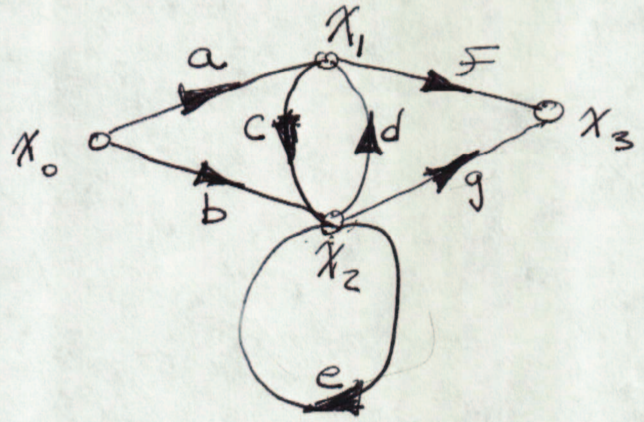
Signal Flow Graph Solution



Using Mason's Formula

$\Delta = 1 - e - cd$

4 paths

$T_1 = aF \qquad \Delta_1 = 1 - e$

$T_2 = bg \qquad \Delta_2 = 1$

$T_3 = acg \qquad \Delta_3 = 1$

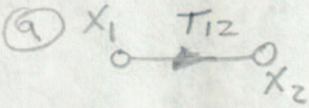
$T_4 = bdF \qquad \Delta_4 = 1$

$$\frac{x_3}{x_0} = \frac{1}{\Delta} \sum_{k=1}^{4} T_k \Delta_k = \frac{aF(1-e) + bg + acg + bdF}{1 - e - cd}$$

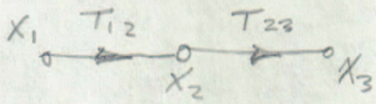
$$\frac{x_3}{x_0} = \frac{aF - aFe + bg + acg + bdF}{1 - e - cd}$$

Signal Flowgraphs

I. ⓐ



ⓑ



$X_2 = T_{12} X_1$

$X_3 = T_{23} X_2$

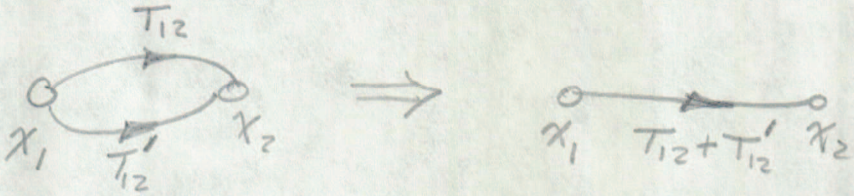
$X_2 = T_{12} X_1$

$X_3 = T_{23}(T_{12} X_1) = T_{12}(T_{23}) X_1 = T_{12} T_{23} X_1$

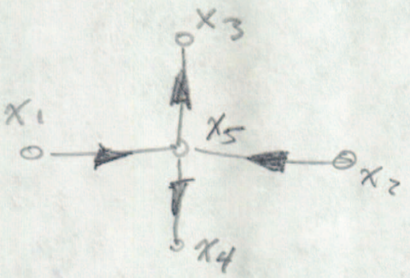
what is $T_{12} \Longrightarrow T_{12} = \dfrac{\partial X_2}{\partial X_1}$

We will deal with linear equations, thus $T_{12} = \text{Const}$

II



III



① $X_5 = T_{15} X_1 + T_{25} X_2$

② $X_1 = X_1$

③ $X_2 = X_2$

④ $X_3 = T_{15} T_{53} X_1 + T_{25} T_{53} X_2$

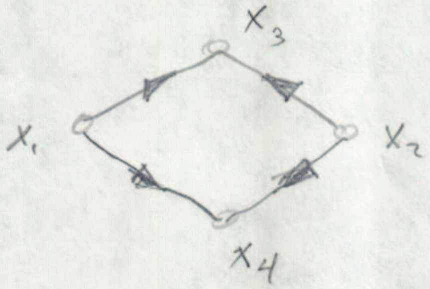
⑤ $X_4 = T_{54} T_{15} X_1 + T_{54} T_{25} X_2$

Sources : X_1 X_2

sinks : X_3 X_4

Source or sink or flow through : X_5

X_5 can be absorbed if not needed directly!



