Coupling Through the Magnetic Field---Faraday's Law

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1. OBJECTIVES

1. To understand mutual coupling through the magnetic field and practical implications of Faraday's law in hardware;
2. To investigate and measure the inductance associated with signal wires;
3. To develop equivalent circuit models for the test configurations, and estimate the values of the unknown parasitic parameters.

2. EQUIPMENT

1. Function Generator
2. Tektronix 2230 Oscilloscope
3. HP4262A LCR Meter
4. Multi-turn wire loops: 11 *cm* (2), 7 *cm*, 5.5 *cm*, and 4 *cm* diameters.

3. THEORY AND DISCUSSION

Faraday's law states that a time-varying magnetic field through a surface bounded by a closed path induces a voltage around the loop. This fundamental principal has important consequences for parasitic coupling in electronic hardware. For example a "noisy" circuit on a printed circuit board may produce a significant time-varying magnetic field as a result of current levels and current loop area. The magnetic field through the loop area of a susceptible or "victim" circuit induces a voltage across the load in the victim circuit. Note that the loop area when considering mutual inductance is the area determined by the closed current path in the victim circuit. Depending upon the current magnitude in the emitter circuit, the source and load impedances and loop area in the victim circuit, distance between the victim and noise emitter circuits, and orientation of the circuits, this coupling can cause faulty operation of the victim circuit. Coupling between circuit loops through the time-varying magnetic field is studied in this laboratory exercise.

As an example, consider a noisy "emitter" circuit with source V_{es} shown in Figure 1 (a) that couples noise through the magnetic field to a "victim circuit". A time-varying magnetic field $\overline{H}_e$ is produced by the current in the emitter circuit. Part of this field penetrates the loop of the victim circuit A_v, and produces an *emf* or *voltage* around the closed path of the victim circuit. This *emf* then drives a noise current in the victim loop. The noise current through the load in the victim circuit produces a noise voltage, which can lead to faulty operation of the victim circuit.

An equivalent circuit for determining the noise voltage at the load in the victim circuit is shown in Figure 1 (b). Analyzing the circuit in the frequency domain, and applying KVL around both of the loops, a second order system of linear equations for the currents in the loops is

