Module 2 – GEARS

Lecture 3 - INVOLUTE SPUR GEARS

Contents

3.1 INTRODUCTION

In lecture 2 various terminologies used in describing a gear and a gear pair in mesh was

dealt in detail. For convenience, the gear nomenclature is reproduced in Fig. 3.1. It can be
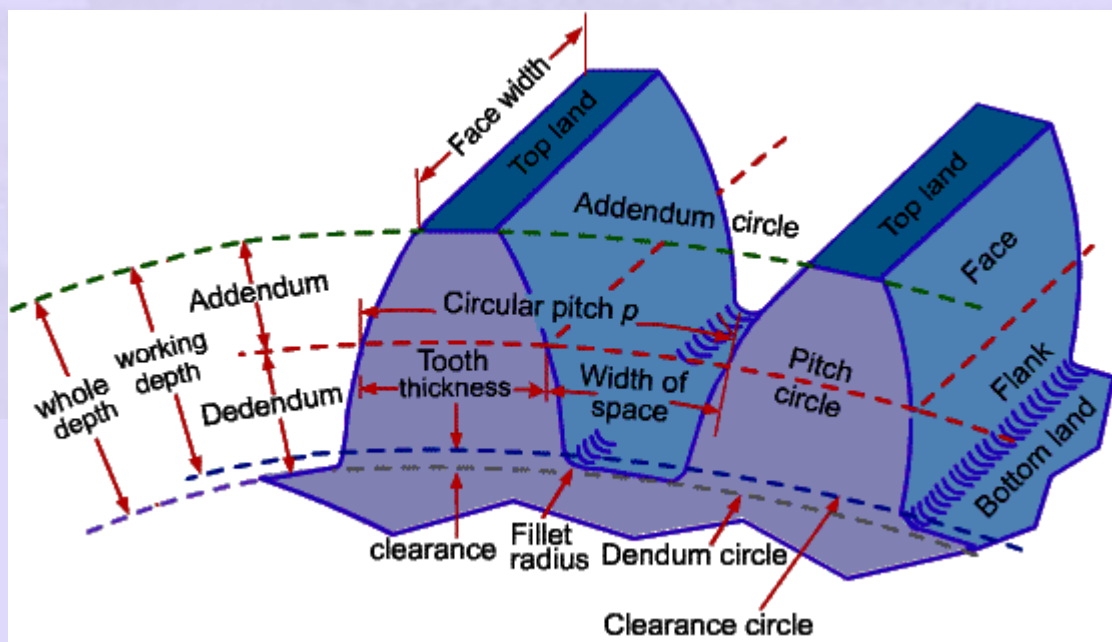

Fig. 3.1 Spur gear nomenclature

observed here that by varying the various parameters, it is possible to get infinite varieties of gears. In practice if we use such large varieties of gears, then the manufacture, quality check, replacement in the case of failure all become more complicated. To overcome this, standard gear systems are evolved and these are dealt in detail here. Earlier the definition of interference in gears was illustrated by many figures and methods of avoiding interference were dealt in detail. For easy recollection of interference, refer to Fig. 3.2 where the interference portion of the teeth is shown in red.
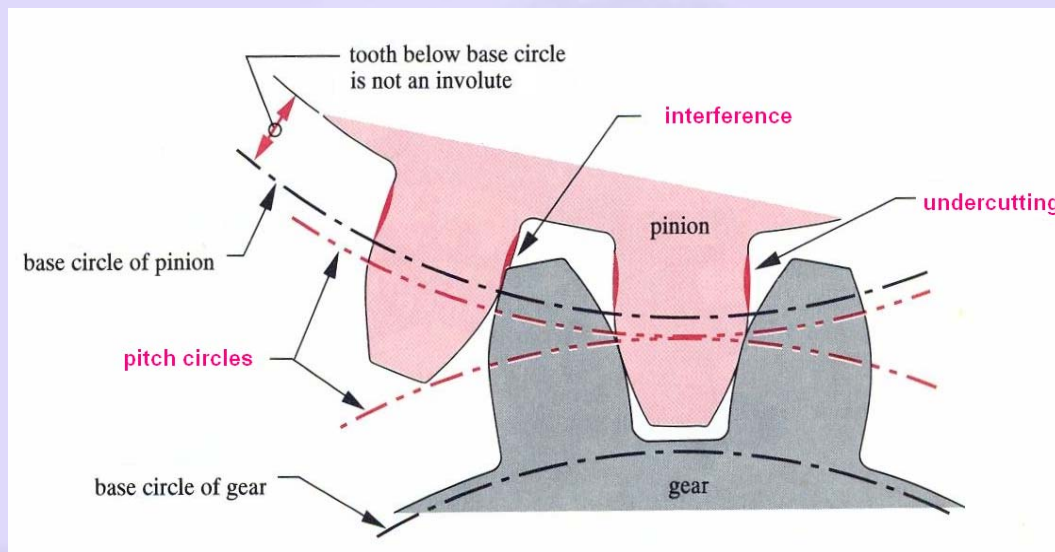
Fig. 3.2 Interference in gears

