NATIONAL UNIVERSITY *of* SINGAPORE

DEPARTMENT OF
ELECTRICAL & COMPUTER ENGINEERING

Course: EE2009 - SIGNALS

EXPERIMENT S2
FREQUENCY MODULATION

LABORATORY MANUAL
(2003/2004)

IMPORTANT! *YOU ARE REQUIRED TO READ THIS MANUAL CAREFULLY AND PREPARE FOR THE EXPERIMENT BEFORE YOU ATTEND THE LABORATORY SESSION. YOU MAY WISH TO TAKE A LOOK AT http://vlab.ee.nus.edu.sg/vlab/, CLICK ON "LABORATORY" TO GET ACCESS TO A SIMILAR WEB-BASED VERSION OF THIS EXPERIMENT.*

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1. Introduction

In this experiment, you will use a *spectrum analyser* to study the waveforms and spectra of *frequency modulated* signals. After completing the experiment, you should have acquired some proficiency on the use of this important but complicated instrument. Also, you should be familiar with the characteristics and design of frequency modulation systems.

To send it over great distance, a message signal has to be suitably modified or modulated to give rise to a travelling wave with wavelength that is comparable to the size of the antenna used. There are many types of modulations. Two common ones are *Amplitude Modulation* (AM) and *Frequency Modulation* (FM). In all these schemes, a high frequency carrier signal is modified so that some of its features are made to change according to the input message signal. In a certain sense, the carrier signal acts as a vehicle for the message signal, while the message signal is the passenger taking the vehicle.

In the case of FM, the frequency of the carrier signal is modified as a function of the message signal. In this experiment, you will investigate the characteristics of FM and see how the modulation is carried out. In particular, you will examine the frequency behaviour, or spectrum, of the resulting FM waveform using an important but rather complicated instrument called the spectrum analyser.

Suppose your have a message signal $x_m(t)$, say from your wireless phone, that has to be transmitted over air. Using a carrier $a_c \cos(2\pi f_c t)$ of amplitude a_c and frequency f_c, a FM system will produce and send the modulated signal

$$x(t) = a_c \cos\left[2\pi f_c t + 2\pi k_c \int_{-\infty}^{t} x_m(\tau)\tau\right],$$

where k_c is the *frequency sensitivity* of the modulator. This, together with the amplitude of the message signal, determines the extent of allowable frequency variation.

The phase of the FM signal is

$$\theta(t) = 2\pi f_c t + 2\pi k_c \int_{-\infty}^{t} x_m(\tau)d\tau \ .$$

The *instantaneous frequency* at time t, given by the derivative of the phase, is

$$f(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt} = \frac{1}{2\pi}\frac{d}{dt}\left[2\pi f_c t + 2\pi k_c \int_{-\infty}^{t} x_m(\tau)d\tau\right] = f_c + k_c x_m(t).$$

Clearly, the instantaneous frequency $f(t)$ is linearly proportional to the message signal $x_m(t)$. Figure 1 shows a sinusoidal message signal and the corresponding frequency modulated signal. As the value of $x_m(t)$ increases, the instantaneous frequency of the carrier increases, and vice versa.

