_{Load} V_{Load}$ 

 $V_s^2/R_{Load}$ , i.e., all the power from the source gets absorbed by the load, regardless of the size of the load.



Figure 9: Source with  $Z_{out} = 0$ . Though  $Z_{out}$  is 0, it has been drawn for completeness; thas really no effect on the circuit.

In reality, a  $Z_{out} = 0$  is something to be wished for but usually is not achievable. Especially when working with AC or high frequency (RF) signals the inductance and capacitance of the cables and connectors will add a finite  $Z_{out}$  to any driver, even an "ideal" one. Under such circumstances, i.e., with a finite  $Z_{out}$ , what is the "optimal" power transfer that can be achieved between the transmitter and the load?



Figure 10: Power transfer with a finite source impedance.

From the circuit above, you can see that the power in the resistive load is:  $P_{Load} = V_s^2 R_{Load}/(R_{Load} + Z_{out})^2$ . Differentiating this with respect to  $R_{Load}$  and setting the result to zero shows that the optimal power transfer is achieved when  $R_{Load} = Z_{out}$ . In such a case, half of the power is "wasted" in the output device and half is transmitted. Any other combination will result in a decrease of the power transmitted to the load.

To recapitulate, ideally we want a device's output impedance to be as low as possible; if  $Z_{out}$  is finite, then the optimal power transfer is achieved if  $Z_{out} = R_{Load}$ .

If you paid close attention, we seemed to have arrived at a contradiction. Borrowing terminology from the previous section, we can consider  $R_{Load}$  as the  $Z_{in}$  of the "receiver." If this is the case, then aren't we saying that in the ideal situation (for power transfer)  $Z_{in} = 0$ , which is exactly the opposite of the conclusion in the previous chapter?

Not if you keep the following fact in mind: in the previous chapter we were not interested in transferring power. Instead, our aim was to "sense" or measure the voltage signal without affecting the signal source, usually a transducer of some sort. For example, in our mechanical analog, we connected another spring as a monitor to our "transmitter" without perturbing the original system. When measuring something, we want the "monitor" to remove as little power from the system as possible, so our argument still holds.

Yet there are some good reasons (as you will see in the next chapter) that some devices have low input impedances. Such devices are only used with others that have identical, i.e., " matched," input an output impedances - an impedance matching value of 50  $\Omega$  is probably the most common.

Therefore, you should be careful when using such devices and never mix them with devices that are not matched. Also keep in mind that for such matched devices, the input signal is always half of the source signal. To confuse things further, some devices, like our HP function generator, will compensate for this by sending out twice the nominal signal when it is set up for matching 50  $\Omega$  impedances! (Generally, we want to disable this setting and use its "High Z" output setting. For more information see: http://mxp.physics.umn.edu/f99/hp%2033120a%20setup.htm)

## F.4. Pulses and Terminating Transmission Lines

So far we have only considered DC or steady state signals, which covers 99% of what you will work with in the MXP lab. Input and output impedance considerations do get a bit more complicated when dealing with transients or short pulses as, for example, are very common in particle experiments.

We use again a mechanical analogy to illustrate this effect. You should be familiar with the mechanical concept of a displacement pulse traveling down a piece of rope. As you may recall, if the end of the rope is rigidly held down, the pulse will be reflected back at the end of rope 180 degrees out phase. If the end is not held down, then the pulse will be returned in phase.

This analogy holds for electrical signals as well. A short voltage pulse will travel down a cable (or transmission line) and then may reflect at the end of the cable. If the end is an open circuit, the reflected signal will be in phase with the original signal and the reflected pulse will maintain the original amplitude. In contradiction to the "slow" signal case, short-circuiting the end of the cable will still reflect the pulse, i.e., it will have the original amplitude, but it is now 180 degrees out of phase.