3.2. STANDARD TOOTH SYSTEMS FOR SPUR GEARS

To reduce the varieties of gears to a manageable numbers, standards are evolved. Standard makes it easy for design, production, quality assurance, replacement etc. Three commonly used pressure angles are 14.5^o, 20° and 25° pressure angle systems as shown in Fig. 3.3. In this, one can have full depth gears or stronger stub tooth gears. In Standard tooth system for metric gears, addendum: a =1m, dedendum: b= 1.25m where as the for the stub tooth gears, addendum a = 0.8m and dedendum: b= 1.0m. The shorter tooth makes it stronger and its load carrying capacity increases. It also helps in avoiding interference in certain cases.
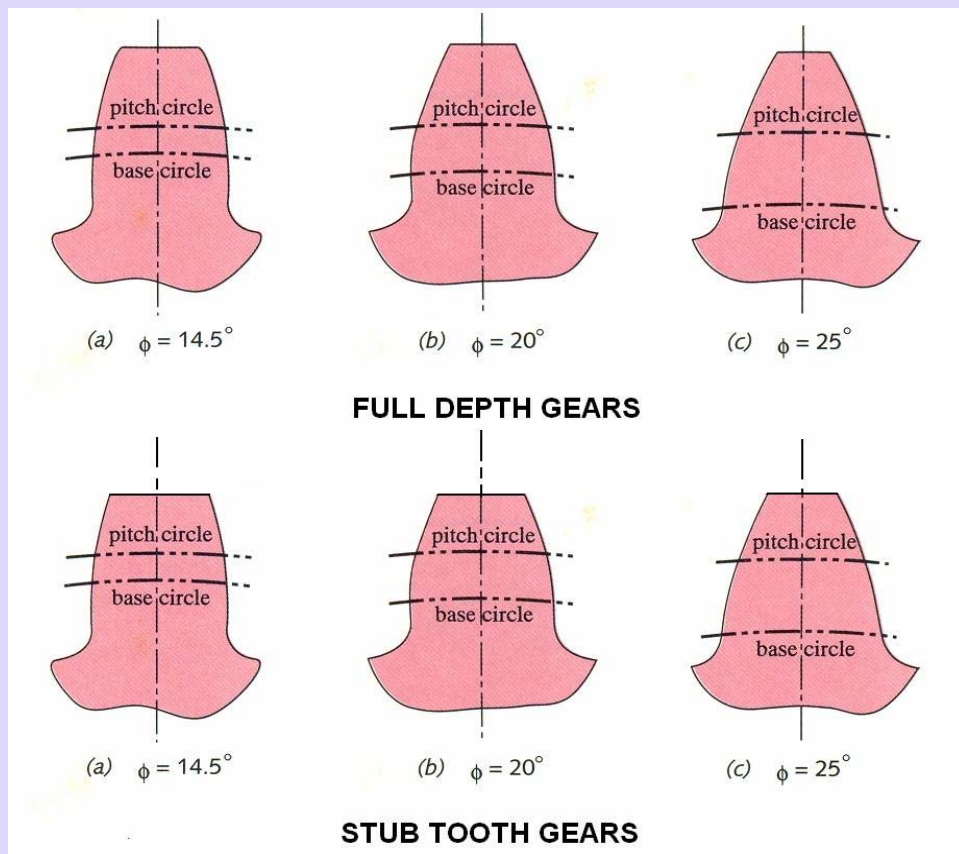
Fig. 3.3 Standard tooth systems

Table 3.1 Standard tooth system for spur gears

Item	20 degree full depth	20 degree Stub	25 degree full depth
Addendum a	1m	0.8m	1m
Dedendum b	1.25 m	1m	1.25m
Clearance c	0.25 m	0.2 m	0.25m
Working dept	2m	1.6m	2m
Whole depth h	2.25m	1.8m	2.25m
Tooth thickness t	1.571m	1.571m	1.571m
Face width f	9m-14m	9m-14m	9m-14m
Fillet radius min.	0.3 m	0.3 m	0.3 m
Top land min.	0.25m	0.25m	0.25m
14.5^o and 25° stub tooth systems have become obsolete now.			

How the various spur gear tooth parameters are fixed in standard gear system is given in Table 3.1

3.3 PROFILE SHIFTED GEARS

Interference in smaller number of tooth pinions can be avoided by having unequal addendum, longer addendum for the pinion and shorter for the gear as seen in the earlier figure. These are called profile shifted gears or non-standard gears (Fig. 3.4). AGMA defines the addendum modification coefficient as x_1 and x_2 which sums up to zero, being equal in magnitude and opposite in sign.
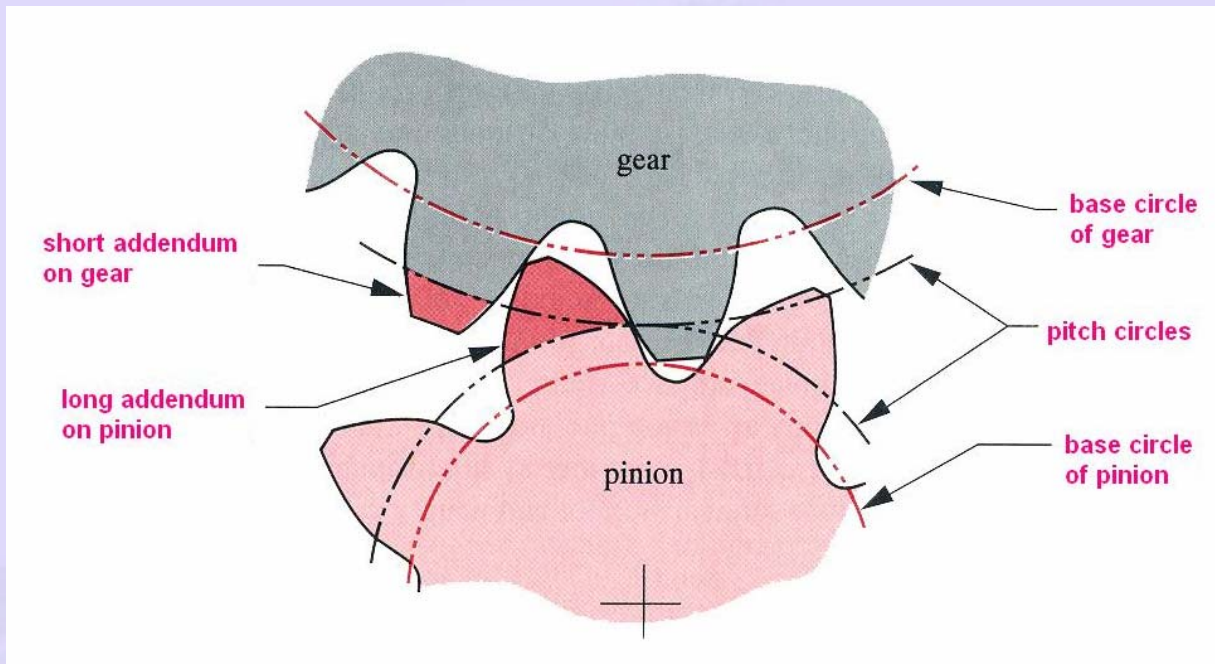
Fig. 3.4 Non Standard gears

