EET 107

Introduction to Circuit Analysis

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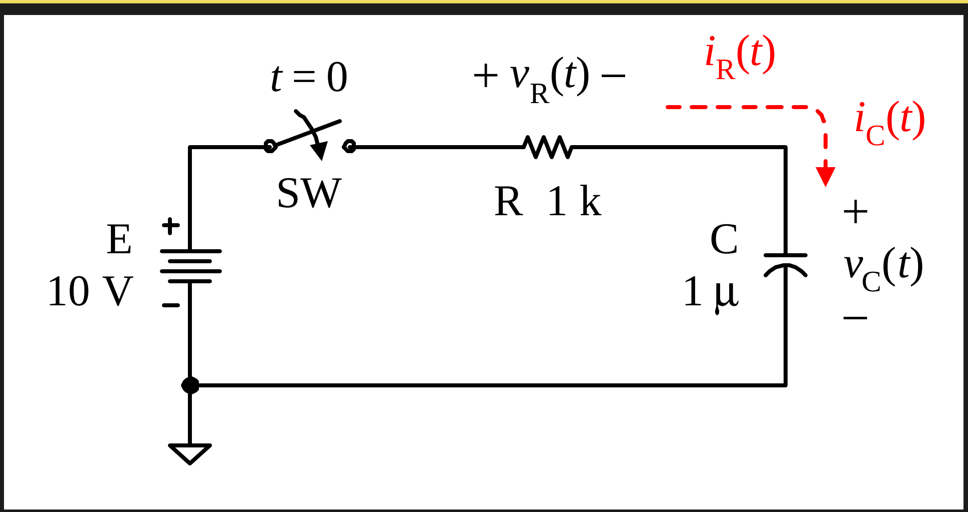
Electrical Engineering Technology

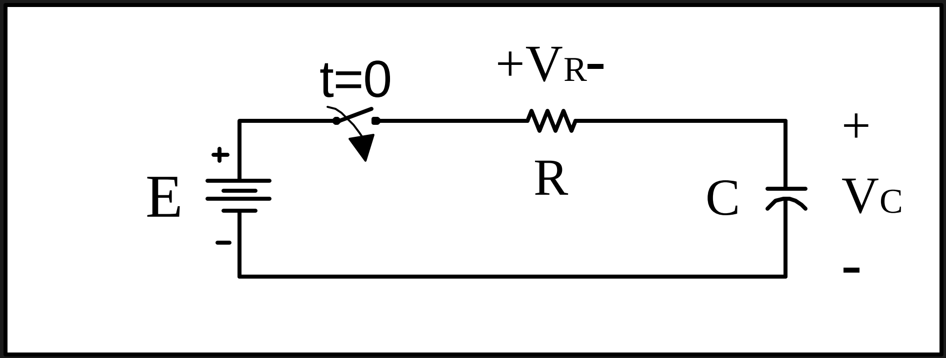
- **Switched RC Transient**
- **Inverse Solution**
- **Multiple RC – Thévenin Model**

RC Circuit - *Sudden DC Change*



$t = 0^-$	Just before switching
$t = 0$	***INITIAL*** - sudden dc change
$0 < t < 5\tau$	***TRANSIENT***
$t = 5\tau$	*Capacitor 99% charged*
$t \rightarrow \infty$	***STEADY STATE*** – final dc

RC Transient DC Circuit



init = initial (t = 0)

ss = steady state (t = ∞)

$v_C(t)$ $i_C(t)$ $v_R(t)$ $i_R(t)$

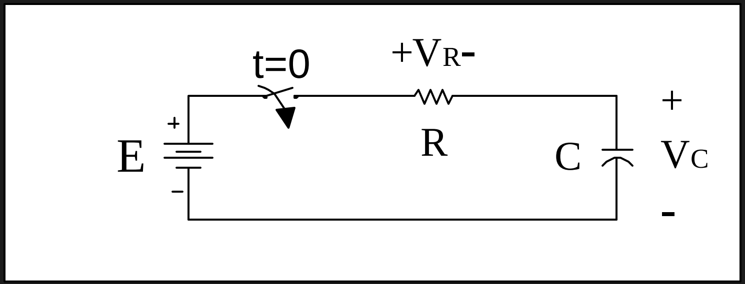
$$v_x(t) = V_{ss} + (V_{init} - V_{ss})\, e^{-t/\tau}$$

$$i_x(t) = I_{ss} + (I_{init} - I_{ss})\, e^{-t/\tau}$$

RC Transient DC Circuit

Analyze circuit and find:

- τ
- Initial value
- Steady state value



Then substitute:

$$v_x(t) = V\mathrm{ss} + (V\mathrm{init} - V\mathrm{ss})\, e^{-t/\tau}$$

RC Circuit – *uncharged capacitor*

$$v_C(t) = V\text{ss} + (V\text{init} - V\text{ss})\, e^{-t/\text{t}}$$

Example:	t = **RC = 10**ms
uncharged	***V*init = 0V**
	***V*ss = 10V**

$$v_C(t) = 10\text{V} + (0\text{V} - 10\text{V})\, e^{-t/10\text{ms}}$$

$$v_C(t) = 10\text{V} - 10\text{V}\, e^{-t/10\text{ms}}$$

RC Circuit – *charged capacitor*

$$v_C(t) = V\text{ss} + (V\text{init} - V\text{ss})\, e^{-t/\text{t}}$$

Example: $\text{t} = \mathbf{RC = 10}\text{ms}$

charged $\mathbf{\mathit{V}init = 3V}$

$\mathbf{\mathit{V}ss = 10V}$

$$v_C(t) = 10\text{V} + (3\text{V} - 10\text{V})\, e^{-t/10\text{ms}}$$

$$v_C(t) = 10\text{V} - 7\text{V}\, e^{-t/10\text{ms}}$$

RC Transient Circuit Analysis

1. Establish ***capacitor voltage*** ***before switch thrown***
2. Evaluate ***time constant*** ***after switch thrown***
3. ***Initial*** model of ***capacitor*** and evaluate circuit
4. ***Steady state*** model of ***capacitor*** and evaluate circuit
5. Apply universal RC equations
6. Sketch resulting equations