This behavior can have troublesome consequences. Consider this example: a particle passes through a detector (typically a scintillator attached to a photomuliplier tube) creating a very short (on the order of a few nanosecond) voltage pulse that is then sent through a cable to a counter. If the input impedance of the counter is infinity, the pulse will be reflected back to the photomuliplier, where depending on its output impedance, it might very well be reflected back into the counter, resulting in erroneous counts. This is a serious problem and we need therefore something to destroy, absorb or "terminate" the pulse once it arrives at the input of the counter.



Detector / Source

Receiver / Counter, Scope etc.

Figure 11: Pulse transmitted from source to a detector. As long as  $Z_{out}$  is not identical to  $R_{Load}$ , pulses will be reflected.

Since reflections are caused whenever a pulse encounters a change in impedance, one solution to this problem is to send the pulse (once it has been received by the counter) down and onto an infinitely long cable from which it never will return. Another, more practical one, is to determine the "**characteristic impedance**,"  $Z_o$ , of such an infinite long cable and then to construct an electronic device that has the same electrical properties as the "infinite" cable. When this device is then attached to the finite cable, it will appear to the pulse as an infinitely long cable and it will annihilate the pulse completely. Note: the characteristic impedance is not the DC resistance of the cable with which we are familiar but the impedance of the cable at a particular frequency based on its resistance, capacitance and inductance:

$$Z_{o} = \frac{V_{x \to \infty}}{I_{x \to \infty}} = \sqrt{\frac{R + \omega L}{G + \omega C}}$$

Where *R* represents the resistance along the cable, *G* the conductance between the two conductors, *C* and *L* the capacitance or inductance respectively. (All quantities are divided by unit length.)

At high frequency, i.e., for very short duration pulses, the characteristic impedance,  $Z_o$ , will approach:

$$Z_o = \sqrt{\frac{L}{C}}$$

i.e., it will become purely resistive. Therefore, the electronic gadget which we mentioned previously that will have the same characteristic as an infinitely long cable, becomes simply (matching) resistor. Below is a table with the characteristic impedances of the most frequently used cables:

Name	Characteristic Impedance (Z <sub>o</sub> )	Type & Usage
RG-58	50 Ω	Typical coaxial cable used, for example, in the lab
RG-59	75 Ω	Coaxial cable used in some High Energy Applications
10BaseT	100 Ω	Twisted (2 wire) pairs in computer networks
Twinlead	300 Ω	Parallel 2 wires used in FM antennas

You might wonder how attaching a 50  $\Omega$  terminating resistor, a "terminator," to an RG-58 fulfils our initial discussion of an ideal device having an infinite input impedance, 50  $\Omega$  being anything but a small impedance. Note that such terminators are only used either when short pulses are present (in which case the output is most of the time at ground anyhow) or when very high frequency signals are used in which case the output is usually matched for an optimal power transfer, with  $Z_{out} = Z_{in} = Z_o = 50 \Omega$ .



Figure: Picture of a BNC 50  $\Omega$ Terminator

## F.5. Terminology: Practical Resistor Values

For all practical purposes, what are considered a low or large resistance in circuits?

Resistances below 10  $\Omega$  are for all practical purposes treated as short circuits, (short) wires, or 0  $\Omega$ . 50 to 100  $\Omega$  resistors are very low resistors and you should be careful when using these because they absorb so much power, they can burn up, literally!

On the other end of the spectrum, resistances above 10 M $\Omega$  are considered infinite resistance. Though you might find resistors that are larger than 10 M $\Omega$ , grease and dirt from your hands, moisture in the air can easily reduce their effective resistance. This is why people tend to avoid these whenever possible.

In conclusion, most circuit designs rely on resistors values between 100  $\Omega$  to 1 M $\Omega$ .

#### F.6. Problems

**F.6.1.** Design a (resistive) power supply that will provide the following output voltages:  $V_3 = +15$ ,  $V_2 + 12$ ,  $V_1 = 5V$ . (See the figure below.) Use an "ideal" power supply with  $Z_{out} = 0$  and use it for the Vs = +15V output.

F.6.1.1. Select values for  $R_1$ ,  $R_2$  and  $R_3$  to obtain above specified output voltages. Furthermore, choose them so that the quiescent power from all the resistors is 1 Watt.