+ve coefficient applied to pinion increases the addendum and –ve coefficient applied to gear decreases the addendum. The net effect is to shift the pitch circles away from the base circle of the pinion and eliminate that non-involute portion of pinion tooth below the base circle.
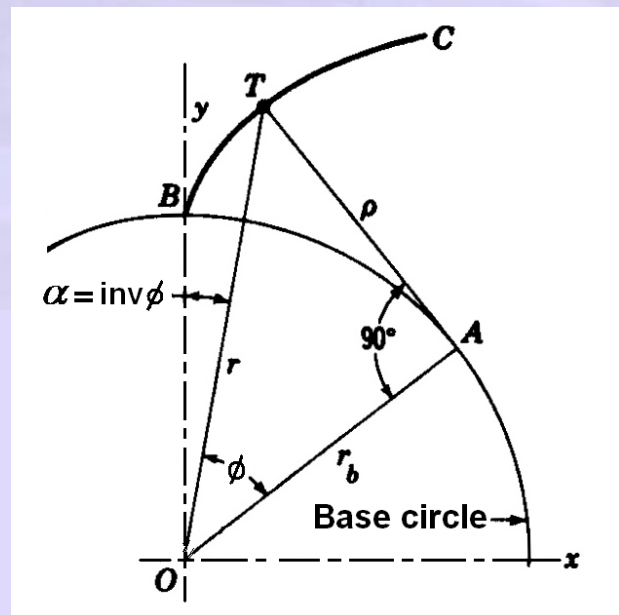
The standard coefficients are ±0.25 and ±0.50 which add and subtract 25% and 50% of the standard addendum, respectively. The limit approach occurs when the pinion tooth becomes pointed. How the tooth appears in profile shifted gears is shown in Fig.3.5.
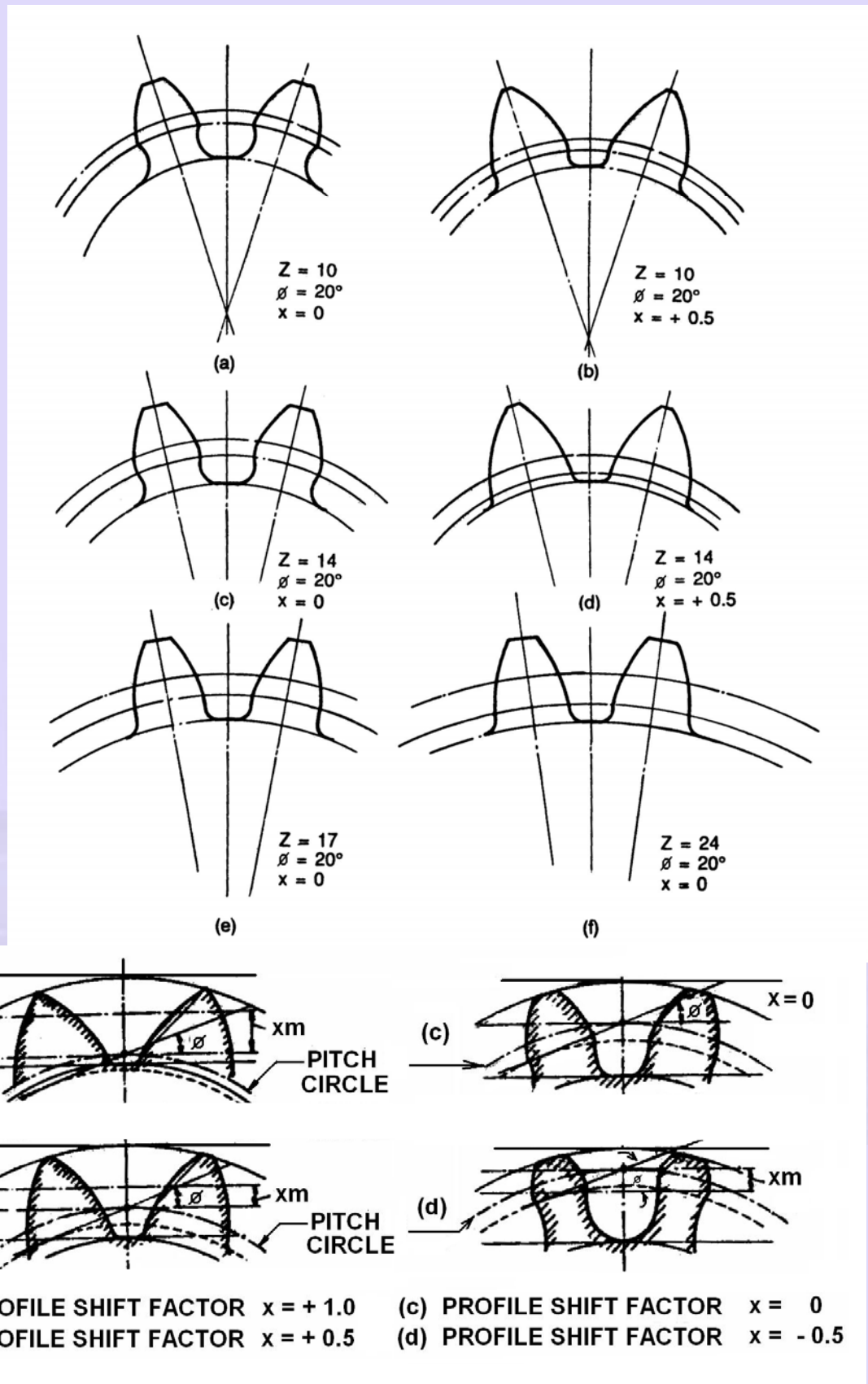
Fig. 3.5 Influence of Profile shift on the shape of gear teeth
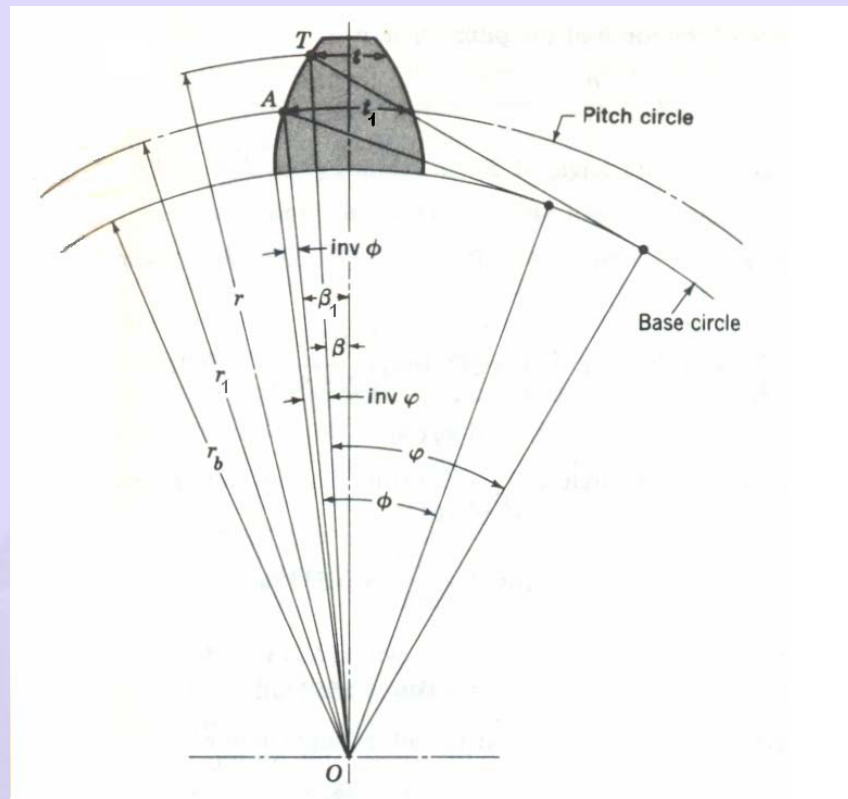
3.3.1 WHAT PROFILE SHIFTING DOES TO GEARS

