

# Impedance matching and the Smith chart – The fundamentals.

*Tried and true, the Smith chart is still the basic tool for determining transmission line impedances.*

*By K-C Chan & A. Harter*

When dealing with the practical implementation of RF applications there are always some tasks that appear nightmarish. One of these is the need to match the different impedances of the interconnected block. Typically these include the antenna to low-noise amplifier (LNA), RF output (RFOUT) to antenna, LNA output to mixer input, etc. The matching task is required for a proper transfer of signal and energy from a "source" to a "load".

At high radio frequencies, the spurious (wires inductances, interlayers capacitances, conductors resistances, etc) elements have a significant, yet unpredictable impact upon the matching network. Above a few tenths of MHz, theoretical calculations and simulations are often insufficient. In-situ RF lab

measurements, along with tuning work, have to be considered for determining the proper final values. The computational values are required to set up the type of structure and target component values.

There are many possible ways to do impedance matching. Some are:

- Computer simulations—Complex to use since such simulators are dedicated for differing design functions and not to impedance matching. The

designer has to be familiar with the multiple data inputs that need to be entered and the correct formats. They also need the expertise to find the useful data among the tons of results coming out. In addition, circuit simulation software is not pre-installed on computers unless they are dedicated to such an application.

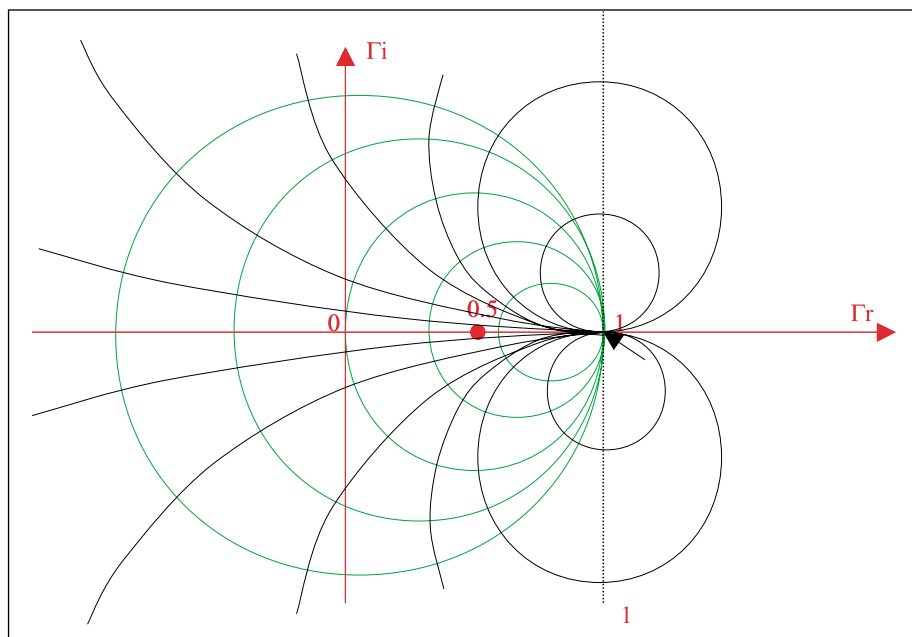
- Manual computations–Tedious due to the length ("kilometric") of the equations and the complex nature of

cal illustrations of parameters such as finding matching network componentS values. Of course, matching for maximum power transfer is not the only thing one can do with Smith charts. They can also help the designer optimize for the best noise figures, insure quality factor impact, asses stability analysis, etc.

## A quick primer

Before introducing the Smith chart utilities, it would be prudent to present a short refresher on wave propagation phenomenon for IC wiring under RF conditions (above 100 MHz). This can be true for contingencies such as RS485 lines, between a PA and an antenna, between a LNA and down-converter/mixer, etc.

It is well known that to get the maximum power transfer from a source to a load, the source impedance must equal the complex conjugate of load impedance, or :



## Fundamentals of impedance and the Smith chart.

the numbers to be manipulated.

- **Instinct**—This can be acquired only after one has devoted many years to the RF industry. In short, this is for the super-specialist!

- **Smith Chart**—Upon which this article concentrates.

The primary objective of this article is to refresh the Smith chart's construction and background, and to summarize practical ways to use it.

Topics addressed will include practi-

$$R_s + jX_s = R_L - jX_L \quad (1)$$

For this condition, the energy transferred from the source to the load is maximized. In addition, for efficient power transfer, this condition is required to avoid the reflection of energy from the load back to the source. This is particularly true for high frequency environments like video lines and RF and microwave networks.

### What it is

A Smith chart is a circular plot with a lot of interlaced circles on it. When correctly used, matching impedances,

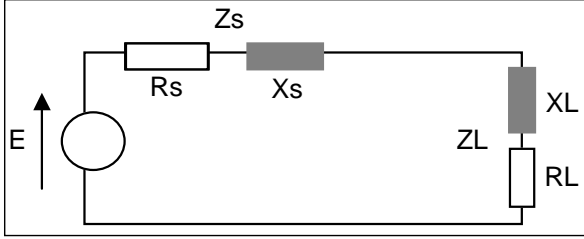


Figure 1. Diagram of  $R_s + jX_s = R_L - jX_L$

with apparent complicate structures, can be made without any computation. The only effort required is the reading and following of values along the circles.

The Smith chart is a polar plot of the complex reflection coefficient (also called gamma and symbolized by  $\Gamma$ ). Or, mathematically defined as the 1-port scattering parameter  $s$  or  $s_{11}$ .

A Smith chart is developed by examining the load where the impedance must be matched. Instead of considering its impedance directly, one expresses its reflection coefficient  $\Gamma_L$ , which is used to characterize a load (such as admittances, gain, transconductances, etc). The  $\Gamma_L$  is more useful when dealing with RF frequencies.