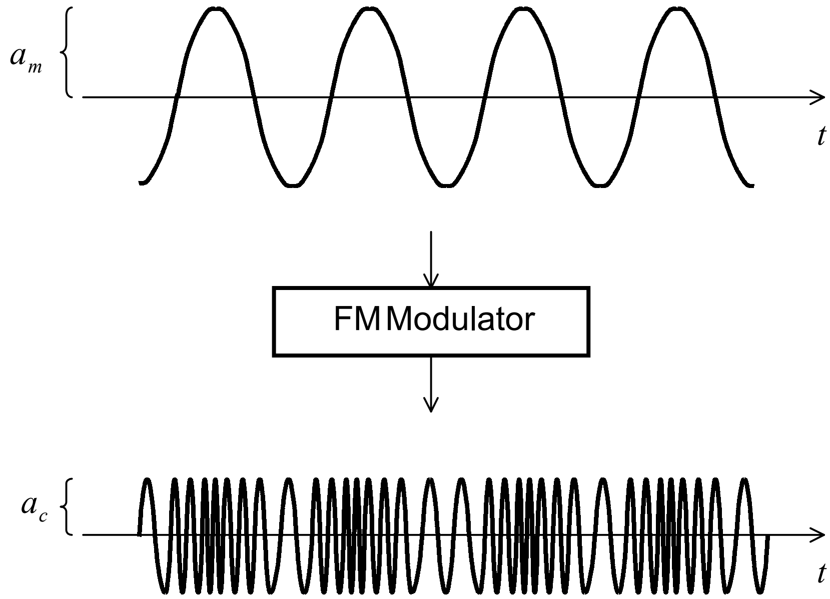


Fig. 1. A sinusoidal message signal and its FM modulated carrier

For the special but interesting case that the message signal is a single tone, such as $x_m(t) = a_m \cos(2\pi f_m t)$, the instantaneous frequency is

$$f(t) = f_c + k_c x_m(t) = f_c + k_c a_m \cos(2\pi f_m t) = f_c + \Delta f \cos(2\pi f_m t).$$

Under this situation, the *peak frequency deviation* (maximum variation in the frequency of the carrier) is $\Delta f = k_c a_m$. Mathematically, $f_c - \Delta f \le f(t) \le f_c + \Delta f$.

The spectrum of FM modulated signal is in general very complicated. Even for the above special single tone case which is often used for testing purposes, the magnitude spectrum of $x(t)$ contains an infinite number of sinusoidal spectral components (called *sidebands*) separated by f_m and symmetrically distributed about f_c. This is shown in Fig. 2.

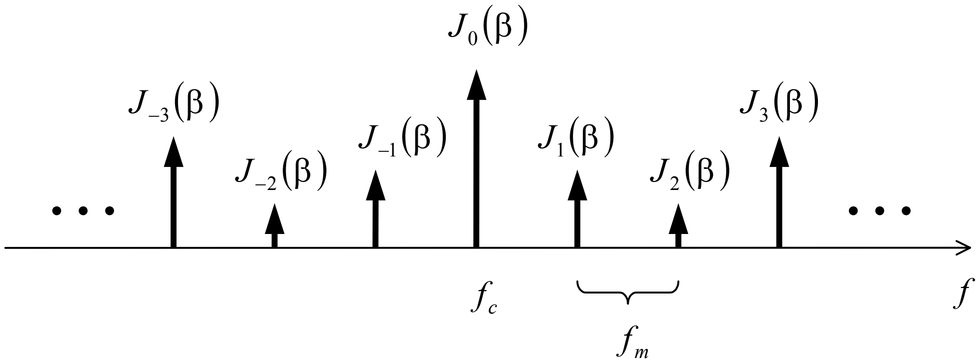


Fig. 2. Magnitude spectrum of a FM modulated signal for a sinusoidal message

The amplitudes of the spectral components are given by Bessel functions of the first kind $J_n(\beta)$, where n is an integer index identifying an individual spectral component and

$$\beta = \frac{\Delta f}{f_m}$$

is the *modulation index*. In general, the modulation index is the ratio of the peak frequency deviation in the carrier, Δf, to the highest frequency component of the message signal, which is f_m for the special case of a sinusoidal test tone. It is thus a measure of the *bandwidth expansion* that results when FM is used, and is an important parameter in any system design.

As illustrated in Fig. 2, the magnitude of the spectral component depends on n and β. Indeed, while not clearly shown, some components may actually have negligible amplitudes. For the very special case of an unmodulated carrier (no message or $x_m(t)=0$), there will be no sidebands or, mathematically, $J_0(\beta)=1$ and $\cdots = J_{-2}(\beta) = J_{-1}(\beta) = J_1(\beta) = J_2(\beta) = \cdots = 0$.