Capacitor Models – Know These

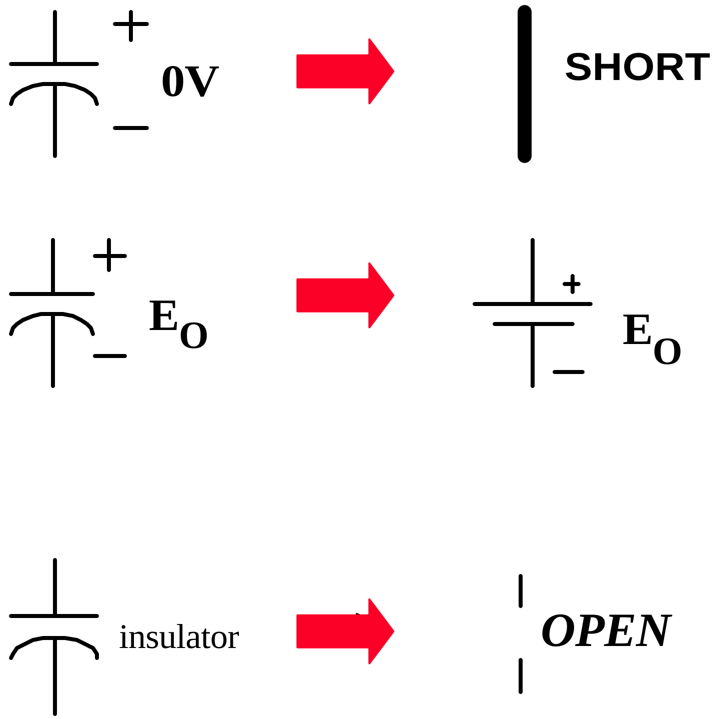
CAPACITOR - stores ***VOLTAGE***

INITIALLY - Un*charged cap*

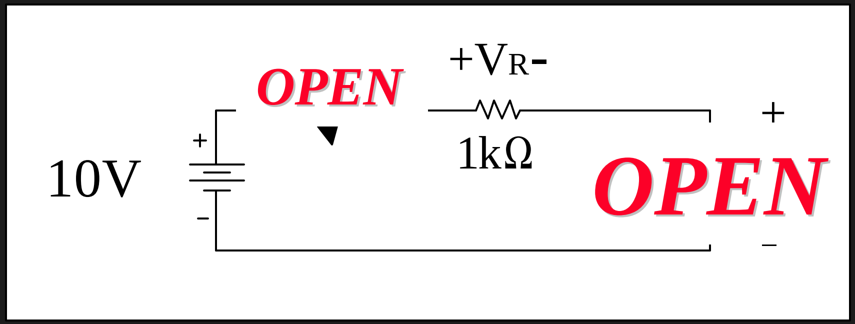
INITIALLY - *charged cap*

E_O inital voltage

STEADY STATE - fully charged *cap*



1. Example - *before switch closed*

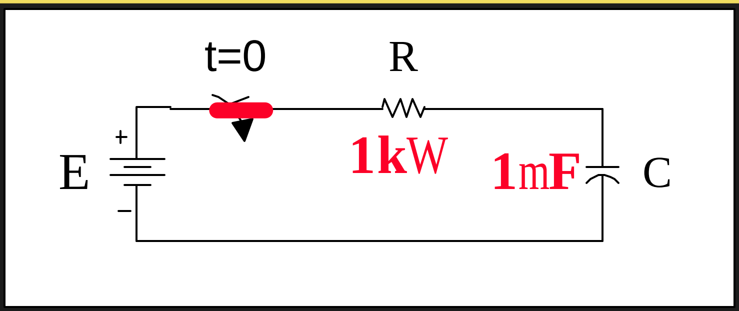


0V

$t = 0^-$

- switch open for a long time is assumed
- steady state capacitor → ***OPEN***
- $V_C = 0V$

2. Example - *time constant*



Switch closed

RC Time Constant when switch closed

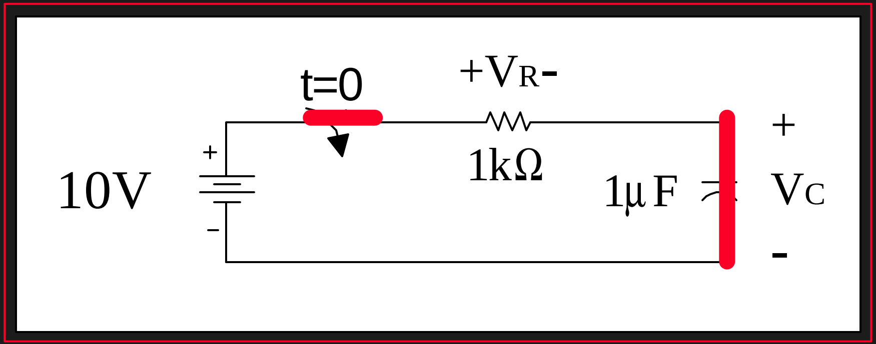
- $\tau = RC = 1k\Omega \times 1\mu F = 1ms$

Cap 63% charged after switch closed

- $5\tau = 5 \times 1ms = 5ms$

Cap 99.3% charged after switch closed

3. Example - *initial circuit*

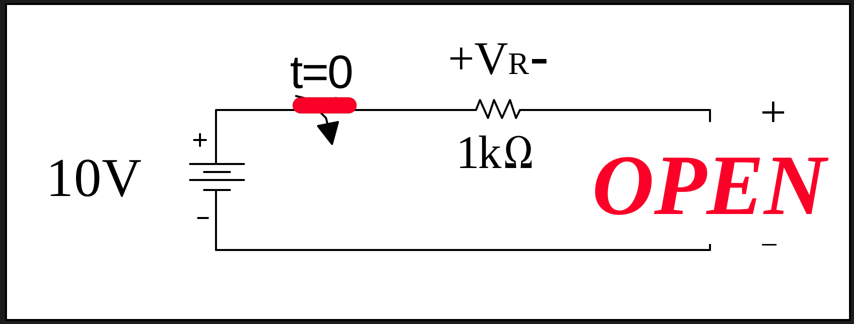


0V from $t = 0^-$

$t = 0$

- $V_C = 0V$ *uncharged cap model*
- Capacitor acts like a ***SHORT***
- $V_R = 10V$ and $I_R = I_C = 10mA$