We know the reflection coefficient is defined as the ratio between the reflected voltage wave and the incident voltage wave :

$$\Gamma = \frac{V_{refl}}{V_{inc}}$$

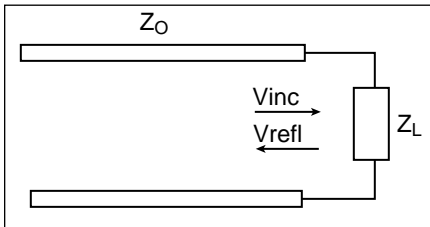


Figure 2. Impedance at the load.

The amount of reflected signal from the load is dependent on the degree of mismatch between the source impedance and the load impedance. Its expression has been defined as follows:

$$\Gamma = \frac{V_{refl}}{V_{inc}} = \frac{Z_L - Z_0}{Z_L + Z_0} = \Gamma_r + j\Gamma_i \quad (2.1)$$

Since the impedances are complex numbers, the reflection coefficient will be a complex number as well.

In order to reduce the number of unknown parameters, it is useful to freeze the ones that appear often and are common in the application. Here  $Z_0$  (the characteristic impedance) is often a constant and a real industry normalized value ie: 50  $\Omega$ , 75  $\Omega$ , 100  $\Omega$ , 600  $\Omega$ , etc. We can then define a normalized load impedance by:

$$z = Z_L/Z_0 = (R + jX) / Z_0 = r + jx \quad (2.2)$$

With this simplification, we can rewrite the reflection coefficient formula as:

$$\begin{aligned} \Gamma_L = \Gamma_r + j\Gamma_i &= \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{\frac{Z_L}{Z_0} - 1}{\frac{Z_L}{Z_0} + 1} \\ &= \frac{z - 1}{z + 1} = \frac{r + jx - 1}{r + jx + 1} \end{aligned} \quad (2.3)$$

Here one can see the direct relationship between the load impedance and its reflection coefficient. Unfortunately the complex nature of the relation is not practically useful, so we can use the Smith chart is a type of graphical representation of the above equation.

To build the chart, the equation must be re-written to extract standard geometrical figures (likes circles or stray lines).

First, equation 2.3 is reversed to give:

$$z = r + jx = \frac{1 + \Gamma_L}{1 - \Gamma_L} = \frac{1 + \Gamma_r + j\Gamma_i}{1 - \Gamma_r - j\Gamma_i} \quad (2.4)$$

and,

$$r = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{1 + \Gamma_r^2 - 2\Gamma_r + \Gamma_i^2} \quad (2.5)$$

By setting the real parts and the imaginary parts of (equation 2.5) equal, we obtain two independent new relationships:

$$r = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{1 + \Gamma_r^2 - 2\Gamma_r + \Gamma_i^2} \quad (2.6)$$

$$x = \frac{2\Gamma_i}{1 + \Gamma_r^2 - 2\Gamma_r + \Gamma_i^2} \quad (2.7)$$

Equation (2.6) is then manipulated, by developing equations (2.8) through (2.13), into the final equation (2.14). This equation is a relationship in the form of a parametric equation  $(x-a)^2 + (y-b)^2 = R^2$ , in the complex plane ( $\Gamma_r, \Gamma_i$ ), of a circle centered at the coordinates  $(r/r+1, 0)$ , and having a radius of  $1/(1+r)$ .

$$r + r\Gamma_r^2 - 2r\Gamma_r + r\Gamma_i^2 = 1 - \Gamma_r^2 - \Gamma_i^2 \quad (2.8)$$

$$\Gamma_i^2 + r\Gamma_r^2 - 2r\Gamma_r + r\Gamma_i^2 + \Gamma_i^2 = 1 - r \quad (2.9)$$

$$(1 + r)\Gamma_i^2 - 2r\Gamma_r + (r + 1)\Gamma_r^2 = 1 - r \quad (2.10)$$

$$\Gamma_r^2 - \frac{2r}{r+1}\Gamma_r + \Gamma_r^2 = \frac{1-r}{1+r} \quad (2.11)$$

$$\begin{aligned} \Gamma_r^2 - \frac{2r}{r+1}\Gamma_r + \frac{r^2}{(r+1)^2} + \Gamma_i^2 \\ - \frac{r^2}{(r+1)^2} = \frac{1-r}{1+r} \end{aligned} \quad (2.12)$$

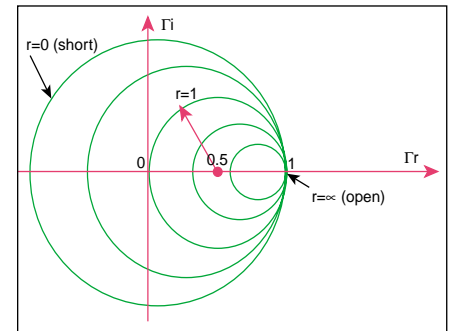


Figure 3. The points situated on a circle are all the impedances characterized by a same real impedance part value. For example, the circle,  $R = 1$ , is centered at the coordinates  $(0.5, 0)$  and has a radius of 0.5. It includes the point  $(0, 0)$  which is the reflection zero point (the load is matched with the characteristic impedance). A short-circuit, as a load, presents a circle centered at the coordinate  $(0, 0)$  and has a radius of 1. For an open-circuit load, the circle degenerates to a single point (centered at 1, 0 and has a radius of 0). This corresponds to a maximum reflection coefficient of 1, at which all of the incident wave is totally reflected.

$$\left(\Gamma_r - \frac{r}{r+1}\right)^2 + \Gamma_i^2 = \frac{1-r}{1+r} + \frac{r^2}{(1+r)^2} = \frac{1}{(1+r)^2} \quad (2.13)$$

$$\left(\Gamma_r - \frac{r}{r+1}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r}\right)^2 \quad (2.14)$$

(See Figure 3 for further details)

When developing the Smith chart, there are certain precautions that should be noted. Among the more important are:

- All the circles have one same, unique intersecting point at the coordinate (1, 0).
- The zero  $\Omega$  circle where there is no resistance ( $r = 0$ ) is the largest one.
- The infinite resistor circle is reduced to one point at (1, 0).
- There should be no negative resistance. If one (or more) should occur, we will be faced with the possibility of oscillatory conditions.
- Another resistance value can be chosen by simply selecting another circle, corresponding to the new value.