From Fig. 2 and noting that the spectrum is symmetric about the carrier f_c, the average power of the FM signal is

$$p = \frac{a_c^{\,2}}{2}\sum_{n=-\infty}^{\infty} J_n^{\,2}(\beta) = \frac{a_c^{\,2} J_0^{\,2}(\beta)}{2} + a_c^{\,2}\sum_{n=1}^{\infty} J_n^{\,2}(\beta).$$

This average power can also be obtained from the time variation of the modulated signal as given by Fig. 1. What is its value? Can you then show that

$$\frac{J_0^{\,2}(\beta)}{2} + \sum_{n=1}^{\infty} J_n^{\,2}(\beta) = \frac{1}{2}.$$

As illustrated in Fig. 2 for a single message tone, a FM modulated signal has a spectrum that covers all frequencies. For this to be sent without any distortion, an infinite bandwidth is required. Bandwidth is however expensive and has to be shared. Most telecommunication companies who wish to use a certain part of the spectrum to set up communication systems for their customers have to bid and pay for it. Also, after having successfully bided for a licence to use part of the radio spectrum (or *channels*), they have to *bandlimit* their signals so that what is being transmitted does not spill over to other channels and cause interference for other users.

Fortunately, in many applications such as voice communication, it turns out that, for any β, a large portion of the FM signal power is confined to the sidebands within some finite bandwidth. Thus, no serious distortion of the signal results if the sidebands outside this bandwidth are lost. Experimentally, it is found that the distortion resulting from bandlimiting a FM signal is tolerable as long as 98% or more of the FM signal power is passed by the bandlimiting (or bandpass) filter.

Using the above 98% rule to determine the bandwidth of a FM signal requires experimental and simulation studies, as the FM spectrum is complicated and depends on the message signal and its statistics. Another simple but not very exact method is based on taking $J_m(\beta)=0$ for $m>\beta$. While this is only very roughly correct for the Bessel function $J_n(\beta)$, the implication is that there are only $2m$ sidebands with significant magnitudes for us to consider in the FM spectrum. From the spectrum of Fig. 2, the FM signal thus has a bandwidth of roughly

$$f_w = 2(1+\beta)f_m = 2(\Delta f + f_m),$$

which is also known as Carson's rule.

For *narrowband FM* where $\beta << 1$, the bandwidth is approximately $2f_m$, the same value as for conventional AM. For *wideband FM* where $\beta >> 1$ and which is more commonly used for radio broadcast, it is approximately $2\Delta f$ or twice the peak frequency deviation.

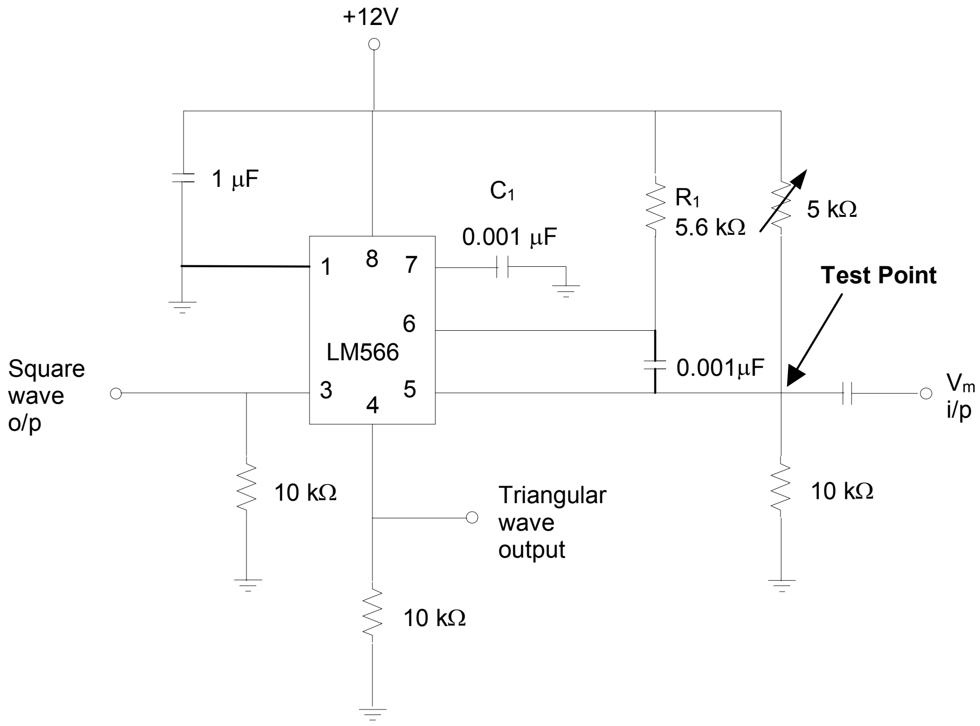
The 98% and Carson's rule are not the only ways for finding out the bandwidth of a FM signal. Others have also been proposed. An example is the 99% rule based on retaining all spectral components whose amplitudes exceed 1% of the amplitude of an unmodulated carrier signal.

