Appendix B
Tuned Circuits

Resonant Circuits

Resonant or *tuned circuits* are not as common these days as they used to be, but they are still essential in many communications devices. They are often called *LC circuits* because they contain inductors (L) and capacitors (C). There is also always some resistance which affects the operation of the circuit as we will see.

Depending on whether the capacitor and inductor are in parallel or series, tuned circuits fall into two categories as shown in Fig. B-1 — *parallel tuned* and *series tuned* circuits. Both of these have a *resonant frequency* which (neglecting the effects of resistance) is the frequency at which the X_L of the inductor is equal to the X_C of the capacitor. We can find this resonant frequency by solving the equation

$$X_L = X_C$$

$$2\,\pi\,f\,L = \frac{1}{2\,\pi\,f\,C}$$

where f is the frequency, L is the inductance, and C is the capacitance, for the frequency:

$$f_{resonant} = \frac{1}{2\,\pi\,\sqrt{L\,C}}$$

The operation of both circuits depends on the fact that the voltage and current in inductors and capacitors are 90 degrees out of phase. You may remember "ELI the ICE man" — this little phrase reminds us that the voltage (E) comes 90 degrees *before* the current (I) in the inductor (the current *lags* behind the voltage), but 90 degrees *after* the current in the capacitor (the current *leads* the voltage).

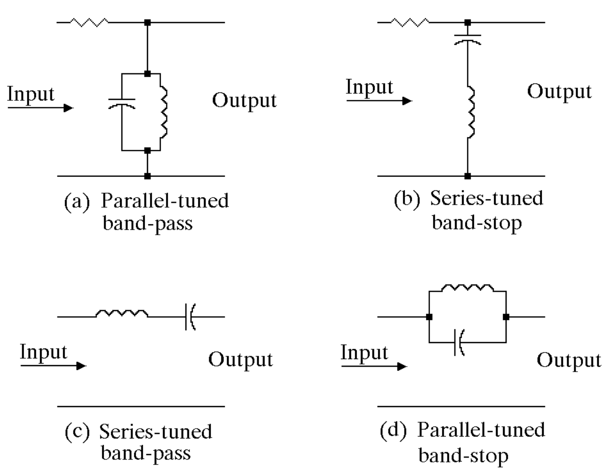

Fig. B-1. Parallel and Series Tuned Circuits

In the parallel-tuned circuit, the capacitor and inductor are in parallel, and they therefore have the same voltage. Now consider what happens at the resonant frequency where $X_L = X_C$; their reactances are equal and so they both have the same current. But because one of these currents leads the voltage by 90 degrees, whereas the other current lags the voltage by 90 degrees, they are 180 degrees apart. They therefore go in opposite directions — when one goes up, the other goes down. Hence the current in the wire which leads to the parallel-tuned circuit must be zero. Since the external current into the tuned circuit is zero, the circuit behaves like an open circuit (which also has a voltage but no current through it.)

The opposite happens in the series-tuned circuit. Here, both the capacitor and the inductor have the same current since they are in series. This time, the voltage across one of them leads the current, while the voltage in the other lags the current by 90 degrees. The two voltages are therefore 180 degrees apart. At resonance (which is another way of saying "at the resonant frequency"), their reactances are equal and so their voltages are equal, but opposite. The total voltage across the series circuit is therefore zero, even though there is a current through it. The circuit therefore behaves like a short circuit (which also has a current but no voltage across it.)

We therefore form the following rules of thumb:

- At resonance, a parallel-tuned circuit behaves like an open
- At resonance, a series-tuned circuit behaves like a short.
- At other frequencies, both circuits have some impedance. Close to the resonant frequency, the circuits are not quite an open (for the parallel-tuned) or short (for the series-tuned), but still fairly close to it. The further we go away from the resonant frequency, the less the circuits behave like an open or short circuit.

Both of these circuits can be used as selective filters to let some frequencies get through, and stop others. They can be connected in one of two basic ways — either the tuned circuit can be connected between a signal and ground as in Fig. B-1 (a) and (b) (in which case it will short some of the signal to ground, depending on the circuit's impedance), or so that a signal has to travel through the filter to get from the input to the output as shown in (c) and (d) (in which case more or less will get through, again depending on the circuit's impedance).

Consider circuit (a), for example. At resonance, the parallel-tuned circuit behaves like an open circuit, and most of the input signal travels right through the resistor to the output. Depending on the load at the output, there

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may be some current in the series resistor and so there may be some voltage loss, but this can be minimized by keeping the load resistance high. Away from resonance, however, the tuned circuit is no longer open; it causes an increased current to flow through the series resistor, and the voltage drop therefore increases. The further away from resonance, the greater this drop, and the smaller the output voltage.