4. Example - *steady state circuit*



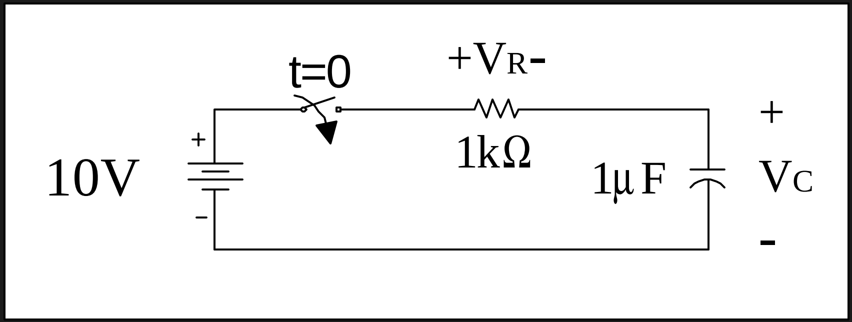
10V

t >> 5t

- capacitor acts like an open: *model as* ***OPEN***

- $V_C = 10V$ *capacitor fully charged*
- $V_R = 0V$
- $I_C = I_R = 0mA$

5. Example - *capacitor voltage equation*



Capacitor Voltage

t = 1ms

*V*init = 0V

*V*ss = 10V

Complete capacitor voltage expression

$$v_C(t) = Vss + (Vinit - Vss)\, e^{-t/\tau}$$

$$v_C(t) = 10V + (0V - 10V)\, e^{-t/1ms}$$

$$v_C(t) = 10V - 10V\, e^{-t/1ms}$$

5. Example - *capacitor voltage equation*

$$v_C(t) = 10\text{V} - 10\text{V}\ e^{-t/1\text{ms}}$$

Alternate form by factoring out 10V

$$v_C(t) = 10\text{V}\ (1 - e^{-t/1\text{ms}})$$

5. Example - *capacitor voltage equation*

Evaluating capacitor voltage - *t in units of ms*

$$v_C(t) = 10V - 10V\, e^{-t/1ms}$$

$$v_C(0) = 10V - 10V\, e^{-0/1ms} = 0V$$

$$v_C(1ms) = 10V - 10V\, e^{-1ms/1ms} = 6.32V$$

$$v_C(2ms) = 10V - 10V\, e^{-2ms/1ms} = 8.65V$$

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$$v_C(5ms) = 10V - 10V\, e^{-5ms/1ms} = 9.93V$$

5. Example - *capacitor voltage equation*

Evaluating capacitor voltage - *t in units of t*

$$v_C(t) = 10\text{V} - 10\text{V}\, e^{-t/\tau}$$

$$v_C(0) = 10\text{V} - 10\text{V}\, e^{-0/\tau} = \underline{0\text{V}}$$

$$v_C(1\tau) = 10\text{V} - 10\text{V}\, e^{-1\tau/\tau} = \underline{6.32\text{V}}$$

$$v_C(2\tau) = 10\text{V} - 10\text{V}\, e^{-2\tau/\tau} = \underline{8.65\text{V}}$$

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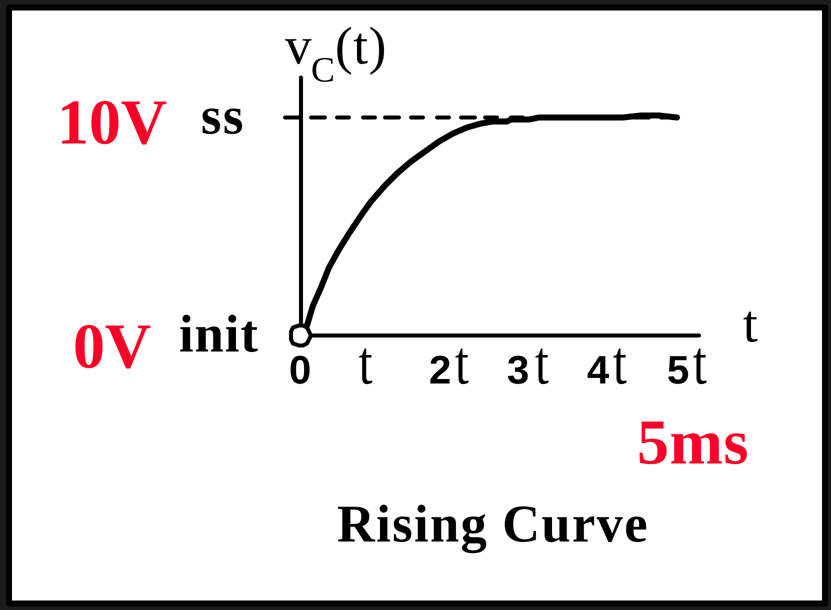
$$v_C(5\tau) = 10\text{V} - 10\text{V}\, e^{-5\tau/\tau} = \underline{9.93\text{V}}$$