Suppose you are designing a FM scheme for a broadcasting station that has to transmit 15 kHz bandwidth audio signal to listeners on a channel called SuperFM95. The radio bandwidth available to you has a bandwidth of 150 kHz. How would you design the FM system or, in order words, what need to be done before you are able to determine the frequency sensitivity k_c? Concentrate on the thinking and the procedure, and assume that technical measurement work will be done by a technical officer working with you.

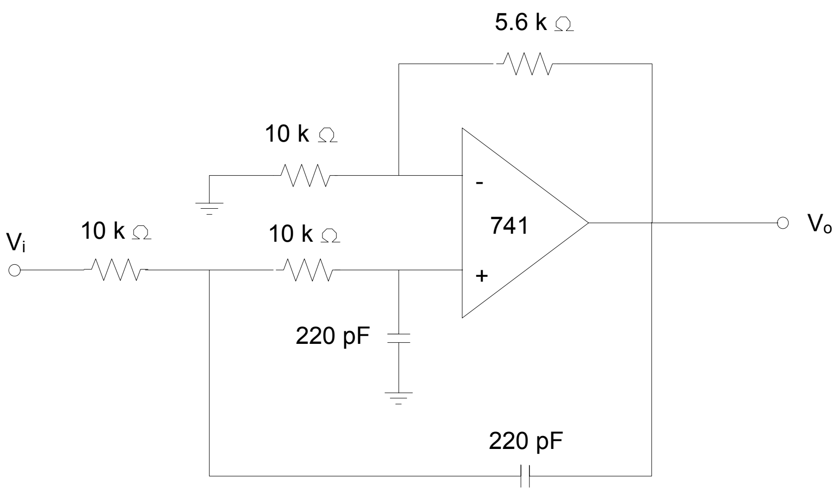
3. Equipment

Frequency modulator circuit board
Frequency meter
Oscilloscope/ Spectrum analyzer
Signal generator
Power supply
Digital voltmeter

The frequency modulator circuit of Fig. 3 consists of a LM566 *Voltage Controlled Oscillator* (VCO) with the associated timing and biasing components. The frequency of oscillation is determined by the value of C1, R1 and the voltage at pin 5. A 0.001 μF capacitor is connected between pins 5 and 6 to prevent parasitic oscillations that may occur during VCO switching. A 5 kΩ variable resistor is also provided for adjusting the VCO centre frequency to the desired carrier frequency. Two output waveforms, a square wave and a triangular wave are available from the VCO. However, a sine wave output can be obtained by passing the triangular wave output through the second order low pass filter provided.



(a) VCO Circuit.



(a) Low pass filter circuit.

Fig. 3 FM modulator circuit.

4. Experiment

4.1 VCO characteristic

We will first measure the VCO characteristic, which controls basically the frequency sensitivity k_c, before studying the spectrum of the FM signal.

Set $a_m = 0$ so that the system produces only the carrier with no modulation. Vary the 5 kΩ resistor, and plot the VCO output frequency as a function of the input voltage at pin 5 over the range 8.5 V to 11.5 V. Both frequency and voltage should be measured accurately with a frequency counter and voltmeter.

Figure out how the value for k_c can be obtained from the slope of the characteristic.

4.2 FM system design

Connect the signal generator to the circuit so that it becomes the message signal. Adjust the various controls such that the carrier frequency is 45 kHz, the message is sinusoidal at 1 kHz, and the modulation index β is 3.

Note that you will need to use the VCO characteristics measured to design your FM system. Also, the modulation index is the ratio of the peak frequency deviation to the maximum input frequency.

4.3 Spectral measurement

Using the spectrum analyser (see the Appendix on how to carry out FFT measurements), measure the spectrum of the FM output signal from the low pass filter. From your measurements, determine the total average power of the FM signal and compare it with that of the unmodulated carrier.