1. The pinion becomes thicker at the base and thus stronger

2. The gear tooth correspondingly weakens since full depth gear tooth is stronger than full depth pinion tooth, thus equalizes the strength.

3. The unequal addendum tooth forms increase the sliding velocity at the tooth tip.

4. Consequently tooth surface stresses increase.

5. The friction losses in the gear mesh also increase at high sliding velocities.

3.3.2 WHEN PROFILE SHIFTING IS USED

1. When the interference is to be avoided

2. When predetermined centre distance has to be attained

3. To increase the strength at the root and flank of the teeth.

4. To improve sliding and contact relation.

5. To shift the beginning of the effective profile away from the base circle.

3.4 INVOLUTOMETRY

Involutometry is the study of involute geometry. In Fig. 3.6, at T, the generating line length, $\rho = r_b \tan\phi$, also $\rho = r_b(\alpha + \phi)$
From the above relationship, $\alpha = \tan\phi - \phi$
Which is also written as $inv\,\phi = \tan\phi - \phi$
From the Fig.3.6 ,

$$r = \frac{r_b}{\cos\phi} \qquad (3.1)$$

Fig. 3.6 Illustration for involutometry

3.4.1 TOOTH THICKNESS AT ANY POINT ON THE TOOTH PROFILE

Fig. 3.7 Tooth layout for finding tooth thickness at any point

In order to derive the relationship between tooth thickness and it distance r from the centre refer to the Fig. 3.7. The half tooth thickness at A and T are given by:

$$\frac{t_1}{2} = \beta_1 r_1 \qquad (3.2), \qquad \frac{t}{2} = \beta r \qquad (3.3)$$