6. Example - *capacitor voltage sketch*

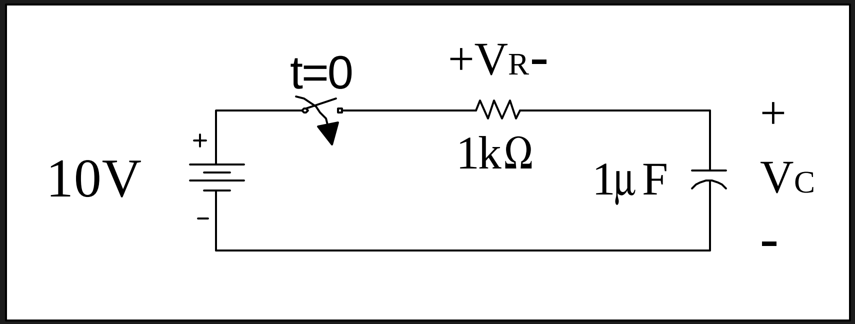
$$v_C(t) = 10V - 10V\, e^{-t/t}$$

t = 1ms
***V*init = 0V**
***V*ss = 10V**

t		$v_C(t)$
0		0.0V
1τ	1ms	6.3V
2τ	2ms	8.7V
3τ	3ms	9.5V
4τ	4ms	9.8V
5τ	5ms	9.9V



5. Example - *capacitor current equation*



Capacitor current

t = 1ms

***I*init = 10mA**

***Iss* = 0mA**

Complete cap current expression

$$i_C(t) = Iss + (Iinit - Iss)\, e^{-t/\tau}$$

$$i_C(t) = 0mA + (10mA - 0mA)\, e^{-t/1ms}$$

$$i_C(t) = 10mA\, e^{-t/1ms}$$

6. Example - *capacitor current sketch*

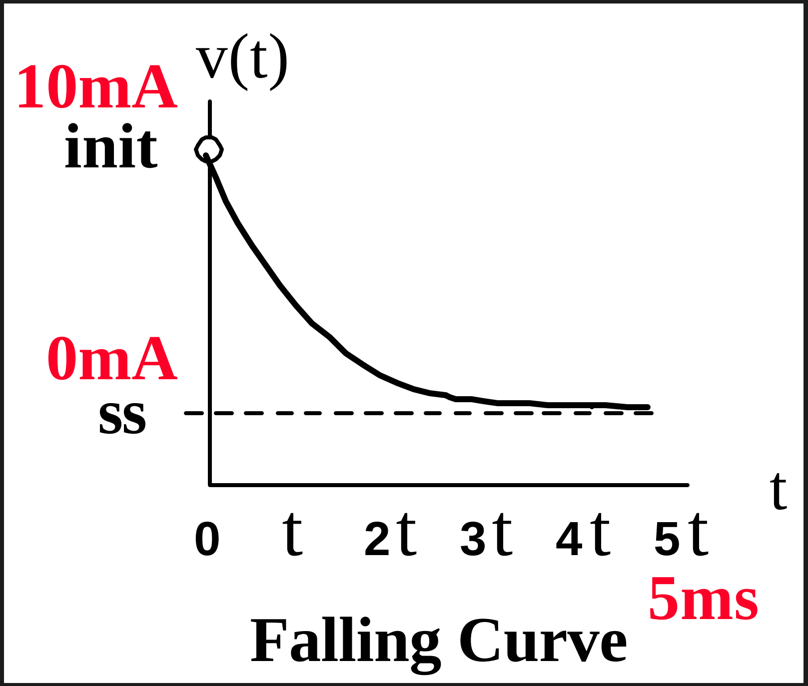
t = 1ms

***I*init = 10mA**

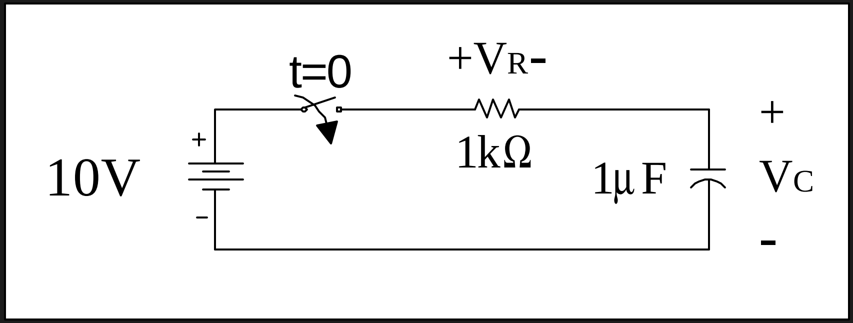
Iss = 0mA

$$i_C(t) = 10mA\ e^{-t/t}$$

t		$i_C(t)$
0		10.0mA
1τ	1ms	3.7mA
2τ	2ms	1.3mA
3τ	3ms	0.5mA
4τ	4ms	0.2mA
5τ	5ms	0.1mA