F.6.1.2. Calculate  $Z_{out1}$  and  $Z_{out2}$ , i.e., the output impedances at  $V_1$  and at  $V_2$ . Assume (as usual) that no load is attached.

F.6.1.3. The design below has one shortcoming: any load applied to  $V_1$  (or  $V_2$ ) will affect  $V_2$  (or  $V_1$ .) To decouple the two outputs we try a different setup consisting of two voltage dividers connected in parallel to an (ideal) power supply. Additionally, we specify that the output impedances of the two voltage dividers are identical, i.e.,  $Z_{out1} = Z_{out2}$ . Calculate the values for the 4 resistors required for this new setup that satisfy all the conditions specified above. Also calculate  $Z_{out1}$  and  $Z_{out2}$ .



**F.6.2.** You are told that a (high impedance) detector's output voltage for a given condition will be about a volt but you observe only a 0.1 V signal on an oscilloscope whose input impedance is 1 M $\Omega$ . When you then disconnect the detector and attach it to a digital volt meter (DVM) with an input impedance of 10 M $\Omega$  the DVM displays 0.5 Volt reading.

F.6.2.1. What is the detector's voltage signal when no load is attached and what is its output impedance?

**F.6.3.** The earths (vertical) electric field is about 100 V / m near sea level. (See also problem 4). We would like to measure it directly with a DVM to study how it changes with weather conditions and as function of day and night. We build a detector consisting of two 1 m<sup>2</sup> metal sheets mounted vertically 1 m apart, held in place with "perfect" insulators. We then attach the sheets to a very good DVM with  $Z_{in}$  = 10 MΩ. The output impedance of the "detector" will be comparable to the resistance of air, which is  $4 \times 10^{13} \Omega$ .

F.6.3.1. Draw the circuit, including the DVM, and indicate the voltages and resistances.

F.6.3.2. What voltage will be displayed by the DVM?

F.6.3.3. Assume that our "perfect" mounts are real and have a resistance of 1 G $\Omega$ . How is the answer in 6.3.2. affected, if at all? (Draw a new diagram.)

**F.6.4.** You have bought the most powerful sound system on the market but find that the lights in your house dim each time you turn it on. You want to measure how much electrical power the system uses and find that with the sound system on, the household voltage drops from 110 VAC to 100 VAC. From your physics class, you remember that a typical output impedance of a household circuit is 1 Ohm.

F.6.4.1. What is the input impedance of your sound system?

F.6.4.2. How much current does it draw?

F.6.4.3. What is its power consumption?

F.6.4.4. What is the maximum current that you could (theoretically) draw from the given output impedance. (Note: "typical" households are rated 100 Amps.)

**F.6.5.** A person finds himself shipwrecked on a tiny, uninhabited island with an advanced game boy, tin cans and an introductory physics E&M textbook. After the batteries to the game boy run out, having no other diversions, he reads the E&M book and learns to his maddening delight that the earths (vertical) electric field near sea level is about 100 V / m. After removing the two dead AA batteries and vertically mounting two flattened tin cans about 3 cm apart on well isolated supports, he then connects two metal strips between the tin cans and to the battery terminals. Frustrated by the lack of success, he tosses the E&M book in the ocean but hangs on to the game boy. What the E&M book neglected to state was that such a system (i.e., air) has a resistivity,  $\rho = 4 \times 10^{13}$  Ohm m. (See the Handbook of Physics and Chemistry.)

F.6.5.1. Assume a realistic value for the surface area of the flattened cans and calculate the  $Z_o$  of this system. (Remember that:  $R = \rho L / A$ , where L is the distance between the cans and A the top (or bottom) surface area.)

F.6.5.2. What's the maximum power to be drawn from this tin can system?

F.6.5.3. What surface area is needed if one wants to get at least 1 mW out of the system, the bare minimum power requirement for some low power chips? Assume that the electric filed does not change as a function of the large number of tin cans required.