### Back to the drawing board

Moving on, we use equations (2.15) through (2.18) to further develop equation (2.7) into another parametric equation. This results in equation (2.19).

$$x + x\Gamma_r^2 - 2x\Gamma_r + x\Gamma_i^2 = 2\Gamma_i \quad (2.15)$$

$$1 + \Gamma_r^2 - 2\Gamma_r + \Gamma_i^2 = \frac{2\Gamma_i}{x} \quad (2.16)$$

$$\Gamma_r^2 - 2\Gamma_r + 1 + \Gamma_i^2 - \left(\frac{2}{x}\right)\Gamma_i = 0 \quad (2.17)$$

$$\Gamma_r^2 - 2\Gamma_r + 1 + \Gamma_i^2 - \left(\frac{2}{x}\right)\Gamma_i + \frac{1}{x^2} - \frac{1}{x^2} = 0 \quad (2.18)$$

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x}\right)^2 = \frac{1}{x^2} \quad (2.19)$$

(See Figure 3a for further details)

Again, 2.19 is a parametric equation of the type  $(x-a)^2 + (y-b)^2 = R^2$ , in the complex plane ( $\Gamma_r, \Gamma_i$ ), of a circle cen-

tered at the coordinates (1,  $1/x$ ) and having a radius of  $1/x$ .

### Get the picture?

To complete our Smith chart, we superimpose the two circle's families. It can then be seen that all of the circles of one family will intersect all of the circles of the other family. Knowing the impedance, in the form of:  $r + jx$ , the corresponding reflection coefficient can be determined. It is only necessary to find the intersection point of the two circles, corresponding to the values  $r$  and  $x$ .

### It's reciprocating too

The reverse operation is also possible. Knowing the reflection coefficient, find the two circles intersecting at that point and read the corresponding values  $r$  and  $x$  on the circles. The procedure for this is as follows:

- Determine the impedance as a spot on the Smith Chart.
- Find the reflection coefficient ( $\Gamma$ ) for the impedance.
- Having the characteristic impedance and  $\Gamma$ , find the impedance.
- Convert the impedance to admittance.
- Find the equivalent impedance.
- Find the components values for the wanted reflection coefficient (in particular the elements of a matching network see Figure 6).

### To extrapolate

Since the Smith chart resolution technique is basically a graphical method, the precision of the solutions depends directly on the graph definitions. Here are some examples that can be represented by the Smith chart for RF applications:

- Example 1: Consider the characteristic impedance of a 50  $\Omega$  termination and the following impedances:

$Z_1 = 100 + j50 \Omega$ ,  $Z_2 = 75 - j100 \Omega$ ,  $Z_3 = j200 \Omega$ ,  $Z_4 = 150 \Omega$ ,  $Z_5 = \infty$  (an open-circuit)  $Z_6 = 0$  (a short circuit),  $Z_7 = 50 \Omega$ ,  $Z_8 = 184 - j900 \Omega$ .

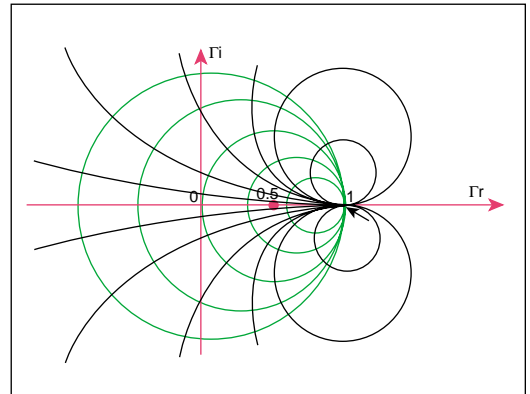


Figure 4. The points situated on a circle are all the impedances characterized by an identical imaginary impedance part value  $x$ . The circle  $x = 1$  is centered at coordinate (1, 1) and has a radius of 1. Furthermore,  $x$  can be positive or negative, which explains the duplicate mirror circles at the bottom side of the complex plane. Note that the zero-reactance circle (a pure resistive load) is just the horizontal axis of the complex plane. The infinite reactance has degenerated to one point situated at (1, 0). All constant reactance circles have the same unique intersecting point at 1, 0. Positive reactances (inductors) are on the circles on the upper half, while negative reactances (capacitors) are on the bottom half.

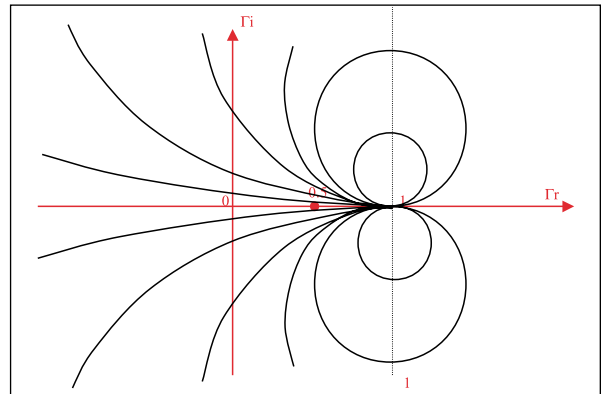


Figure 3a. The points situated on a circle are all the impedances characterized by a same imaginary impedance part value  $x$ . For example, the circle  $x = 1$  is centred at coordinate (1, 1) and having a radius of 1. All circles (constant  $x$ ) include the point (1, 0). In Differing with the real part circles,  $x$  can be positive or negative. This explain the duplicate mirrored circles at the bottom side of the complex plane. All the circles centers are placed on the vertical axis, intersecting the point 1.

Then, normalize and plot (see Figure 5.) The points are plotted as follows:

$z_1 = 2 + j$ ,  $z_2 = 1.5 - j2$ ,  $z_3 = j4$ ,  $z_4 = 3$ ,  $z_5 = 8$ ,  $z_6 = 0$ ,  $z_7 = 1$ ,  $z_8 = 3.68 - j18S$ .