5. Example - *resistor current equation*



Resistor current

t = 1ms

***I*init = 10mA**

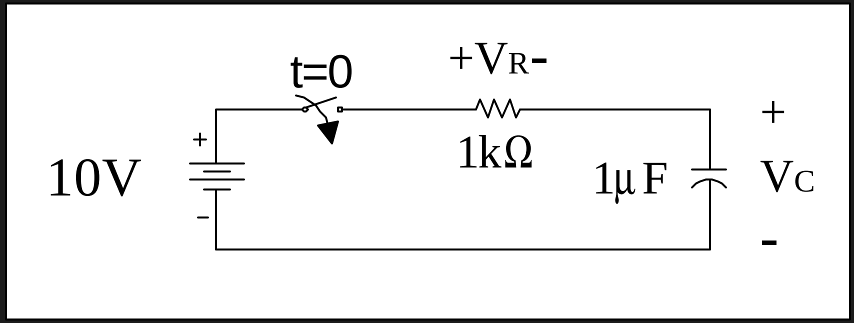
***I*ss = 0mA**

Note: quick solution, ***same*** as the capacitor current

$$i_R(t) = i_C(t) = 10\text{mA}\ e^{-t/1\text{ms}}$$

Same sketch

5. Example - *resistor voltage equation*



Resistor voltage

t = 1ms

***V*init = 10V**

Vss = 0V

Quick solution: *Ohm's Law*

$$i_R(t) = 10\text{mA } e^{-t/1\text{ms}}$$

$$v_R(t) = 1\text{k}\Omega \times 10\text{mA } e^{-t/1\text{ms}}$$

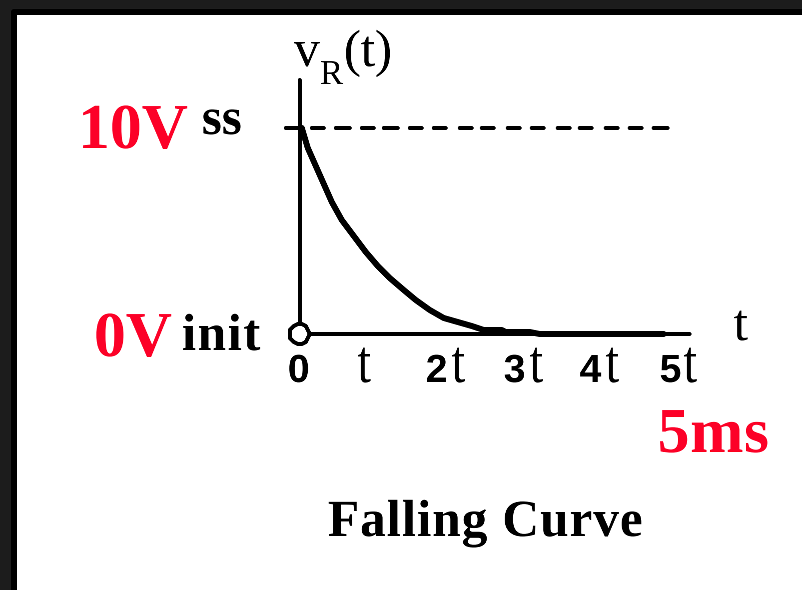
$$\mathbf{v_R(t) = 10V\ e^{-t/1ms}}$$

6. Example - *resistor voltage sketch*

$$v_R(t) = 10V\ e^{-t/t}$$

t = 1ms
*V*init = 10V
*V*ss = 0V

t		$V_R(t)$
0		10.0V
1τ	1ms	3.7V
2τ	2ms	1.3V
3τ	3ms	0.5V
4τ	4ms	0.2V
5τ	5ms	0.1V



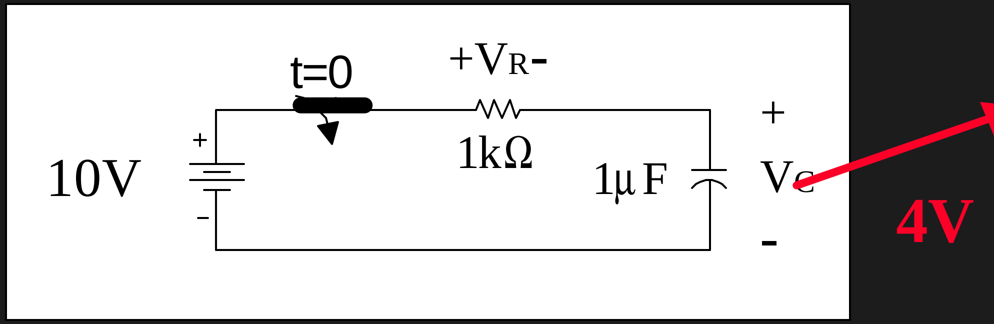
Resistor Sketches

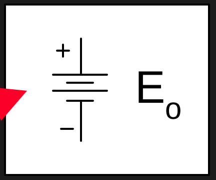
Resistor voltage and current sketch must have the ***same*** *shape.*

Ohm's Law $v(t) = \mathrm{R}\, i(t)$

Related by a constant !

What if Charged Cap - *initial circuit*





t = 0 *Intial Circuit*

- What if cap was previously charged to 4V
- Initial model of cap would be a 4V supply
- $V_R = 10V - 4V = 6V$
- $I_R = I_C = 6V/1k\Omega = 6mA$

Electrical Engineering Technology

- **Switched RC Transient**
- **Inverse Solution**
- **Multiple RC – Thévenin Model**

RC Circuit - *universal equation*

$$v_C(t) = V\text{ss} + (V\text{init} - V\text{ss})\, e^{-t/t}$$

Example

$t = 10\text{ms}$

$V\text{init} = -4\text{V}$

$V\text{ss} = 10\text{V}$

$$v_C(t) = 10\text{V} + (-4\text{V} - 10\text{V})\, e^{-t/10\text{ms}}$$

$$v_C(t) = 10\text{V} - 14\text{V}\, e^{-t/10\text{ms}}$$

RC Circuit - *universal equation*

Evaluate $v(t)$ at $t = 18\mu s$

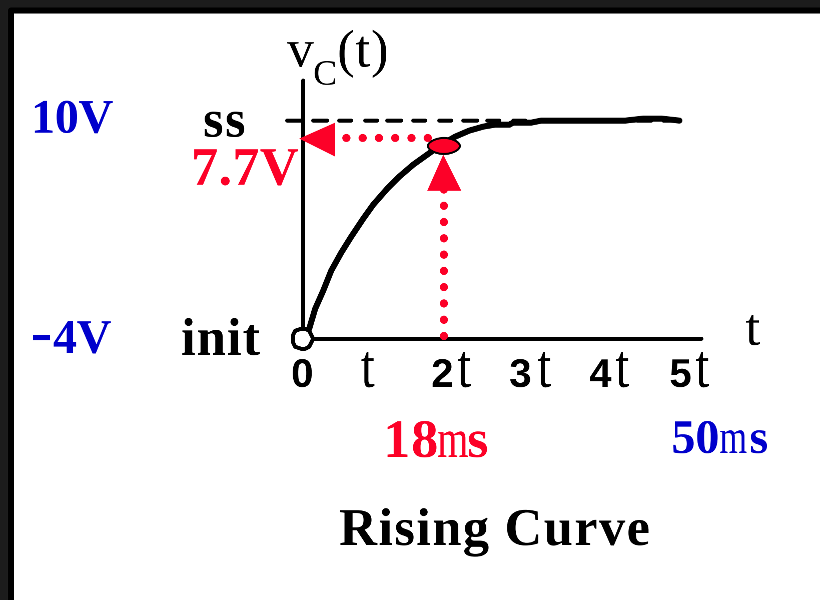
Plug in a *time* and find a *voltage*.

$$v_C(\mathbf{18}\text{ms}) = 10\text{V} - 14\text{V}\, e^{-\mathbf{18}\text{ms}/10\mu\text{s}}$$

$$v_C(\mathbf{18}\text{ms}) = 7.686\text{V}$$

RC Circuit - *capacitor voltage sketch*

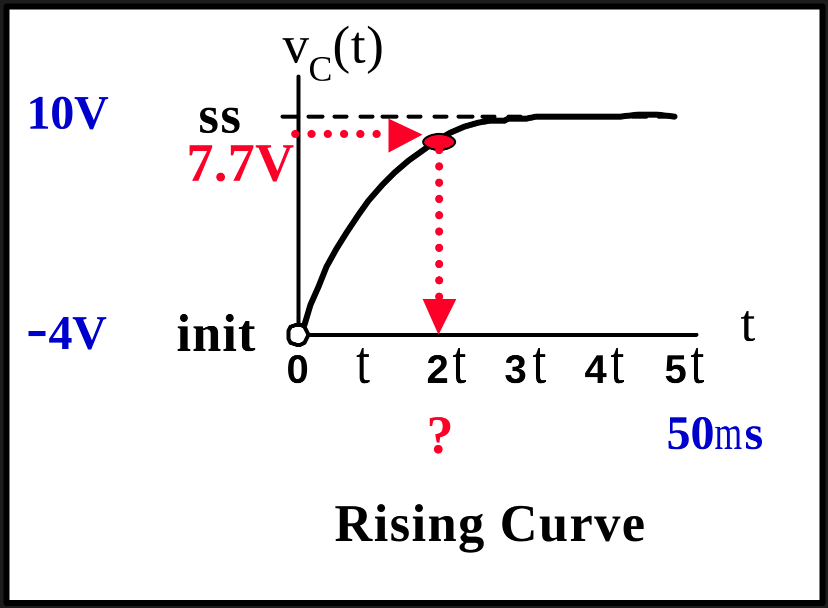
At $t = 18$ms, $V_C = 7.686$V



t = 10ms

RC Circuit - *capacitor voltage sketch*

Given a voltage, find the corresponding time.



t = 10ms

RC Circuit - *universal equation*

How much time "*t*" does it take to charge to a particular voltage ?