$$(j\omega L_e + Z_{se} + Z_{Le})I_e + j\omega M I_v = V_{es}$$

$$j\omega M I_e + (j\omega L_v + Z_{sv} + Z_{Lv}) I_v = 0 \qquad (1)$$

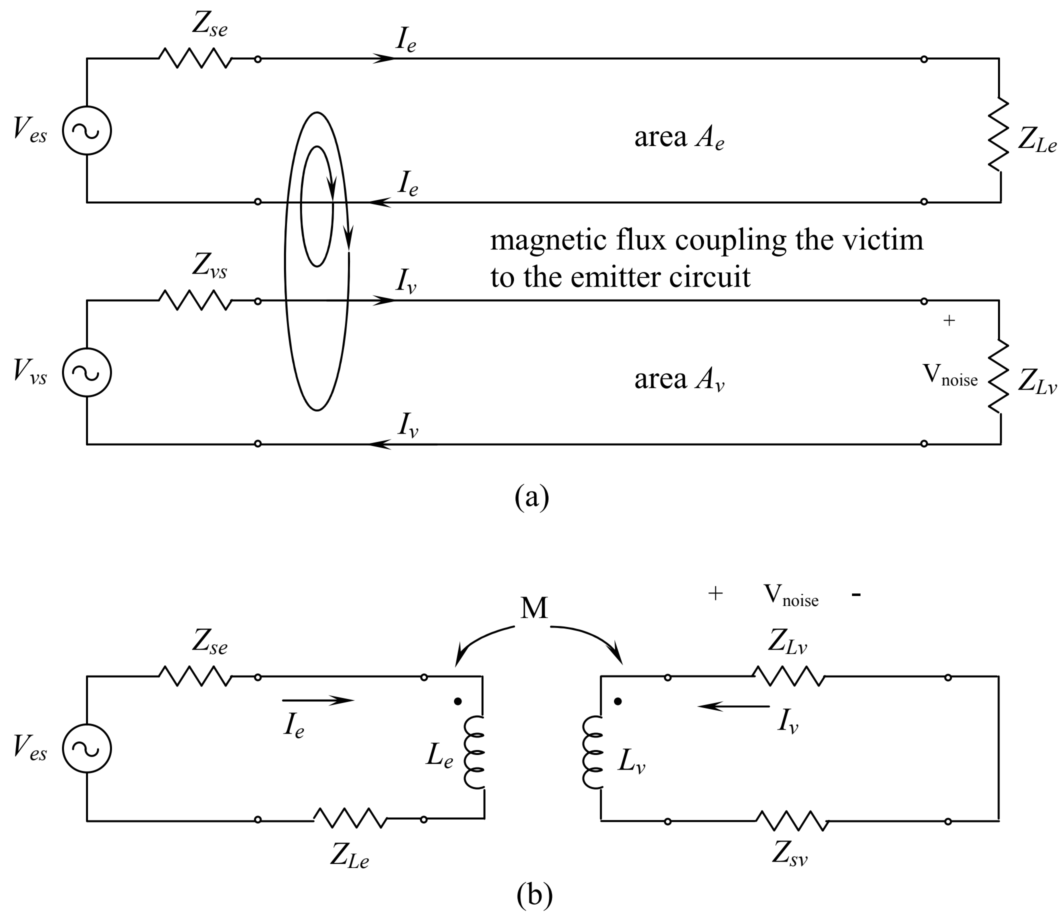


Figure 1: (a) Schematic representation of noise coupling through a time-varying magnetic field, and the induced noise voltage at the load in a victim circuit. (b) Equivalent circuit diagram for the noise coupling.

Note that superposition has been employed to obtain the above equations. Namely, since the noise voltage in the victim circuit as a result of magnetic field coupling from the emitter circuit is of concern, the intended voltage source V_{vs} in the victim circuit has been set to zero. The total voltage at the load in the victim circuit will then be the sum of the induced noise voltage and the voltage resulting from the source V_{vs}.

The above system of equations can be solved for $I_v = I_{noise}$, the noise current that results from the *emf* induced around the victim circuit loop by the time-varying magnetic field set up by the current I_e in the emitter loop. The noise current in the victim circuit is

$$I_{noise} = I_v = -\frac{j\omega M I_e}{j\omega L_v + Z_{sv} + Z_{Lv}}, \qquad (2)$$

and the noise voltage at the load in the victim circuit is

$$V_{noise} = \frac{j\omega MI_e}{j\omega L_v + Z_{sv} + Z_{Lv}} Z_{Lv} . \tag{3}$$

In general it is desirable to minimize L_v. As a result $\left|j\omega L_v\right| \leq \left|Z_{sv} + Z_{lv}\right|$ in many cases. The noise voltage at the victim circuit load is then

$$V_{noise} \approx -\frac{j\omega MI_e}{Z_{sv} + Z_{Lv}} Z_{Lv} . \tag{4}$$

Coupling through a time-varying magnetic field will then be larger for larger currents in the emitter circuit, larger mutual inductance *M* between the two loops, and a larger victim circuit load impedance.

The mutual coupling *M* between the circuits is defined as

$$M = \frac{1}{I_e}\int_{A_v} \overline{B_e}(\bar{r}) \cdot \overline{dS_v} = \frac{1}{I_e}\int_{A_v} \mu_0 \overline{H_e}(\bar{r}) \cdot \overline{dS_v} \tag{5}$$

Where $\overline{H_e}$ is the magnetic field resulting from the current in the emitter circuit I_e in the absence of the victim circuit. The mutual inductance can be minimized, by minimizing the loop area A_v of the victim circuit, and/or minimizing the magnetic field $\overline{H_e}$ produced by the emitter circuit through the area A_v of the victim circuit loop. The magnetic field $\overline{H_e}$ through A_v can be reduced by increasing the separation between the emitter and victim circuits, as well as placing the signal and signal return currents in the emitter loop very close to each other. The latter case results in two currents in opposite directions being very near each other. The magnetic fields from each current are in opposite directions, resulting in significant cancellation as shown in Figure 2.