- It is now possible to directly extract the reflection coefficient  $\Gamma$  on the Smith chart of Figure 5. Once the impedance point is plotted (the intersection point of a constant resistance circle and of a constant reac-



tance circle), simply read the rectangular coordinates projection on the horizontal and vertical axis. This will give  $\Gamma_r$ , the real part of the reflection coefficient) and  $\Gamma_i$ , the imaginary part of the reflection coefficient (see Figure 6).

- It is also possible to take the eight cases presented in Example 1 and extract their corresponding  $\Gamma$  directly from the Smith chart of Figure 5. The numbers are:

$$\Gamma_1 = 0.4 + 0.2j, \Gamma_2 = 0.51 - 0.4j, \Gamma_3 = 0.875 + 0.48j, \Gamma_4 = 0.5, \Gamma_5 = 1, \Gamma_6 = -1, \Gamma_7 = 0, \Gamma_8 = 0.96 - 0.1j.$$

### Working with admittance

The Smith chart is built by considering impedance (resistor and reactance). Once the Smith chart is built, it can be used to analyze these parameters in both the series and parallel worlds. Adding elements in a series is straightforward. New elements can be

added and their effects determined by simply moving along the circle to their respective values. However, summing elements in parallel is another matter. It requires considering additional parameters. Often, it is easier to work with parallel elements in the admittance world.

We know that, by definition,  $Y = 1/Z$  and  $Z = 1/Y$ . The admittance is expressed in mhos or  $\Omega^{-1}$  (in earlier times it was expressed as Siemens or

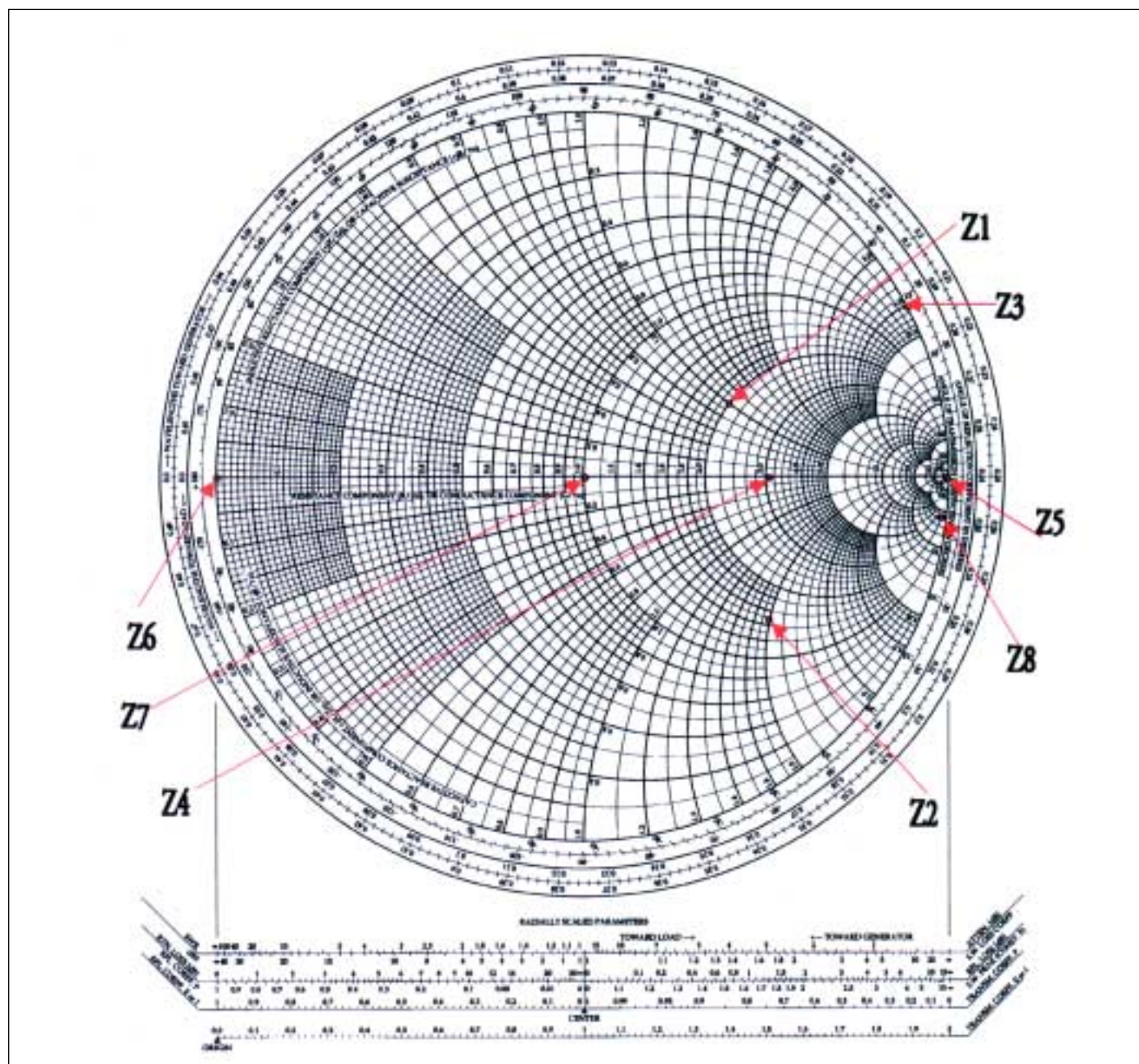


Figure 5. Points plotted on the Smith chart.

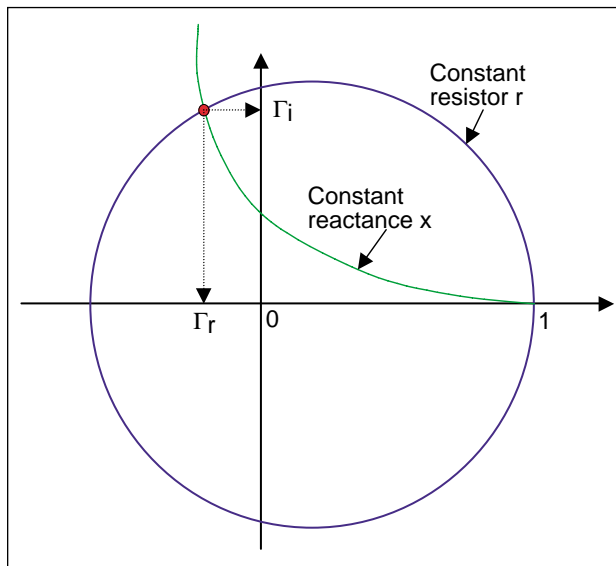


Figure 6. Direct extraction of the reflected coefficient  $\Gamma$ , real and imaginary along the X-Y axis.