For example, how much time "*t*" is needed to charge the capacitor to 3V ?

RC Circuit - *universal equation*

$$v_C(t) = \underline{3V}$$

$$t = ?$$

$$\underline{3V} = 10V - 14V\, e^{-t/10\mu s}$$

$$-7V = -14V\, e^{-t/10\mu s}$$

$$0.5 = e^{-t/10\mu s}$$

RC Circuit - *universal equation*

$$\mathbf{Ln}\,(\,0.5\,) = \mathbf{Ln}\,(\,e^{-t/10\mu s}\,)$$

exponent pops out

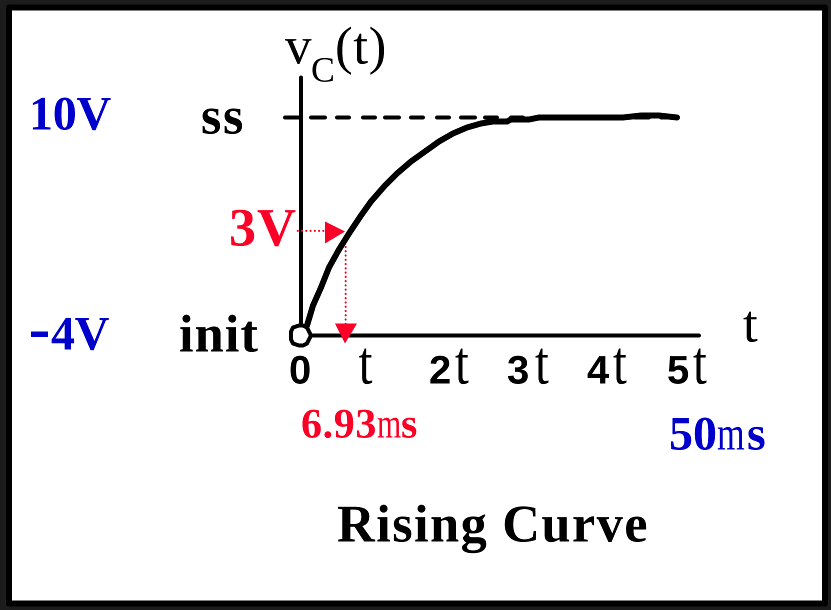
$$-0.693 = -t\,/\,10\mu s$$

$$t = 0.693 \times 10\mu s = \underline{\mathbf{6.93}\mathit{ms}}$$

RC Circuit - *capacitor voltage sketch*

How much time is required to charge the capacitor to 3V?

$t =$ **6.93**ms

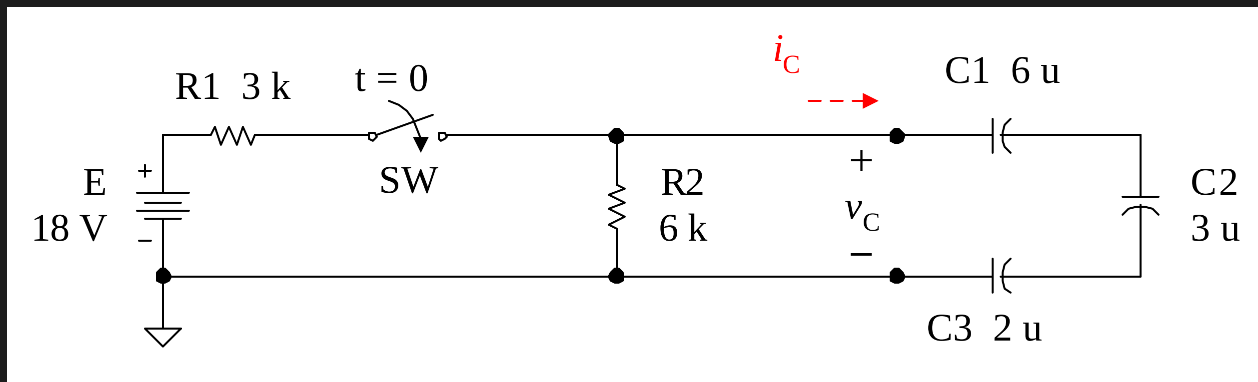


t = **10**ms

Electrical Engineering Technology

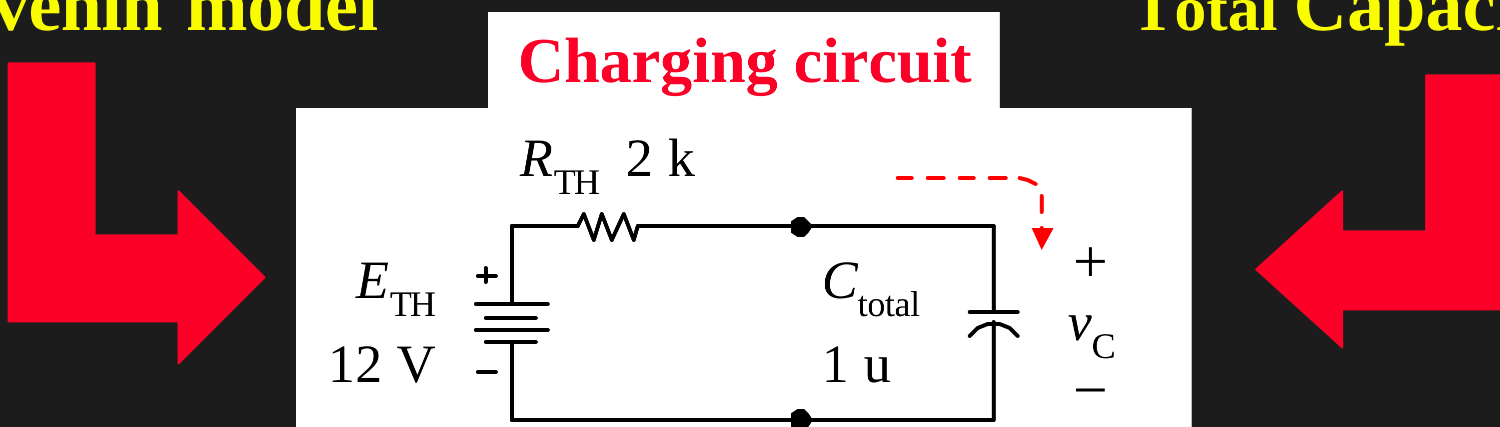
- **Switched RC Transient**
- **Inverse Solution**
- **Multiple RC – Thévenin Model**