If we keep the input voltage constant but vary the frequency, and then plot the output voltage vs. frequency, we get a plot similar to Fig. B-2 (a). We see that the peak in output occurs at the resonant frequency, and there is a dropoff on both sides. There is a band of frequencies around resonance that do get through, while frequencies far away from resonance are reduced (though not entirely stopped). Since there is this band of frequencies that get through, this is called a *band-pass* filter. Circuit (c) in Fig. B-1 is also a band-pass filter; since the series-tuned circuit is a short circuit at resonance, frequencies at (and near) resonance get through, while frequencies farther away are reduced because the series-tuned circuit now has some reactance.

Circuits (b) and (d) do the opposite — at resonance, they stop the signal; circuit (b) does it by shorting the signal, whereas circuit (d) does it by opening the path between the input and output. Even near resonance they reduce the signal, so they stop (or reduce) a band of frequencies, as shown in Fig. B-2 (b). They are therefore called *band-stop* or *band-reject* filters.

Bandwidth

Returning to Fig. B-2, we're interested in measuring how wide a band of frequencies gets through the filter;

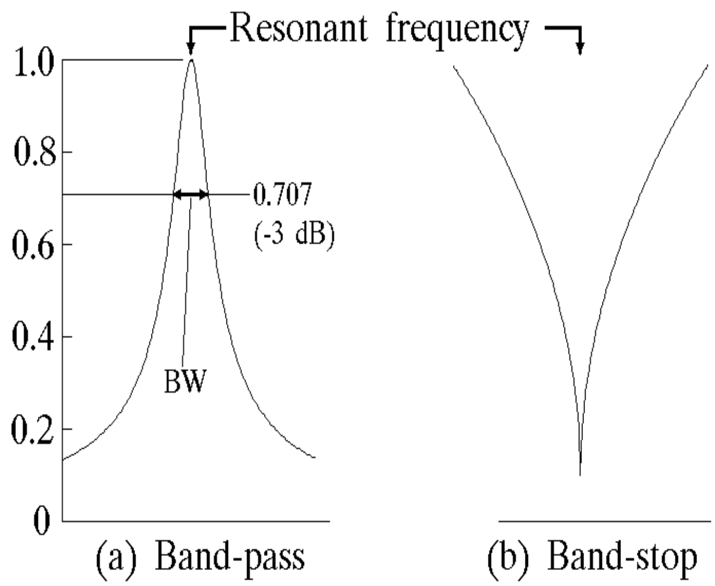

Fig. B-2. Band-pass and Band-stop filter responses

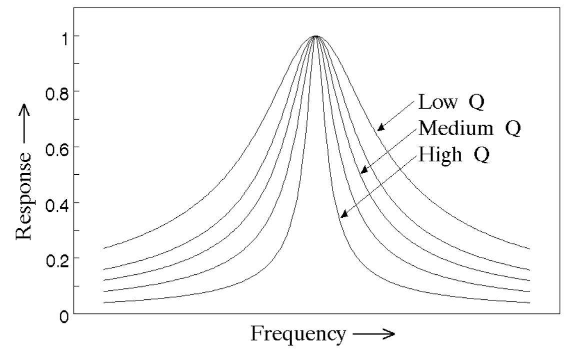

Fig. B-3. Effect of Q on bandwidth

that is, we want to know the *bandwidth.* Clearly the width of the curve depends on where you measure it; the customary point is to measure the bandwidth at the point where the height of the curve is 70% of the maximum height; this is labelled BW in Fig. B-2. (To be exact, the amplitude is $1/\sqrt{2}$ or 0.707 of the maximum.) This point also happens to be 3 dB below the maximum, as we can see from

$$20 \log_{10} \frac{0.707}{1} = 20 \times -0.15 = -3 \text{ dB}$$

and so it is often called the "–3 dB point" or "half-power point" (see the very end of Appendix A for an explanation.).

Quality or "Q"

The narrower the bandwidth, the "better" the circuit, and so we define the *Q* or *Quality* of a tuned circuit as

$$Q = \frac{\text{resonant frequency}}{\text{bandwidth}}$$

For example, if a circuit resonant at 1 MHz has a bandwidth of 50 kHz (at the –3 dB point), then the Q would be 20; if the bandwidth is only 40 kHz, then the Q would be 25, which would be considered "better" or "higher quality" for some applications. Fig. B-3 shows how the Q affects the response and bandwidth of a tuned circuit.