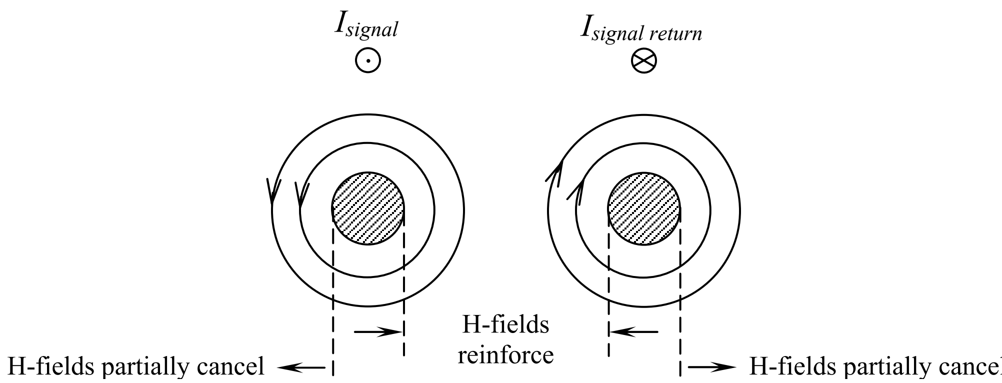


Figure 2: Partial magnetic field cancellation as a result of closely spaced parallel currents in opposite directions.

Some general design principles related to minimizing noise coupling through the magnetic field can then be formulated.

1. Minimize circuit loop areas. The loop area is determined by the complete current path from the source to load, and the return to the source. (Recall that current always returns to its source.)
2. Route signal return conductors (sometimes referred to as *reference* or *ground*) close to the signal conductor to take advantage of the partial cancellation of the magnetic field outside the current loop.
3. Attempt to separate as much as possible circuits that are expected to have large currents and act as emitters, from high impedance circuits that might be susceptible to noise induced in them through magnetic field coupling.

4. PROCEDURE

The test configuration for the following experiments is shown in Figure 3.

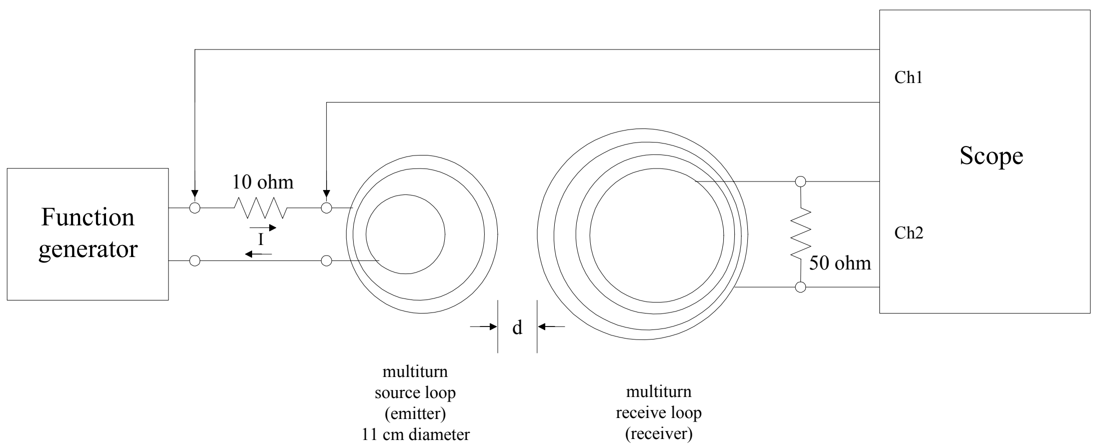


Figure 3: Experimental configuration for studying coupling between circuit loops through a time-varying magnetic field

1. For the four diameters of victim loops, 11, 7, 5.5 and 4 *cm*, and a separation between the loops of $d = 0$, measure the voltage developed across the terminal of the victim loop (secondary). Adjust the amplitude of the source to obtain approximately 3 $V_{p\text{-}p}$ measured across the 10 Ω resistor in the source loop. Use a sinusoidal source with frequency of 1 *MHz*. Plot the peak-to-peak voltage $V_{p\text{-}p}$ (ordinate) at the terminals of the victim loop versus area (abscissa). Is this function linear? Is it expected to be? Discuss this.
2. For the 11 *cm* diameter victim loop, plot $V_{p\text{-}p}$ at the terminals of the victim loop (ordinate) versus distance d (abscissa). Acquire no more than six data points. Discuss the functional variation of this curve?