So that

$$\beta_1 = \frac{t_1}{2r_1} \qquad (3.4), \qquad \beta = \frac{t}{2r} \qquad (3.5)$$
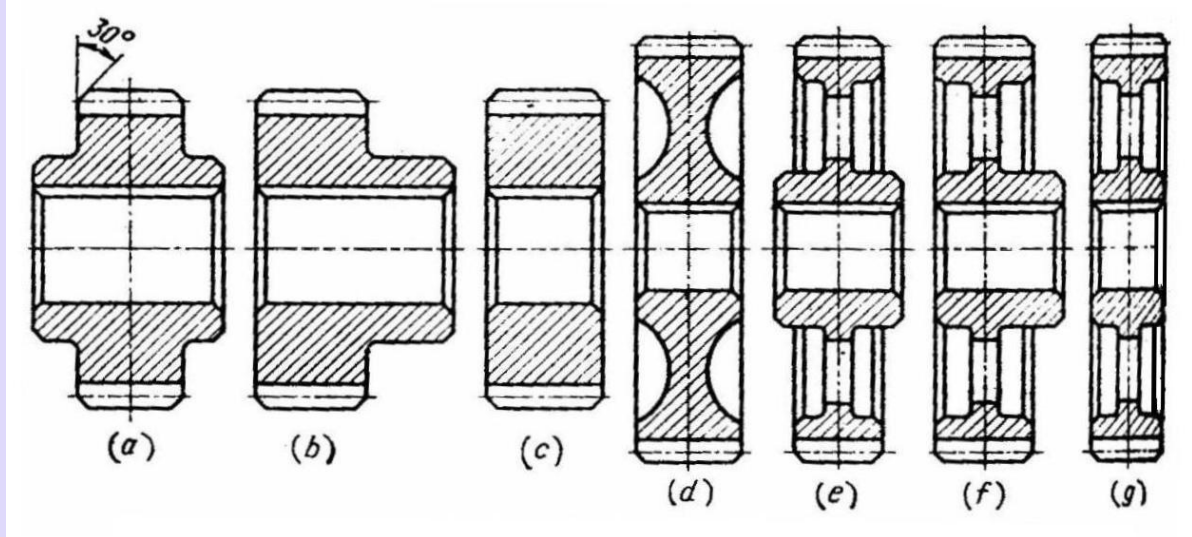
Now we can write

$$inv\varphi - inv\phi = \beta_1 - \beta = \frac{t_1}{2r_1} - \frac{t}{2r} \qquad (3.6)$$

Rearranging the terms we get

$$t = 2r\left(\frac{t_1}{2r_1} + inv\phi - inv\varphi\right) \qquad (3.7)$$

3.5 DESIGN OF GEAR BLANKS

Fig. 3.8 Design of gear blanks for small diameter gears d < 200 mm

The design of gear blank depends on the size, load carrying capacity, speed of operation, space limitations and application. Small gears up to a pitch diameter of 200 mm are normally made of solid blanks as in Fig. 3.8, (a) to (c). However in these gears, sometimes to reduce the weight and inertia in higher sizes, material is removed in the web portion where the stress is low by turning process Fig. 3.8 (d) or turning and drilling process Fig.3.8 (e) to (g) especially for high speed operation. In multi-speed gear boxes to make the arrangement compact, cluster and sliding gears as shown in Fig. 3.9 (a) to (e) are used. The gaps between the gears should be adequate to relieve the gear cutter.
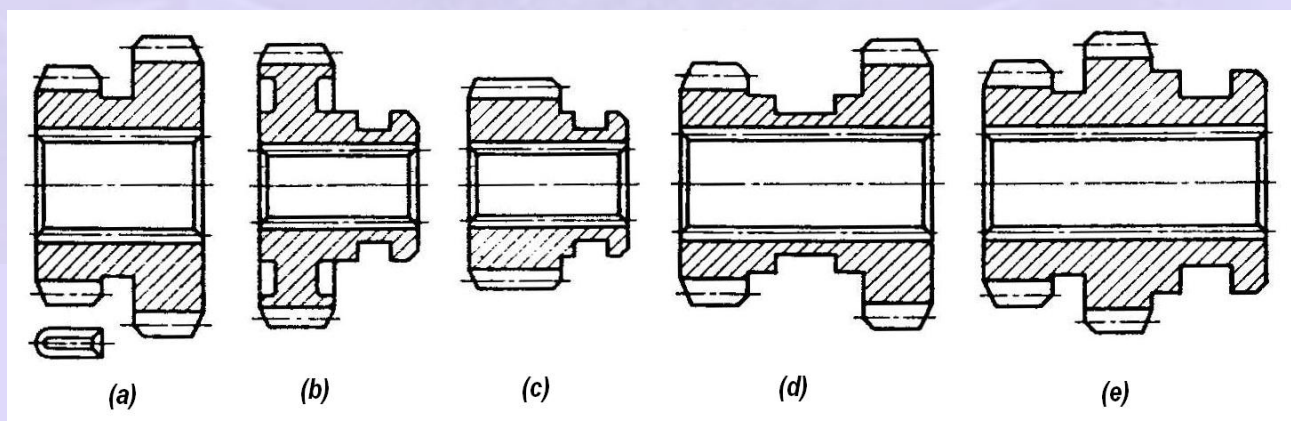