The Q, in turn, depends on the resistance in the circuit. Ideally, a tuned circuit would consist of only capacitance and inductance; with no added resistance; the Q would be infinite because the circuit would be ideal. In practice, however, there is always some resistance in a circuit, and this degrades the quality.

The added resistance could be in one of two places: it could be in series with the in-

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ductor (for instance, every inductor consists of wire that has some resistance), or it could be in parallel with it (such as the resistance of whatever other circuitry the tuned circuit is connected to). If X_L is the reactance of the inductor at the resonant frequency, then the Q is

$$Q_{series} = \frac{X_L}{R_{series}}$$

if the resistance is in series with the inductor, and it is

$$Q_{parallel} = \frac{R_{parallel}}{X_L}$$

if the resistance is in parallel. Note that, since R and X_L are both in ohms, Q has no units.

In most cases, there are resistances both in series and in parallel with the inductor. In that case, we need to take both effects into our calculations. There are some fairly complex equations used in circuit theory which give us the total Q, but a much easier way of finding the equivalent total Q is

$$Q_{total} = \frac{Q_{series}\, Q_{parallel}}{Q_{series} + Q_{parallel}}$$

You may recognize that this is the same format as the "product over the sum" formula for parallel resistors.

Example

Let's do a sample calculation: what is the bandpass for the circuit in Fig. B-4? First, find the resonant frequency:

$$f = \frac{1}{2\,\pi\,\sqrt{L\,C}} = \frac{1}{2 \times 3.14159 \times \sqrt{10^{-3} \times 10^{-6}}} = 5033 \text{ Hz}$$

Actually, if we used the full equation that many textbooks give for the resonant frequency when there is a resistor inside the tuned circuit, we would find that the resonant frequency is about 5039 Hz. But the difference is less than 1⁄10%, and so our equation is perfectly adequate.

Next, find X_L at this frequency:

$$X_L = 2\,\pi\,f\,L = 2 \times 3.14159 \times 5033 \times 10^{-3} = 31.623\ \Omega$$

There is a 1-ohm resistor in series with the inductor, so the series Q is

$$Q_{series} = \frac{X_L}{R_{series}} = \frac{31.623}{1} = 31.623$$

What about the parallel resistance? If we assume that the signal generator has zero output resistance and imagine that we are looking outward from the inductor at whatever resistances are outside, we would see the 500-ohm resistor in parallel with the inductor. (We could prove that more rigorously by applying Thevenin's theorem). Hence we use 500 ohms in the parallel Q formula:

$$Q_{parallel} = \frac{R_{parallel}}{X_L} = \frac{500}{31.623} = 15.811$$

The total Q is therefore

$$Q_{total} = \frac{Q_{series}\, Q_{parallel}}{Q_{series} + Q_{parallel}} = \frac{31.623 \times 15.811}{31.623 + 15.811} = 10.54$$

Note how the total Q is a combination of the series and parallel Q. The fact that the equation has the same form as the "product over the sum" equation for parallel resistors means that we can apply similar reasoning to the Q as we can to parallel resistors. That is,

- When two resistors are in parallel, the total parallel resistance is always smaller than the smaller resistor. Likewise, the total Q is always smaller than either the parallel Q or the series Q.
- If one of two parallel resistors is much larger than the other, we can approximate the total parallel resistance by ignoring the larger resistor. The same applies to the Q. For example, if the series and parallel Q were 5 and 50, the total Q would be 4.55, which is very close to 5, the smaller Q. Hence it is the smaller Q that plays a major role in setting the total Q.