$\Longrightarrow$

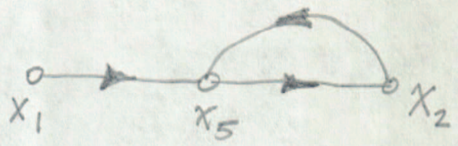
$T_{13} = T_{15} T_{53}$

$T_{14} = T_{15} T_{54}$

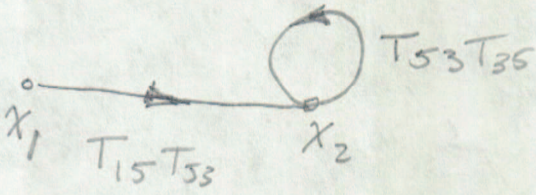
$T_{23} = T_{25} T_{53}$

$T_{24} = T_{25} T_{54}$

IV Special Case of III with $x_2 = x_3$ and $T_{54} = 0$



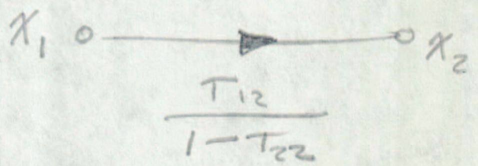
apply node absorption



$T_{12} = T_{15}\,T_{53}$

$T_{22} = T_{53}\,T_{35}$

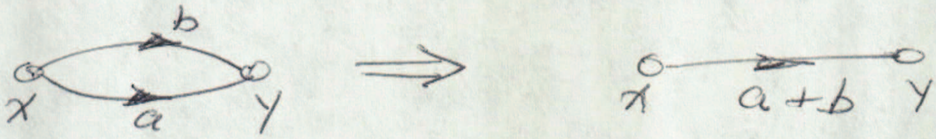
apply feedback rule



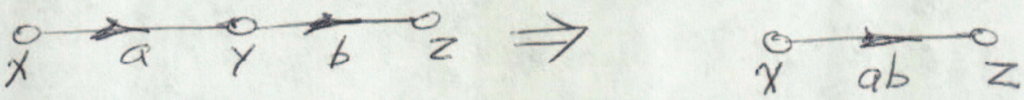
① In node absorption retain all individual path transmissions through the absorbed node

② In eliminating a self loop with transmission T at the node, the branch transmissions at all branches entering must be divided by $1-T$

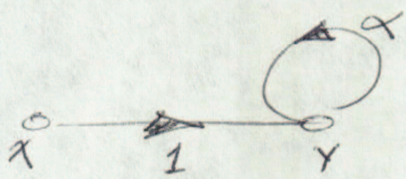
Sum Rule



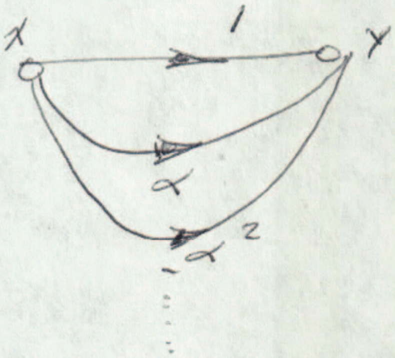
Multiplication Rule



Feedback Rule



Find Y/X



$$Y/X = 1+\alpha+\alpha^2 \cdots = \sum_{i=0}^{\infty}\alpha^i = \frac{1}{1-\alpha}$$

ELE443 Electronics II

Simplification Rules for Flowgraphs

1. Summation rule: Parrallel Branches
2. Product rule: Series Branches
3. Node Absorption:

   a) Retain all individual path transmittances through the absorbed node.

   b) In elimination a self loop, with transmission T at the node, the branch transmissions at all branches entering must be divided by 1-T.

How to Create a Flowgraph

1. Start with the output node and work back toward sources
2. Use node equations to define the output in terms of other intermediate nodes or input nodes.
3. Continue until all nodes which are not sources have been completely defined.
4. A node equation consists of all terms involving transmissions toward that node
5. Sources have transmissions only away from them and do not have to be defined in the flowgraph.
6. Once a flowgraph is completed, any node can be considered an output node.

Mason's formula for the reduction of general SFGs

The transmittance T_{ij} from an independent variable (source) x_i to any other variable x_j is given by:

$$\boxed{T_{ij} = \frac{x_j}{x_i} = \frac{1}{\Delta} \sum_k T_k \Delta_k}$$

where: Δ = 1 − (sum of all individual loop transm.)
+ (sum of the products of loop transm. of all possible nontouching feedback loops taken two at a time)
− (sum of the products of loop transm. of all possible nontouching feedback loops taken three at a time)
+ (...

Δ_k = value of Δ for that part of the graph not touching the kth open path

T_k = path transmittance of the kth open path

Def. A path leading from a source node x_i to another node x_j without passing through any node more than one is called an open path.