3. Investigate the coupling for the orientations of emitter and receiver loops shown in Figure 4 for two 11 *cm* diameter loops. For Case 1 and 5, the measured voltage in the victim circuit is 180° out of phase. Record the magnitude and phase of the voltage (relative to the voltage measured across the 10 Ω resistor). Develop an equivalent circuit model for each case showing the correct dot convention for the measured voltage. Do not determine any of the parameter values.

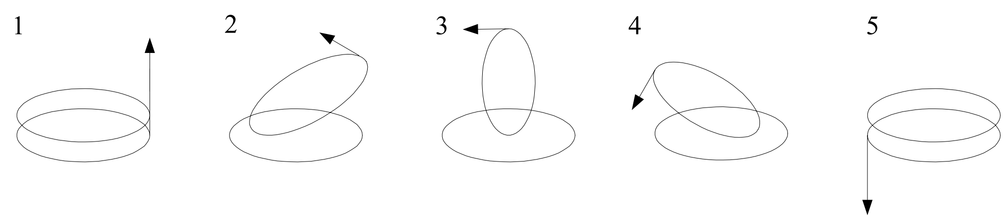
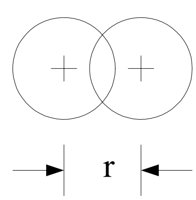


Figure 4: Orientation of emitter and receiver loops

4. Using the 11 *cm* receiver loop, place this on top of the emitter loop (axes coincident), and plot the measured voltage $V_{p\text{-}p}$ in the victim loop as a function of the distance r between the two loop centers as the victim loop is moved off the emitter loop as shown below. Explain why the peak positive voltage occurs where it does, as well as the peak negative voltage, and the zero voltage.



5. For the emitter and victim loops coincident with $r = 0$, as in Part 4 above, apply a square wave from the source, and investigate the relationship between the source signal (Channel 1 of the scope) and the victim loop signal (Channel 2). Use the 11 *cm* loops for both the emitter and receiver. Let the frequency be 100 *kHz*, and the rise time of the square wave be approximately 6 *ns*. How are the signals in the victim and emitter circuits related? Is this expected? Develop an equivalent circuit model for the emitter and victim circuits. Measure the inductance in each loop (emitter and victim) with the LCR meter. Finally, employ a sinusoidal source at 1 *MHz* to determine M for values of r=0, 11, and 16.5 *cm*. Note that M can be determined from Eq. (4), where $\left|\dfrac{Z_{sv} + Z_{lv}}{Z_{lv}}\right| = 1$, since, $Z_{sv} = 0$ and $Z_{lv} = 50\Omega$ is the load impedance of the victim loop.

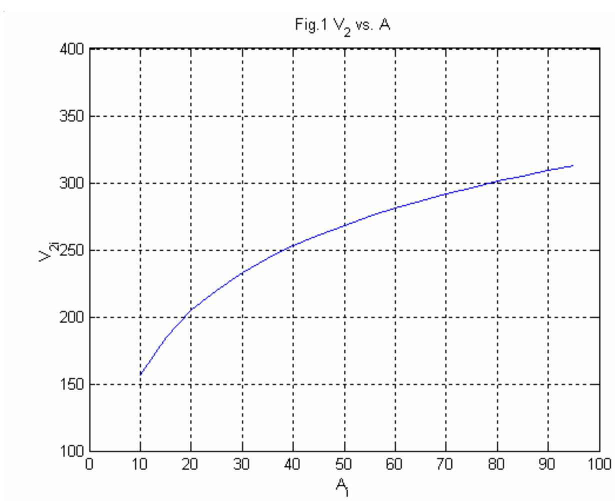
5. SUMMARY AND CONCLUSIONS

The lab summary should include all the data taken above, plotted in a clear manner, brief discussions to all questions, posed, and the equivalent circuit models indicated. The summaries need only be clear, neat, legible, and concise. It is not necessary to type the summary, or use a plotting or drawing package to produce the plots or diagrams. The discussions should be clear, but brief. The emphasis should be on demonstrating a clear understanding of the fundamental principles, and not lengthy or fancy summaries.