We now find the bandwidth from

$$Q = \frac{\text{resonant frequency}}{\text{bandwidth}}$$

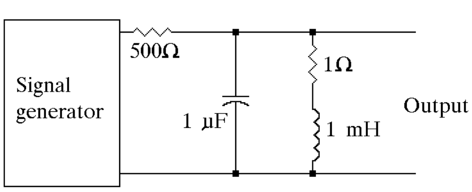

Fig. B-4. Example for calculating the Q

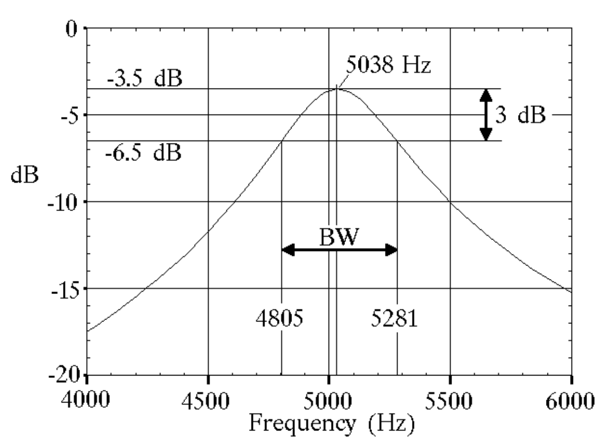

Fig. B-5. Response of circuit in Fig. B-4

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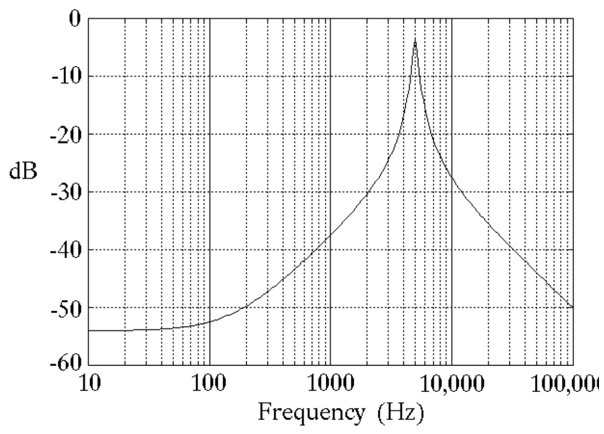

Fig. B-6. Frequency response plotted on log paper

$$\text{bandwidth} = \frac{\text{resonant frequency}}{Q} = \frac{5033\text{ Hz}}{10.54} = 477\text{ Hz}$$

Fig. B-5 shows the actual bandpass as measured in the laboratory and then plotted. Note how we customarily use a dB scale for the amplitude. At the peak (which occurs at the resonant frequency) there is a loss of approximately 3.5 dB in the circuit, so the peak plots at –3.5 dB.

The measured resonant frequency is 5038 Hz, the lower 3-dB frequency (the lower frequency at which the gain drops by 3 dB, from –3.5 dB to –6.5 dB) is 4805 Hz, and the upper 3-dB frequency is 5281 Hz. The measured bandwidth is thus

$$5281\text{ Hz} - 4805\text{ Hz} = 476\text{ Hz}$$

which is quite close to the calculated value.

Incidentally, the frequency scale in Fig. B-5 is linear; that is, frequency increments are evenly spaced left to right, so that the distance from 4000 Hz to 5000 Hz is equal to the distance from 5000 to 6000 Hz. This is fairly common when only a small range of frequencies is plotted; when plotting large spans of frequencies, however, we generally use a logarithmic frequency scale, as shown in Fig. B-6. Here equal spacing along the frequency axis is given to each *decade* (10-to-1 frequency ratio), so that the spacing from 10 to 100 Hz is the same as from 100 to 1000 Hz, and so on. While this looks as though it unduly compresses the high frequencies, a given percentage change in frequency occupies the same amount of space at all points on the graph.

Conclusion

You may wonder why we've reviewed resonant circuits to such depth, when you have probably already studied them in a previous course. The reason is that resonance and bandwidth are important phenomena in many analog communications circuits. For example, Fig. B-7 shows the coupling circuit in a typical small transistor AM radio.

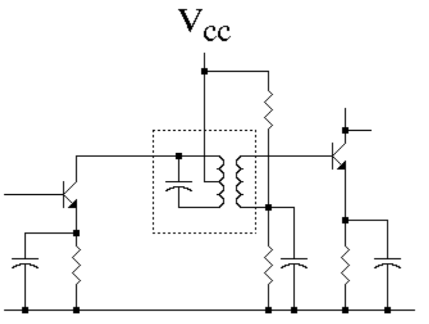

Fig. B-7. Radio receiver coupling circuit

You will note that there is a transformer which couples the signal from one transistor stage to the next. But the primary of the transformer is connected to a capacitor, which makes the primary resonant. The purpose of this circuit in the radio is to help select the one station you wish to listen to, and reject other stations on the dial. If the bandwidth were too large (meaning that the Q is too small) the circuit would let through undesired adjacent stations.

Just as the 500-ohm resistor in Fig. B-4 was in parallel with the tuned circuit, so the collector output impedance of the left-hand transistor in Fig. B-7 is in parallel with the tuned circuit, thus reducing the Q.

You will note, however, that all of the transformer primary is used with the tuned circuit, but only part of it actually connects between the transistor and the V_{cc}power line. Since only part of the primary is actually used in the collector circuit, this reduces the inductance (and therefore the X_L) in the collector circuit. Since the parallel Q is

$$Q_{parallel} = \frac{R_{parallel}}{X_L}$$

reducing the X_L increases the Q.

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