Mason's Formula for the Reduction of general SFGs

The transmittance T_{ij} from an independent node variable (source) X_i to any other node variable X_j in the same SGF is given by:

$$T_{ij} = \frac{X_j}{X_i} = \frac{1}{\Delta} \sum_k T_k \Delta_k$$

where:

$\Delta = 1-$ (sum of all individual loop transmittances).

$+$ (sum of the products of loop transmittances of all possible *nontouching* feedback loops taken *two* at a time).

$-$ (sum of the products of loop transmittances of all possible *nontouching* feedback loops taken *three* at a time).

$+$ (. . .).

Δ_k = value of Δ for that part of the graph *not* touching the *kth open path.*

T_k = path transmittance of the *kth open path.*

Def.:

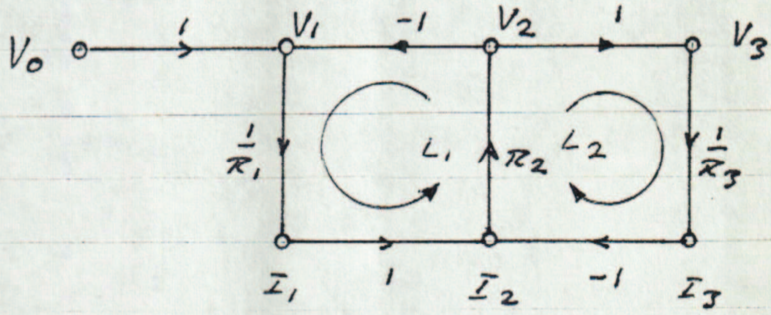
A path leading from a source node X_i to another node X_j without passing through any other node more than once is called an *open path.*

Δ is also called the <u>graph determinant</u>

Δ_k is also called <u>cofactor</u> or <u>path factor</u> for the kth open path.

<u>Note:</u> According to Mason's formula, all transmittances from a given source to an arbitrary node variable of a network exhibit the <u>same denominator</u>, the graph determinant Δ.
(In some cases, numerator and denominator comprise common factors which can be canceled out).

Back to Ex ④:
original SFG



SFG comprises 2 Loops: $L_1: -\frac{R_2}{R_1}$, $L_2: -\frac{R_2}{R_3}$

$$\Rightarrow \quad \underline{\underline{\Delta}} = 1 - L_1 - L_2 = \underline{\underline{1 + \frac{R_2}{R_1} + \frac{R_2}{R_3}}}$$

Let's calculate:

a) $\frac{V_3}{V_0}$: open path from V_0 to V_3

$$\underline{T_1} = 1 \cdot \frac{1}{R_1} \cdot 1 \cdot R_2 \cdot 1 = \underline{\underline{\frac{R_2}{R_1}}}$$

Cofactor (path factor) for this path:

$\underline{\Delta_1 = 1}$

$$\Rightarrow \quad \frac{V_3}{V_0} = \frac{1}{\Delta} \cdot T_1 \cdot \Delta_1 = \frac{\pi_2/\pi_1}{1 + \pi_2/\pi_1 + \pi_2/\pi_3}$$

b) $\frac{I_3}{V_0}$: open path from V_0 to I_3

$\underline{T_1 = \frac{1}{\pi_1} \cdot \pi_2 \cdot \frac{1}{\pi_3}}$

Cofactor for this path:

$\underline{\Delta_1 = 1}$

$$\Rightarrow \quad \frac{I_3}{V_0} = \frac{\frac{1}{\pi_3} \cdot \pi_2/\pi_1}{1 + \pi_2/\pi_1 + \pi_2/\pi_3}$$

c) $\frac{V_1}{V_0}$: open path from V_0 to V_1

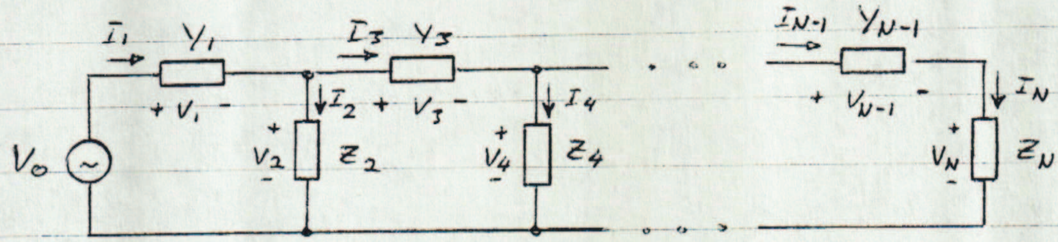
$T_1 = 1$

Cofactor for this path:

$\Delta_1 = 1 - L_2 = 1 + \pi_2/\pi_3$

$$\Rightarrow \quad \frac{V_1}{V_0} = \frac{1 + \pi_2/\pi_3}{1 + \pi_2/\pi_1 + \pi_2/\pi_3}$$

Ex ⑦ general ladder network (N even)



$I_1 = Y_1 \cdot V_1$

$V_1 = V_0 - V_2$

$V_2 = Z_2 \cdot I_2$

$I_2 = I_1 - I_3$

$I_3 = Y_3 \cdot V_3$

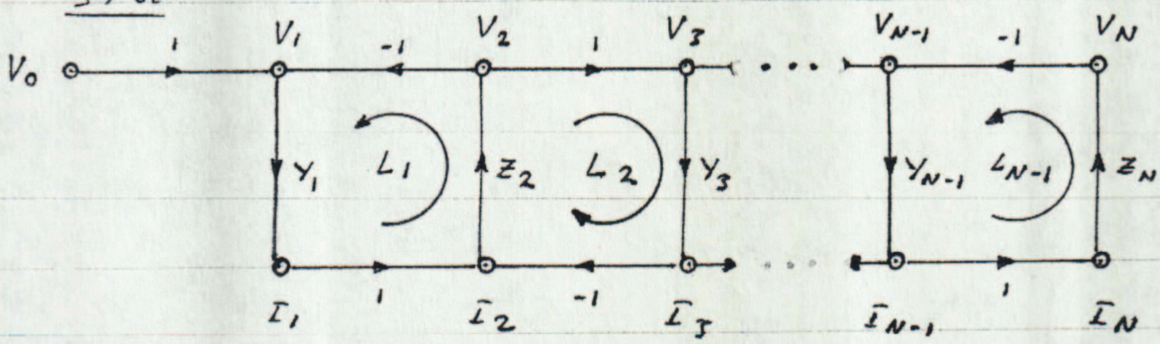
$\vdots$

$V_{N-1} = V_{N-2} - V_N$

$V_N = Z_N \cdot I_N$

$I_N = I_{N-1}$

SFG:



$N = 4 \quad \Rightarrow \quad \Delta = 1 - L_1 - L_2 - L_3 + L_1 L_3$

$$= 1 + Y_1 Z_2 + Z_2 Y_3 + Y_3 Z_4 + Y_1 Z_2 Y_3 Z_4$$

e.g. $\dfrac{V_4}{V_0} = \dfrac{Y_1 \cdot Z_2 \cdot Y_3 \cdot Z_4}{1 + Y_1 Z_2 + Z_2 Y_3 + Y_3 Z_4 + Y_1 Z_2 Y_3 Z_4}$

Homework: Calculate the input impedance of a ladder network ($N = 4$) if

$Y_1 = sC$

$Z_2 = Z_4 = R$

$Y_3 = 1/R$

Solution: $Z_{in} = \dfrac{V_0}{I_1}$

from Ex (4) $\Rightarrow \dfrac{I_1}{V_0} = \dfrac{Y_1\,(1 + Z_2 Y_3 + Y_3 Z_4)}{1 + Y_1 Z_2 + Z_2 Y_3 + Y_3 Z_4 + Y_1 Z_2 Y_3 Z_4}$

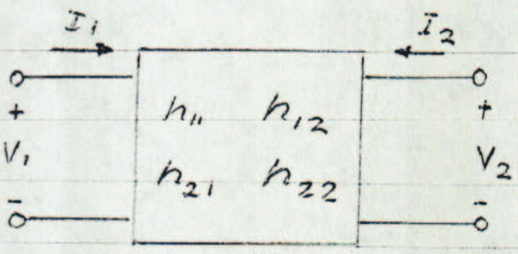
$$\Rightarrow Z_{in} = \frac{1}{Y_1}\,\frac{(1 + Y_1 Z_2 + Z_2 Y_3 + Y_3 Z_4 + Y_1 Z_2 Y_3 Z_4)}{(1 + Z_2 Y_3 + Y_3 Z_4)}$$