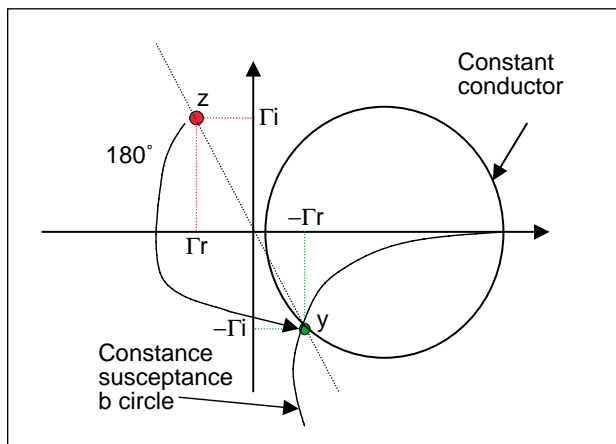


Figure 7. Results of the 180° rotation.

S). And, since  $Z$  is complex,  $Y$  must also be complex.

Therefore,  $Y = G + jB$ , (2.20) where  $G$  is called "conductance" and  $B$  the "susceptance" of the element. One must exercise caution, though. By following the logical assumption, one may conclude that  $G = 1/R$  and  $B =$

$1/X$ . This, however, is not the case. If this assumption is used, the results will be incorrect.

When working with admittance, the first thing that one must do is normalize  $y = Y/Y_0$ . This results in  $y = g + jb$ . So, what happens to the reflection coefficient? By working through the following :

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{\frac{1}{Y_L} - \frac{1}{Y_0}}{\frac{1}{Y_L} + \frac{1}{Y_0}} = \frac{Y_0 - Y_L}{Y_0 + Y_L} = \frac{1 - y}{1 + y} \quad (2.21)$$

It turns out that the expression for  $G$  is the opposite, in sign, of  $z$ , and  $\Gamma(y) = -\Gamma(z)$ .

If we know  $z$ , we can invert the signs of  $\Gamma$  and find a point situated at the same distance from  $(0, 0)$ , but in the opposite direction. This same result can be obtained by rotating an angle 180° around the center point (see Figure 7).

Of course, while  $Z$  and  $1/Z$  do represent the same component, the new point appears as a different impedance (the new value has a different point in the Smith chart and a different reflection value, etc.). This occurs because the plot is an impedance plot. But the new point is, in fact, an admittance. Therefore, the value read on the chart has to be read as mhos.

While this method is sufficient for

making conversions, it does not work for determining circuit resolution when dealing with elements in parallel.

### The admittance Smith chart

In the previous discussion we saw that every point on the impedance Smith chart can be converted into its admittance counterpart by taking a 180° rotation around the origin of the  $\Gamma$  complex plane. Thus, an admittance Smith chart can be obtained by rotating the whole impedance Smith chart by 180°. This is extremely convenient since it eliminates the necessity to build another chart. The intersecting point of all the circles (constant conductances and constant susceptances) is at the point  $(-1, 0)$  automatically. With that plot, adding elements in parallel also becomes easier. Mathematically, the construction of the admittance Smith chart is created by:

$$\Gamma_L = \Gamma_r + j\Gamma_i = \frac{1 - y}{1 + y} = \frac{1 - g - jb}{1 + g + jb} \quad (3.1)$$

then, reversing the equation:

$$y = g + jb = \frac{1 - \Gamma_L}{1 + \Gamma_L} = \frac{1 - \Gamma_r - j\Gamma_i}{1 + \Gamma_r + j\Gamma_i} \quad (3.2)$$

$$g + jb = \frac{(1 - \Gamma_r - j\Gamma_i)(1 + \Gamma_r - j\Gamma_i)}{(1 + \Gamma_r + j\Gamma_i)(1 - \Gamma_r + j\Gamma_i)} = \frac{1 - \Gamma_r^2 - \Gamma_i^2 - j2\Gamma_i}{1 + \Gamma_r^2 + 2\Gamma_r + \Gamma_i^2} \quad (3.3)$$

Next, by setting the real and the imaginary parts of Equation 3.3 equal, we obtain two new, independent relationships:

$$g = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{1 + \Gamma_r^2 + 2\Gamma_r + \Gamma_i^2} \quad (3.4)$$

$$b = \frac{-2\Gamma_i}{1 + \Gamma_r^2 + 2\Gamma_r + \Gamma_i^2} \quad (3.5)$$

By developing Equation 3.4 we get:

$$g + = g\Gamma_r^2 + 2g\Gamma_r + g\Gamma_i^2 = 1 - \Gamma_r^2 - \Gamma_i^2 \quad (3.6)$$

$$\Gamma_r^2 + g\Gamma_r^2 + 2g\Gamma_r + g\Gamma_i^2 + \Gamma_i^2 = 1 - g \quad (3.7)$$

$$(1 + g)\Gamma_r^2 + 2g\Gamma_r + (g + 1)\Gamma_i^2 = 1 - g \quad (3.8)$$

$$\Gamma_r^2 + \frac{2g}{g+1}\Gamma_r + \Gamma_i^2 = \frac{1-g}{1+g} \quad (3.9)$$

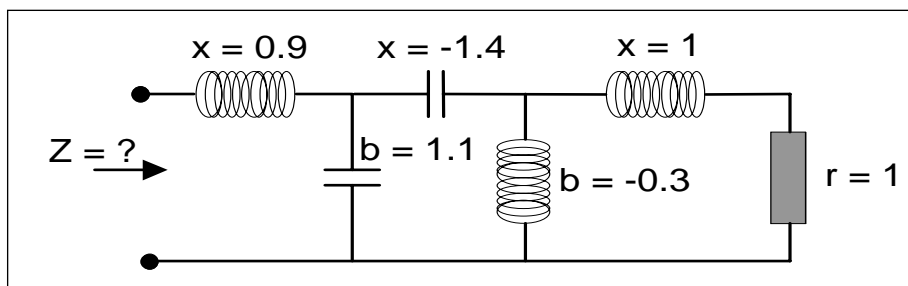


Figure 8. A multi-element circuit.