Multiple RC – simplify to Thévenin

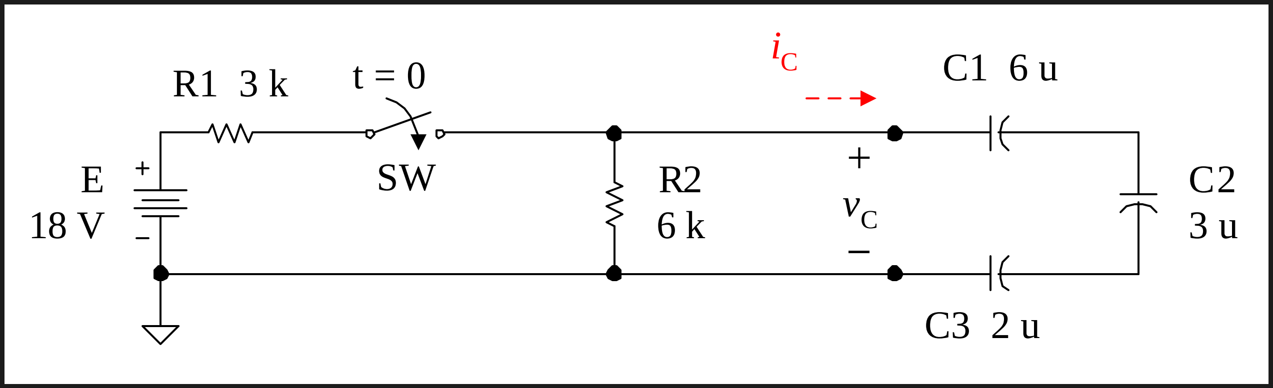


Thévenin model

Total Capacitance

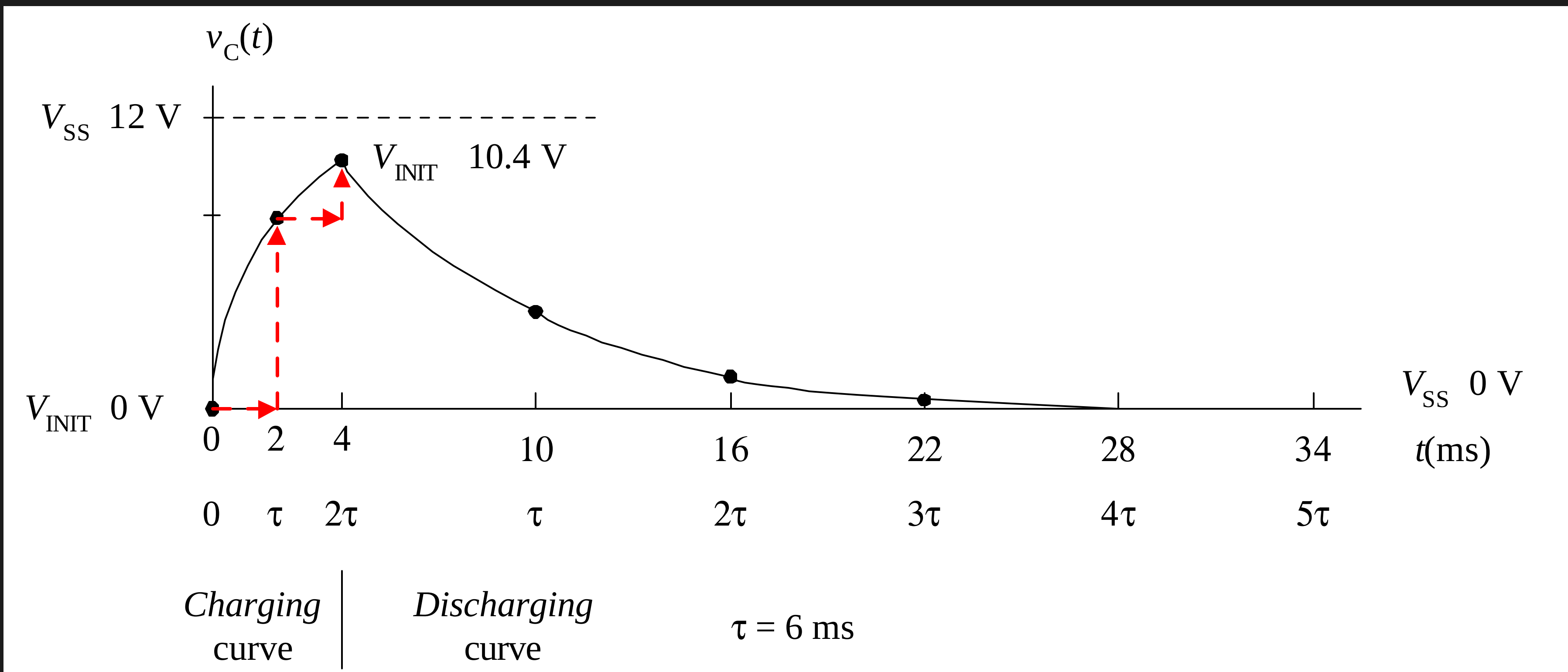


Textbook Example 14-5



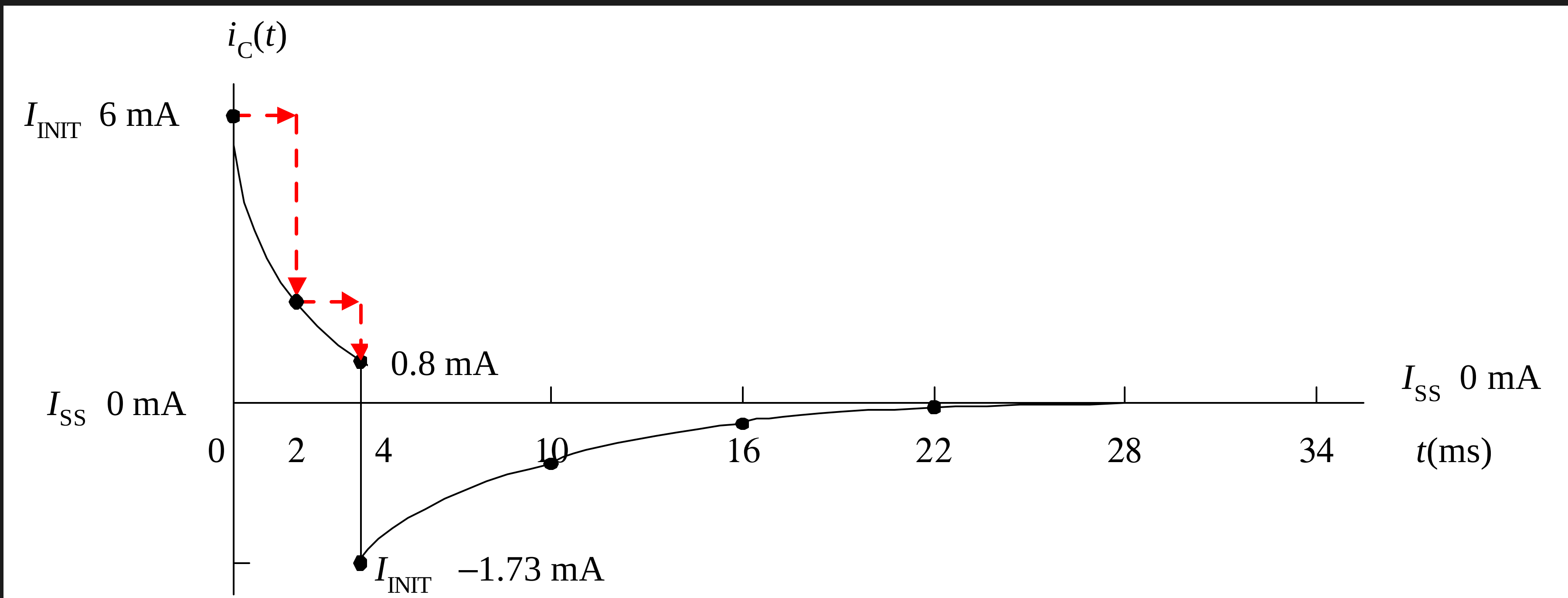
- **Uncharged capacitors before switch closed**
- **Close switch for 4 ms then reopened**
- **Draw $v_C(t)$ and $i_C(t)$**

Textbook Example 14-5



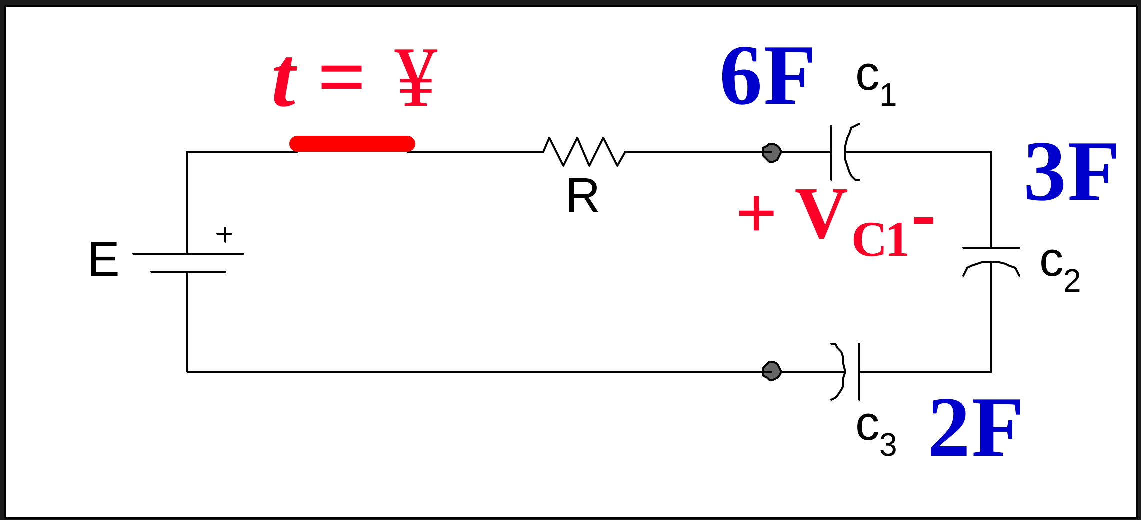
t = 2 ms **t = 6 ms**