Fig.3.9 Cluster and sliding gears

Many a times, the tool relief does not result in compact drive. Hence, to further make it compact, glue jointed or shrink fitted composite gears shown in Fig. 3.10 (a) and (b) are often used.
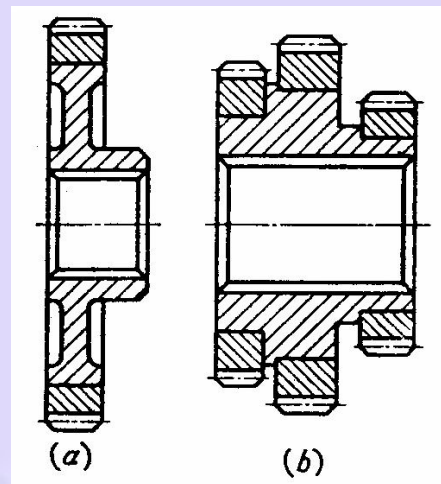
**Fig. 3.10 Glue-jointed / shrink fitted composite gears**

In the case of medium sized gears normally forging process is used. The wheels are made solid or cored as shown in Fig. 3.11 (a). Cast wheel with crossed I shaped spokes shown in Fig. 3.11 (b) are used when the diameter is <1000 mm and f<200 mm.
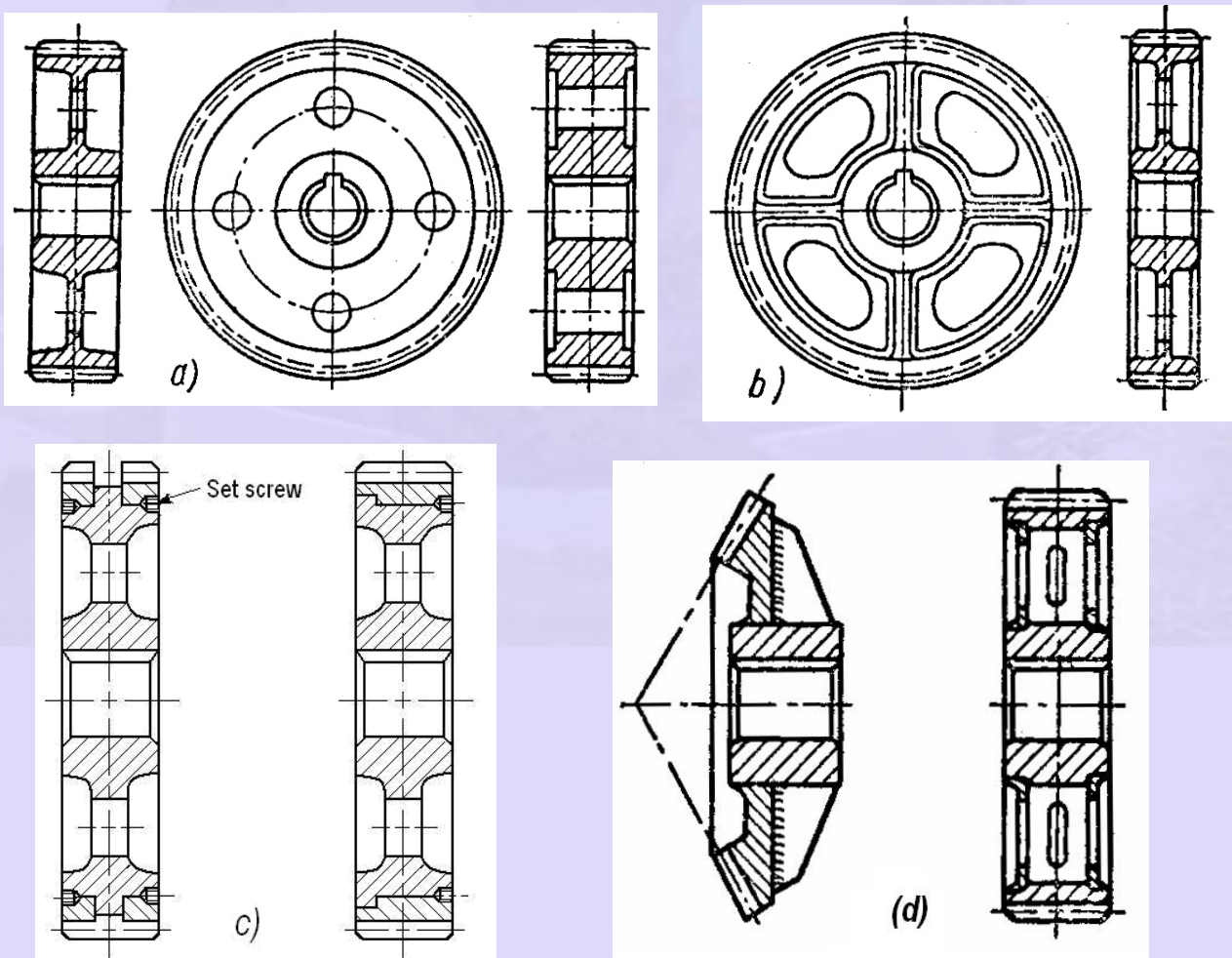
**Fig. 3.11 Design of medium size gear blanks**

Many a times to reduce the cost of the gears, rimmed construction is used. Here the central portion of the gear wheel is made of lower grade steel as the stress encountered here is less and the gear portion rim is press fitted or shrunk on the central portion. The rim is prevented from loosening by grub screws as seen in Fig. 3.11 (c)