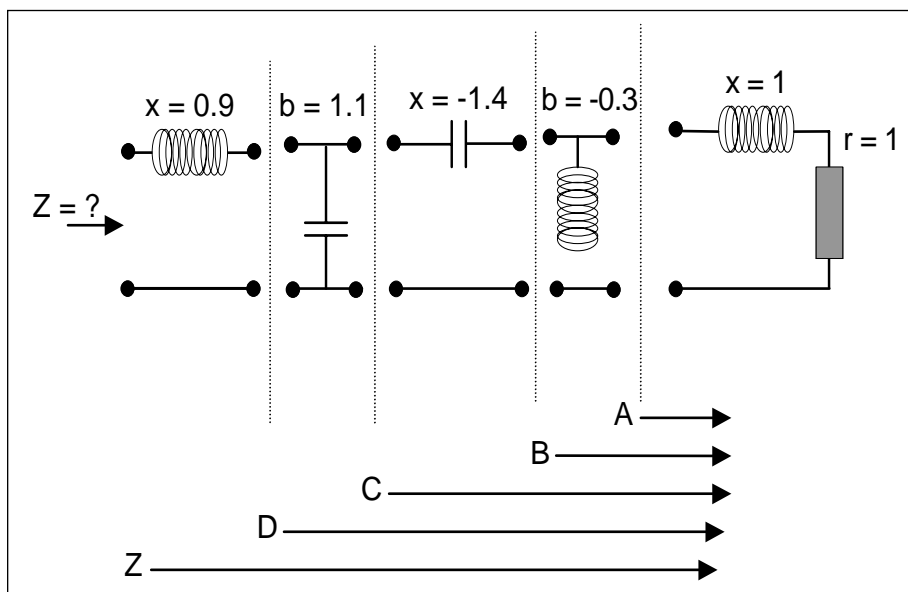


Figure 9. The network of Figure 8 with its elements broken out for analysis.

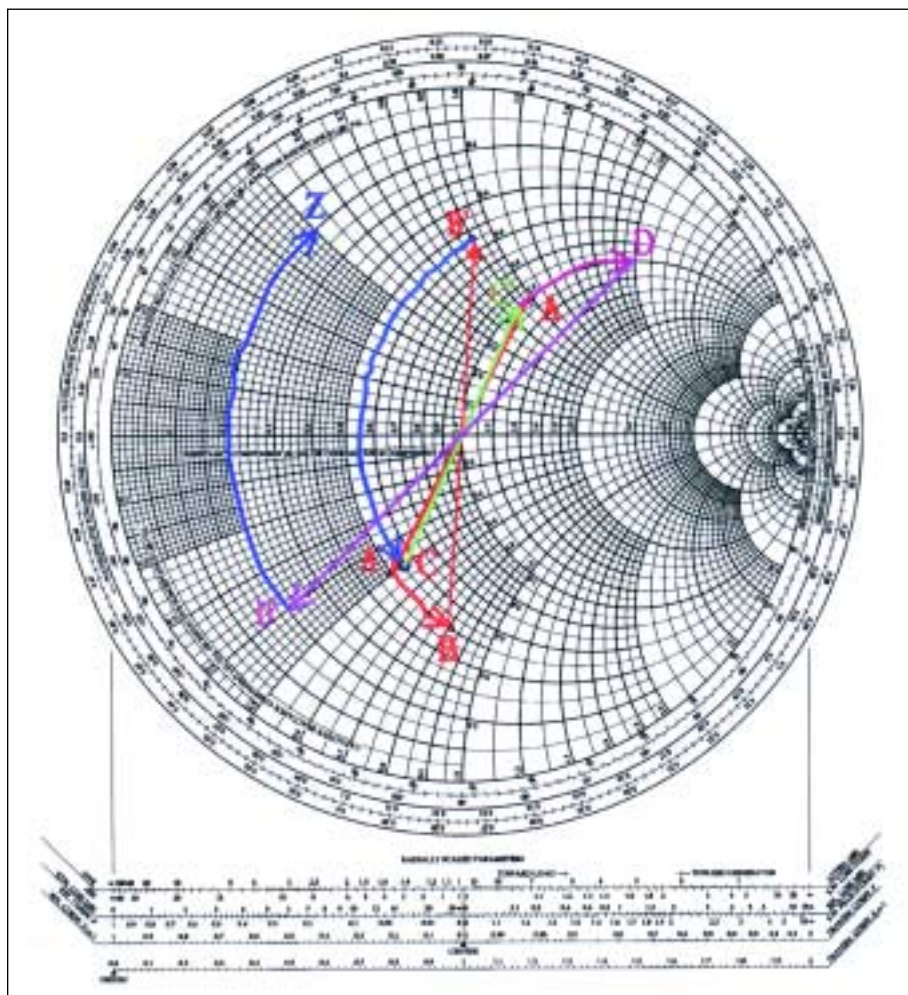


Figure 10. The network elements plotted on the Smith chart.

$$\Gamma_r^2 + \frac{2g}{g+1} \Gamma_r + \frac{g^2}{(g+1)^2} + \Gamma_i^2 - \frac{g^2}{(g+1)^2} = \frac{1-g}{1+g} \quad (3.10)$$

$$\left( \Gamma_r + \frac{g}{g+1} \right)^2 + \Gamma_i^2 = \frac{1-g}{1+g} + \frac{g^2}{(1+g)^2} = \frac{1}{(1+g)^2} \quad (3.11)$$

$$\left( \Gamma_r + \frac{g}{g+1} \right)^2 + \Gamma_i^2 = \frac{1}{(1+g)^2} \quad (3.12)$$