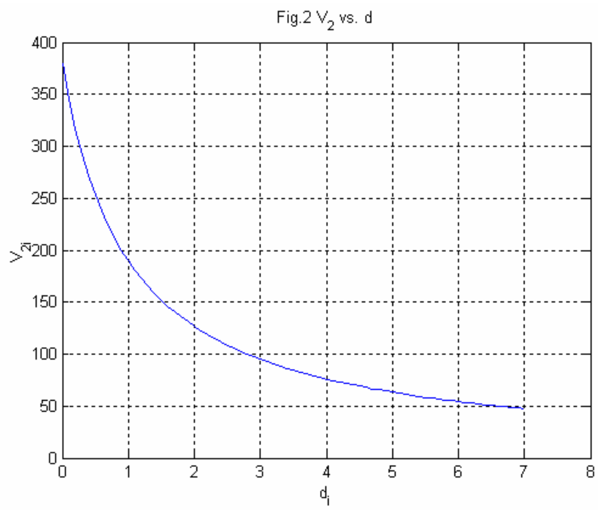
1. The plot of the measured voltage in the victim circuit as a function of the area of the victim loop is not a linear function, i.e., doubling the area of the victim loop does not double the measured noise voltage. The only way that the function would be linear is if the flux density resulting from the current in the emitter circuit that penetrates the victim loop were constant. This is the same as saying the magnetic field resulting from the current in the emitter circuit is constant over space. Clearly this is not true. The magnetic field drops off rapidly as an observer moves away from the emitter loop, as is seen from exercise 2 in the experiment. Initially increasing the area in the victim loop from the 4 *cm* diameter loop to the 5.5 *cm* diameter loop results in a relatively rapid increase in the measured noise voltage in the victim circuit. However, as the loops become larger, while more flux is coupled to the victim loop as a result of the increasing area, the additional flux is weaker, because the added new area is getting farther from the emitter circuit where the magnetic field is decreasing rapidly. Faraday's law is

$$emf = -\frac{d}{dt}\iint_A \overline{B}(\overline{r},t)\cdot\overline{dS} \qquad (6)$$

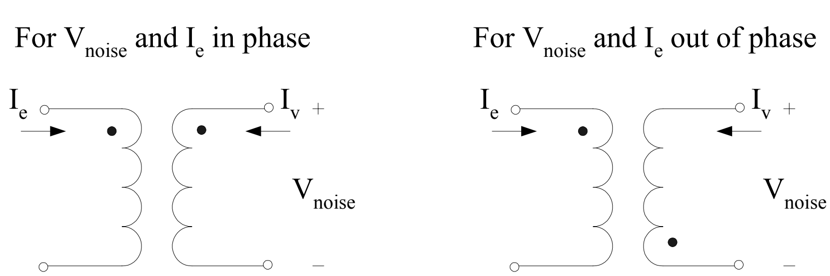
Since $\overline{B}(\overline{r},t)$ is a function of space, i.e. $\overline{r}$, doubling the area does not double the *emf*, because the flux is the integral over space of $\overline{B}(\overline{r},t)$.



2. As indicated in Exercise 1, the magnetic field resulting from the current in the emitter loop decreases as the distance from the loop increases. Then, as the separation between the emitter and victim loops increases, the coupled flux gets weaker, and the measured noise voltage decreases. Beyond a few centimeters, this decay is approximately $\frac{1}{r}$. Note however, that very near the emitter loop, the shape of the current loop (emitter) is important in determining the magnetic field, and it is not a simple $\frac{1}{r}$ decay.