Large gears are normally cast as in Fig. 3.12 (a) to (d) with web straight or inclined. To reduce the weight of the gears, non stressed portion is made hollow by keeping cores. When a small number of high quality gears are required, the gears made with rimmed, bolted or welded construction as shown in Fig. 3.11 (e) to (g). To save alloy steel, large wheels are made with fretting rings. Wheel central portions are made from Cast iron or cast steel. The ring is forged or roll expanded from special steel of tooth material. For f > 500 mm two Fretting rings are hot-fitted on the centers. Set screws to prevent loosening are also provided. When wheels are made in small quantities, to reduce the weight, welding is employed. Care should be taken to ensure adequate rigidity in these cases.
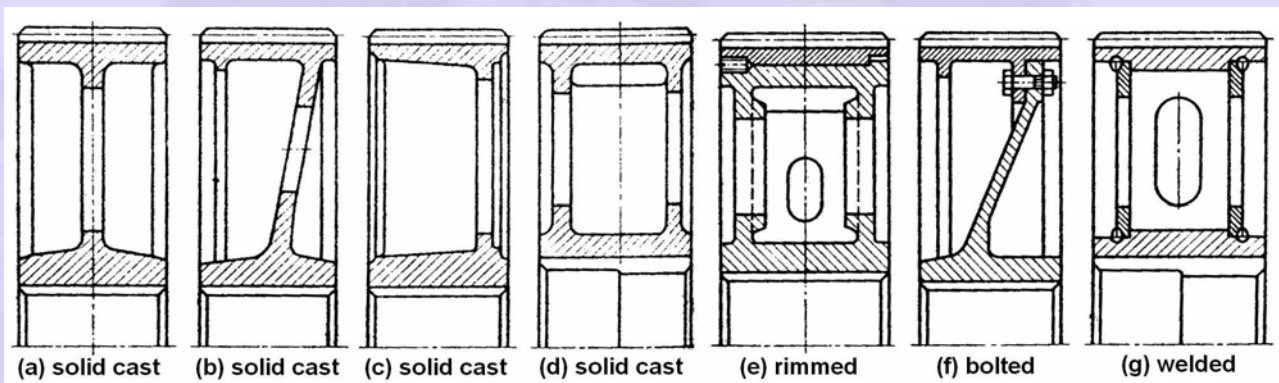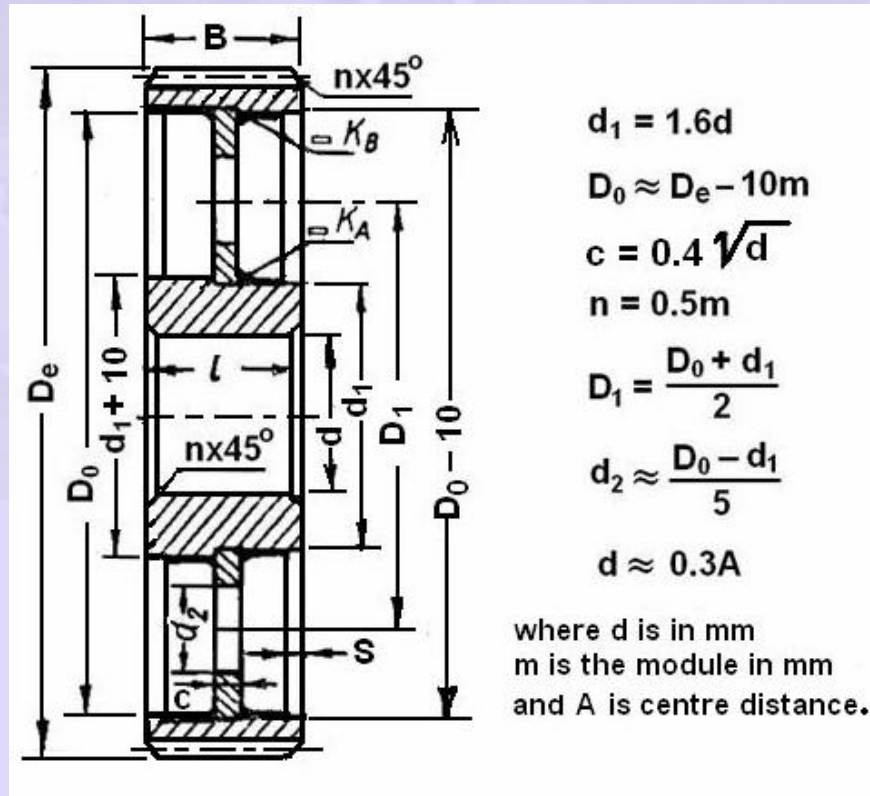

Fig. 3.12 Design of large gear blanks

The empirical formulae for finding proportions of spur gear wheel are given in Fig.3.13 (a) and that for a bevel gear wheel is given in Fig. 3.13 (b).