Textbook Example 14-5



t = 2 ms **t = 6 ms**

Textbook Example 14-5



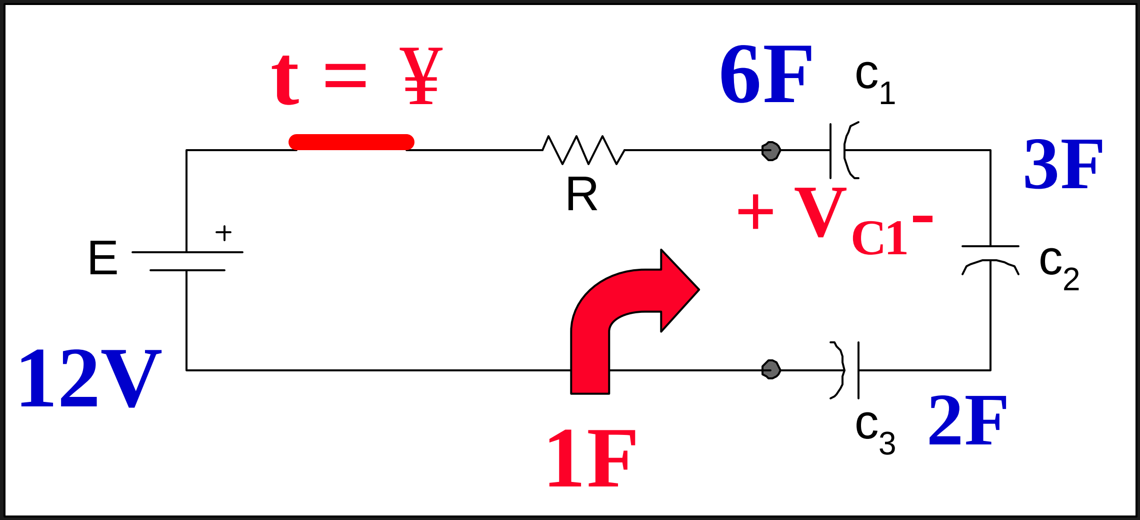
Larger C

Smaller X_C

Less voltage drop

6F drops <u>least</u> voltage

Capacitance - *VDR in Steady State*



R → 1/C

VDR steady state

$$V_{C1} = \frac{C_T}{C_1} E \quad = \frac{1F}{6F} 12V = 2V$$

Electrical Engineering Technology

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- **Inverse Solution**
- **Multiple RC – Thévenin Model**