$$= \frac{1}{sC}\,\frac{(1 + sCR + 1 + 1 + sCR)}{(1 + 1 + 1)}$$

$$Z_{in} = \frac{1}{sC} + \frac{2}{3}R$$

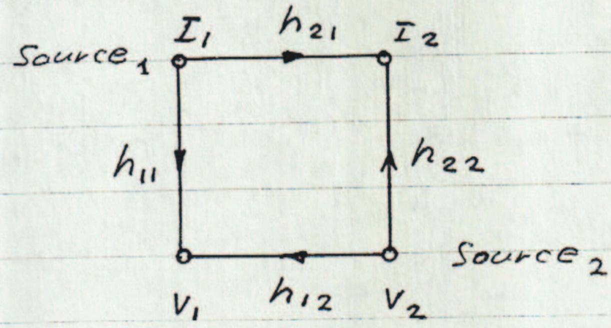
SFG representation of general Twoports

a) Twoport is given by its [h] Parameters



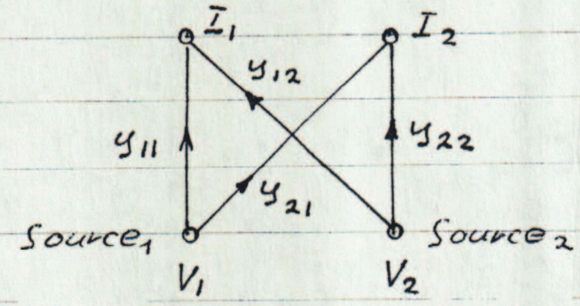
$$\left| \begin{aligned} V_1 &= h_{11} I_1 + h_{12} V_2 \\ I_2 &= h_{21} I_1 + h_{22} V_2 \end{aligned} \right|$$

SFG representation:



b) [Y] Matrix representation

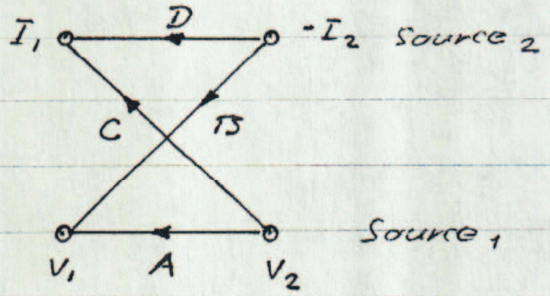
$$\left| \begin{aligned} I_1 &= y_{11} V_1 + y_{12} V_2 \\ I_2 &= y_{21} V_1 + y_{22} V_2 \end{aligned} \right|$$



$$\left| \begin{aligned} y_{11} &= \frac{1}{h_{11}} \qquad & y_{12} &= -\frac{h_{12}}{h_{11}} \\ y_{21} &= \frac{h_{21}}{h_{11}} \qquad & y_{22} &= h_{22} - \frac{h_{12} h_{21}}{h_{11}} \end{aligned} \right|$$

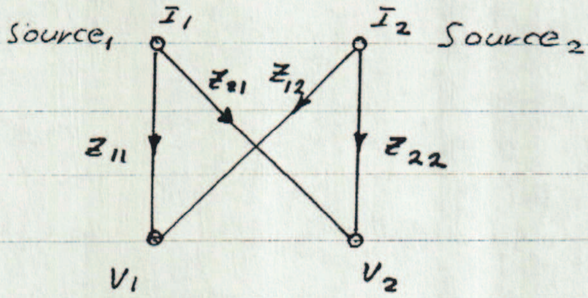
c) [ABCD] Matrix representation (Chain-Matrix)

$$\left| \begin{aligned} V_1 &= A \cdot V_2 - B \cdot I_2 \\ I_1 &= C \cdot V_2 - D \cdot I_2 \end{aligned} \right|$$



$$\left| \begin{aligned} A &= h_{12} - \frac{h_{11} h_{22}}{h_{21}} \qquad & B &= -\frac{h_{11}}{h_{21}} \\ C &= -\frac{h_{22}}{h_{21}} \qquad & D &= -\frac{1}{h_{21}} \end{aligned} \right|$$

d) [Z] Matrix representation



$$\left| \begin{array}{l} V_1 = Z_{11} \cdot I_1 + Z_{12} \cdot I_2 \\ V_2 = Z_{21} \cdot I_1 + Z_{22} \cdot I_2 \end{array} \right|$$

$$\left| \begin{array}{ll} Z_{11} = h_{11} - \dfrac{h_{12} h_{21}}{h_{22}} & Z_{12} = \dfrac{h_{12}}{h_{22}} \\ Z_{21} = - \dfrac{h_{21}}{h_{22}} & Z_{22} = \dfrac{1}{h_{22}} \end{array} \right|$$

If we have to calculate $T_{12} = \frac{V_2}{V_1}$ from the above SFG, we don't have to invert the path Z_{11} to make V_1 a source, since

$$T_{12} = \frac{V_2}{V_1} = \frac{V_2/I_1}{V_1/I_1} = \frac{Z_{21}}{Z_{11}}$$