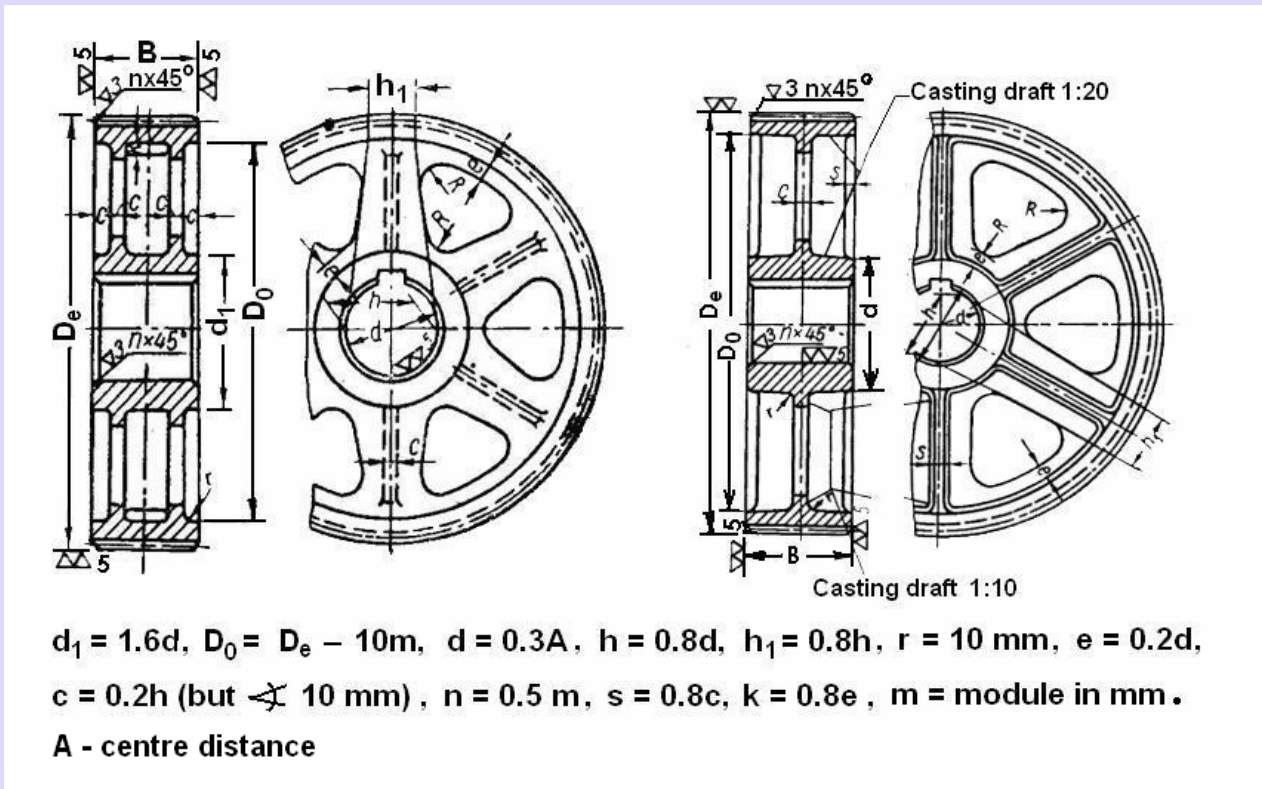(a)   (b)
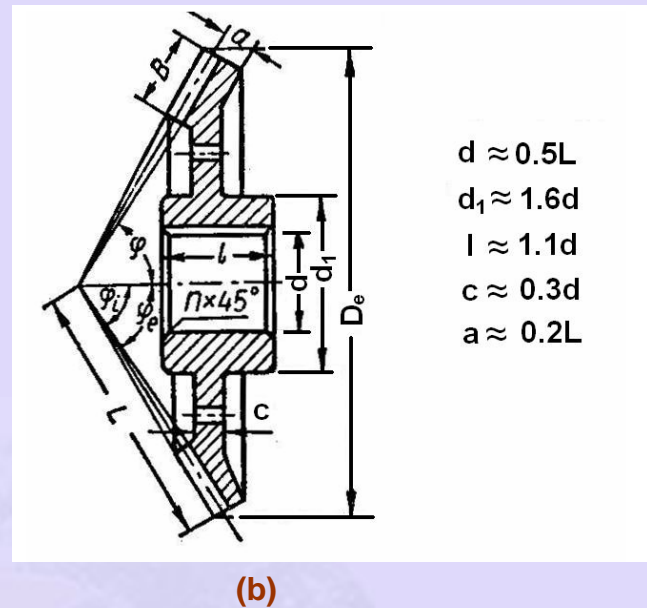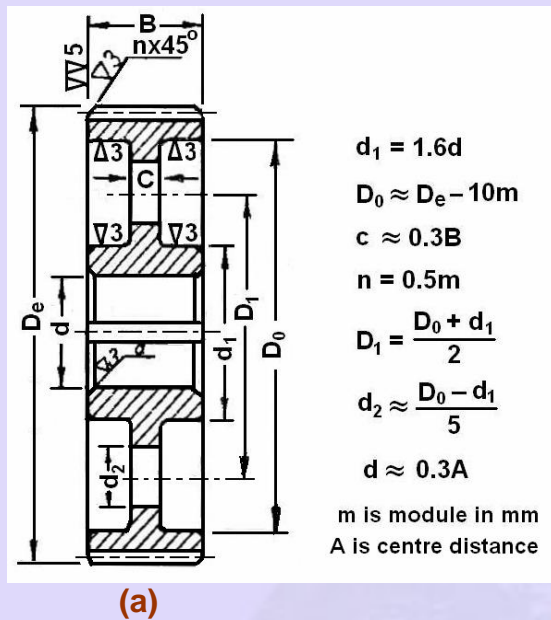
Fig. 3.13 Gear wheel proportions (a) spur gear (b) bevel gear

The empirical formulae for finding proportions of welded wheel elements are shown in Fig. 3.14

Fig. 3.14 Spur gear wheel proportions for weld construction.

The empirical formulae for finding proportions of wheel elements of cast spur and helical gears are given in Fig. 3.15.

$d_1 = 1.6d$, $D_0 = D_e - 10m$, $d = 0.3A$, $h = 0.8d$, $h_1 = 0.8h$, $r = 10$ mm, $e = 0.2d$, $c = 0.2h$ (but $\not< 10$ mm), $n = 0.5$ m, $s = 0.8c$, $k = 0.8e$, $m =$ module in mm.

A - centre distance

Fig. 3.15 Spur gear proportions for cast wheels.