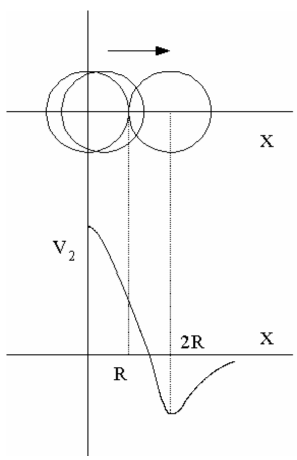
3. The two orientations of the loops for Cases 1 and 5 result in a coupled noise voltage that is approximately in phase, and 180° out of phase with the current in the emitter loop ($I_e = \frac{V_{10\Omega}}{10}$). This results in dots that are opposite in the equivalent circuit model to yield the sign difference as shown below.

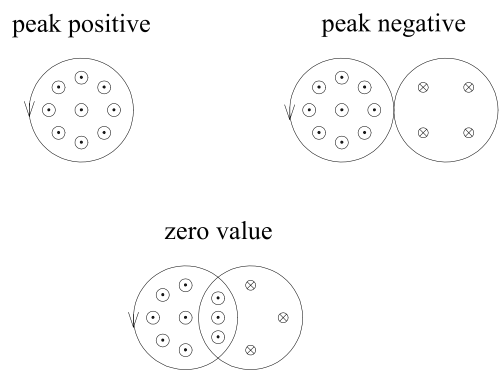


For V_{noise} and I_e in phase: $V_{noise} = j\omega L_v I_v + j\omega M I_e$

For V_{noise} and I_e out of phase: $V_{noise} = j\omega L_v I_v - j\omega M I_e$

4. For the two loops with the axes coincident, and the orientation of the victim loop such that the measured noise voltage is positive, the maximum positive voltage occurs. In this case, nearly all of the flux created by the current in the emitter loop is coupled through the area of the victim loop, since it is lying directly on top of the emitter loop as shown below. However, as the victim loop axis is moved off that of the emitter loop, flux is coupled to the victim loop in opposite directions in different areas as shown below. When the net positive flux equals the net negative flux coupling the victim loop, the measured noise voltage is zero. The peak measured negative voltage occurs when all of the flux coupled to the victim circuit is in the same direction, and the separation between the loops is zero. As the separation between the two loops increases, the magnetic field resulting from the current in the emitter circuit drops off rapidly, and the measured noise voltage decreases.





5. The measured noise voltage in the victim circuit is proportional to the derivative of the current in the emitter circuit as in Figure 5 below.

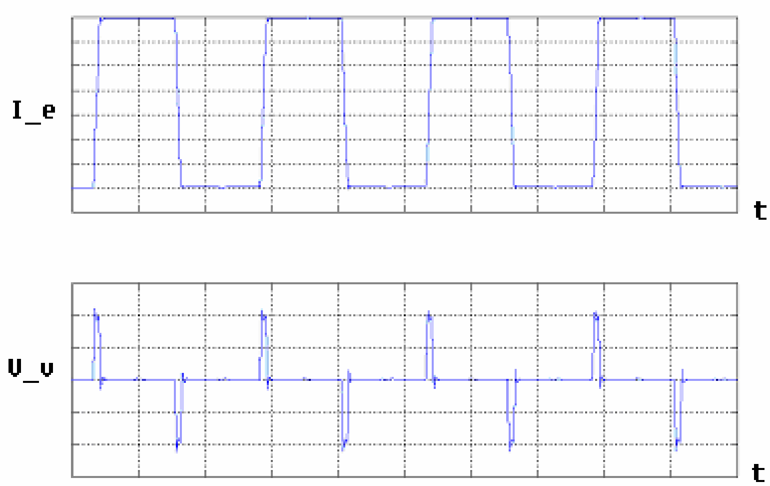


Figure 5

The mutual inductance can be obtained from Eq. (3) or a good approximation Eq. (4). Using Eq. (4)

$$M \approx \frac{V_{noise}}{j\omega I_e} \qquad (7)$$

Note that V_{noise} and I_e are complex. Further, the mutual inductance is $M = k\sqrt{L_e L_v}$, where k is the coupling coefficient, and L_e and L_v are approximately $4.6\mu H$. Approximate values for M at several distances are then

r (cm)	M (μH)
0	2.6
11	0.38
16.5	0.048