Which again is a parametric equation of the type  $(x-a)^2 + (y-b)^2 = R^2$  (Equation 3.12), in the complex plane  $(\Gamma_r, \Gamma_i)$ , of a circle with its coordinates centered at  $(-g/g+1, 0)$  and having a radius of  $1/(1+g)$ . Furthermore, By developing (Equation 3.5), we show that :

$$b + b\Gamma_r^2 + 2b\Gamma_r + b\Gamma_i^2 = -2\Gamma_i \quad (3.13)$$

$$1 + \Gamma_r^2 + 2\Gamma_r + \Gamma_i^2 = \frac{-2\Gamma_i}{b} \quad (3.14)$$

$$\Gamma_r^2 + 2\Gamma_r + 1 + \Gamma_i^2 + \frac{2}{b} \Gamma_i = 0 \quad (3.15)$$

$$\Gamma_r^2 + 2\Gamma_r + 1 + \Gamma_i^2 + \frac{2}{b} \Gamma_i + \frac{1}{b^2} - \frac{1}{b^2} = 0 \quad (3.16)$$

$$(\Gamma_r + 1)^2 + \left( \Gamma_i + \frac{1}{b} \right)^2 = \frac{1}{b^2} \quad (3.17)$$

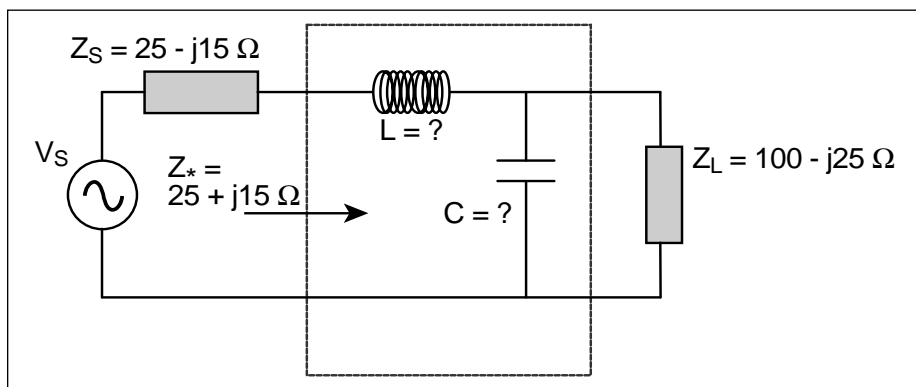
which is again a parametric equation of the type  $(x-a)^2 + (y-b)^2 = R^2$  (Equation 3.17).

### Equivalent impedance resolution

When solving problems where elements in series and in parallel are mixed together, one can use the same Smith chart and rotate it around any point where conversions from  $z$  to  $y$  or  $y$  to  $z$  exist.

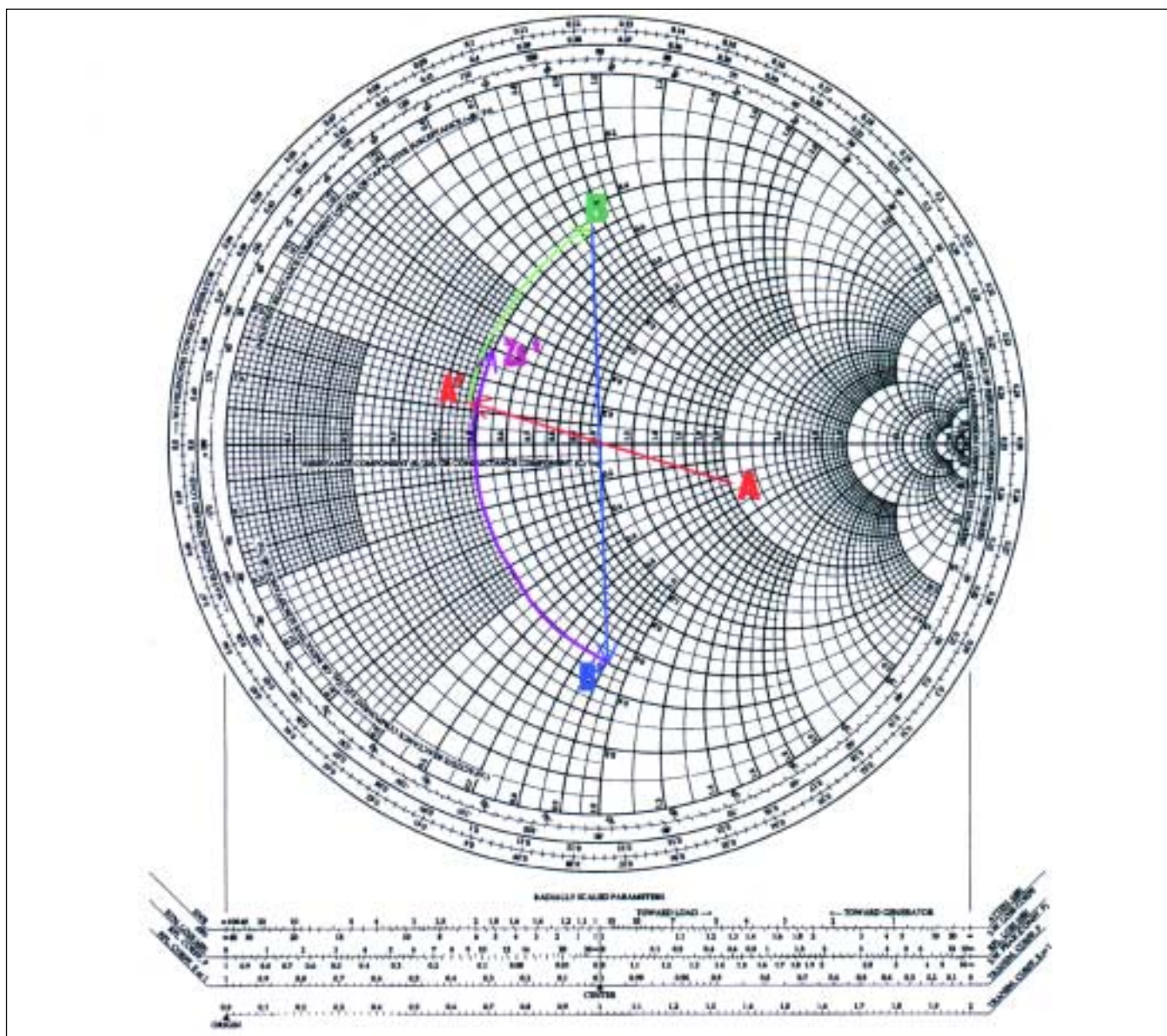
Let's consider the network of Figure 8 (the elements are normalized with  $Z_0 = 50 \Omega$ ). The series reactance ( $x$ ) is positive for inductance and negative for capacitors. The susceptance ( $b$ ) is positive for capacitance and negative for inductance.