Verify the measured spectrum by using the following table on the Bessel function $J_n(\beta)$:

	β					
n	1	2	3	4	5	6
0	.7652	.2239	-.2601	-.3971	-.1776	.1506
1	.4401	.5767	.3391	-.06604	-.3276	-.2767
2	.1149	.3528	.4861	.3641	.04657	-.2429
3	.01956	.1289	.3091	.4302	.3648	.1148
4	.002477	.0340	.1320	.2811	.3912	.3576
5		.00704	.04303	.1321	.2611	.3621
6		.001202	.01139	.04909	.1310	.2458
7			.002547	.01518	.05338	.1296
8				.004029	.01841	.05653
9					.005520	.02117
10					.001468	.006964
11						.002048

What is the bandwidth needed for 98% power transmission? Compare this with the value using Carson's rule.

Note that the spectral peaks are measured in terms of $\mathrm{dBV} = 20\log(v_{rms})$, and that the average power of a signal is the square of its root mean square value v_{rms}.

Repeat 4.2 and the above for $\beta = 0.2$. Comment on your results. Write down anything interesting you see (be it on the shape and behaviour of the spectra) and any deduction or thinking behind your observation.

4.4 Other message signals

Observe and think about the spectra for a square wave message or modulating signal of 1 kHz and a peak frequency deviation of 6 kHz. While keeping to the scheduled laboratory session, select any other message signals that you feel interesting and find out what happens. Assume that you have been commissioned to investigate how the FM spectrum changes under different situations. Give some brief concluding remarks on your investigation.

5. References

[1] S. Haykin, An Introduction to Analog & Digital Communications, John Wiley, 1989, pp.322 - 368.

[2] M. Schwartz, Information Transmission, Modulation and Noise, 3rd Edition, McGraw-Hill International, 1980, pp. 259 - 288.

[3] H. Taub and D.L. Schilling, Principles of Communication Systems, McGraw-Hill, 1971, pp. 113 - 156.

Appendix – Fast Fourier Transform (FFT) Measurements

The following steps explain how to measure the amplitude spectrum of a signal using the HP 54600-Series oscilloscope. This oscilloscope captures a segment of the input signal and computes the signal spectrum using the Fast Fourier Transform (FFT).

Preliminary Settings:

1. Press ± .
2. Set the **Function 2** softkey located in front of the oscilloscope screen to **On**.
3. Press the **Function 2 Menu** softkey.
4. Set the **Operand** softkey to take the source signal from channel **1**.
5. If **FFT** has not yet been selected, press the **Operation** softkey until **FFT** is selected.
6. Press the **FFT Menu** softkey to activate the FFT menu display.
7. Set the **Autoscale FFT** to **On**. This will automatically set the vertical sensitivity (Units/div) and reference level (RefLevl) to properly display the FFT output.

Measuring the FFT:

When the FFT is displayed on the screen, you can centre the peak, readjust the frequency span (to zoom-in on the peak), determine the location of the peak(s) and their power levels in dbV.

1. Press the **Cursors** key to activate cursors. A new cursor menu display will appear at the bottom of the display.
2. Ensure that the **Source** is set to **F2**.
3. Press **Find Peaks** softkey to lock on the largest and second largest peaks. Note that the display will also show the locations of the two respective peaks, denoted as **V1(F2)** and **V2(F2)** and the frequency separation between them.
4. Press **Move f1 to Center** softkey to centre the largest peak in the middle of the display.
5. You can next zoom-in on the peak by resetting the frequency span. First press the ± softkey to bring back the FFT menu. Then ensure that the **Freq Span** softkey is set **On**. Then turning the knob nearest the **Cursors** key.
6. Next, you can determine the power levels of the spectral peaks. First you go back to the cursors menu by pressing the **Cursors** key. Then set the **V1** softkey under **Active Cursors** to **On**. The power levels of the 2 largest peaks are then displayed below the plots. Note that two dashed horizontal lines appear on the display when this softkey is pressed.
7. Turning the knob nearest the **Cursors** key, the horizontal dashed line can be moved. This will help you determine the power levels of other peaks in the display. Note that as you turn the knob the horizontal dashed line moves and the power level of this line changes accordingly.

8. Finally, to print the display, press the **Print/Utility** key, followed by the $\boxed{\textbf{Print Screen}}$ softkey on the left below the display.