The circuit needs to be simplified (see Figure 9). Starting at the right



**Figure 11. The representative circuit with know impedances and unknown components.**

side, where there is a resistor and inductor with a value of 1, we plot a series point where the  $r$  circle = 1 and the  $l$  circle = 1. This becomes point A. Since the next element is an element in shunt (parallel), we switch to the admittance Smith chart (by rotating the whole plane  $180^\circ$ ). To do this, however, we need to convert the previous point into admittance. This becomes A'. We then rotate the plane by  $180^\circ$ . We are now in the admittance mode. The shunt element can be added by going along the conductance circle by a distance corresponding to 0.3. This



**Figure 12. The network of Figure 11 with its points plotted on the Smith chart.**



must be done in a counter-clockwise direction (negative value) and gives point B. Next, we have another series element. We again switch back to the impedance Smith chart. Before doing this, it is again necessary to reconvert the previous point into impedance (it was an admittance). After the conversion, we can determine B'. Using the previously established routine, the chart is again rotated  $180^\circ$  to get back to the impedance mode. The series element is added by following along the resistance circle by a distance corresponding to 1.4 and marking point C. This has to be done counter-clockwise (negative value). For the next element, the same operation is performed (conversion into admittance and plane rotation). Then move the prescribed distance (1.1), in a clockwise direction (since the value is positive), along the constant conductance circle. We mark this as D. Finally, we reconvert back to impedance mode and add the last element (the series inductor). We then determine the required value,  $z$ , located at the intersection of resistor circle 0.2 and reactance circle 0.5. Thus  $z$  is determined to be  $0.2 + j0.5$ . If the system characteristic impedance is  $50\ \Omega$ , then  $Z = 10 + j25\ \Omega$  (see Figure 10).

### Matching impedances by steps

Another function of the Smith chart is the ability to determine impedance matching. This is the reverse operation of finding the equivalent impedance of a given network. Here, the impedances are fixed at the two access ends (often the source and the load) as shown in Figure 11. The objective is to design a network to insert between them so that proper impedance matching occurs.

At first glance, it appears that is is no more difficult than finding equivalent impedance. But the problem is that an infinite number of matching network component combinations can exist that create similar results. And, other inputs may need to be considered as well (filter type structure, quality factor, limited choice of components, etc.).

The approach chosen to accomplish this calls for adding series and shunt elements on the Smith chart until the desired impedance is achieved. Graphically, it appears as finding a way to link the points on the Smith chart.

Again, the best method to illustrate the approach is to address the requirement as an example.

The objective is to match a source impedance ( $Z_S$ ) to a load ( $Z_L$ ) at the working frequency of 60 MHz (see Figure 11). The network structure has been fixed as a lowpass, L type (an alternative approach is to view the problem as how to force the load to appear like an impedance of value  $= Z_S$  (a complex conjugate of  $Z_S$ ). Here is how the solution is found.

The first thing to do is to normalize the different impedance values. If this is not given, choose a value that is in the same range as the load/source values. Assume  $Z_0$  to be  $50\ \Omega$ . Thus  $z_S = 0.5 - j0.3$ ,  $z_S^* = 0.5 + j0.3$  and  $Z_L = 2 - j0.5$ .

Next, position the two points on the chart. Mark A for  $z_L$  and D for  $Z_S^*$ .

Then identify the first element connected to the load (a capacitor in shunt) and convert to admittance. This gives us point A'.

Determine the arc portion where the next point will appear after the connection of the capacitor C. Since we don't know the value of C, we don't know where to stop. We do, however, know the direction. A C in shunt means move in the clock-wise direction on the admittance Smith chart until the value is found. This will be point B (an admittance). Since the next element is a series element, point B has to be converted to the impedance plane. Point B' can then be obtained. Point B' has to be located on the same resistor circle as D. Graphically, there is only one solution from A' to D, but the intermediate point B (and hence B') will need to be verified by a "test-and-try" setup. After having found points B and B', we can measure the lengths of arcs A' - B and arc B' - D. The first gives the normalized susceptance value of C. The second gives the normalized reactance value of L. The arc A' - B measures  $b = 0.78$  and thus  $B = 0.78 \times Y_0 = 0.0156$  mhos. Since  $\omega C = B$ , then  $C = B/\omega = B/(2\pi f) = 0.0156/(2\pi \times 60^7) = 41.4\ \text{pF}$ . The arc B - D measures  $x = 1.2$ , thus  $X = 1.2 \times Z_0 = 60\ \Omega$ . Since  $\omega L = X$ , then  $L = X/\omega = X/(2\pi f) = 60/(2\pi \times 60^7) = 159\ \text{nH}$ .

### Conclusion

Given today's wealth of software and accessibility of high-speed, high-power computers, one may question

the need for such a basic and fundamental method for determining circuit fundamentals.

In reality, what makes an engineer a real engineer is not only academic knowledge, but the ability to use resources of all types to solve a problem. It is easy to plug a few numbers into a program and have it spit out the solutions. And, when the solutions are complex and multifaceted, having a computer handy to do the grunt work is "back-savily" handy.

However, knowing underlying theory and principles that have been ported to computer platforms, and where they came from, makes the engineer or designer a more well rounded and confident professional—and makes the results more reliable. **RF**

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