Construction Rules for SFG's

The *variables* of an equation system corespond to *nodes,* the *coefficients* to *branches* of the SFG.

Signals are transmitted only in the direction of the *arrows.*

Each signal transmitted along a branch is *multiplied* with the *transmittance* of the branch.

Each (node) *variable* is equal to the algebraic *sum* of all incoming signals.

Each (node) variable is transmitted along each branch *leaving* the node.

1. Draw circuit schematic
2. Draw equivalent circuit
3. Write down all equations (if needed)
4. Draw SFG for desired variabes.
5. Use masons gain formula or simplification rules

I. Signal-Flow-Graphs (SFG)

S. J. Mason: "Feedback Theory - Some Properties of Signal Flow Graphs," Proc. of the Inst. of Radio Eng., 41, pp. 1144-1156; September 1953

S. J. Mason: "Feedback Theory - Further Properties of Signal Flow Graphs," Proc. of the Inst. of Radio Eng., 44, pp. 920-926; July 1956

The classical Method to analyze a network is to describe the network by means of a system of equations and to solve this system afterwards.
Excellent method for computers but we humans readily get confused if too many variables (e.g. > 4) are involved. Alternative methods which provide more systematic:

- Matrix - Analysis (e.g. Admittance Matrix)
- Signal Flow Graph Analysis

Signal Flow Graphs are nothing but graphical representations of equation systems.
In contrast to the equations, SFG's provide also insight into the flow of signals inside a given Network.

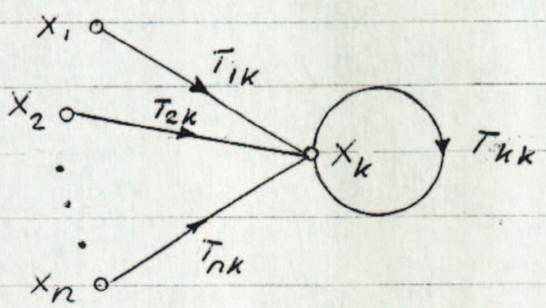
Def A SFG is a diagram consisting of nodes which are connected by directed branches.

Each node is associated with a system variable (e.g. V_1, I_3)

Each directed branch jk from node j to node k has an associated branch transmittance T_{jk}.

The nodes sum up all signals that enter them, and transmit the sum signals to all outgoing branches.

Ex ①



$\Downarrow$

$$X_k = \sum_i T_{ik} X_i$$

In the above example T_{kk} is called a self-loop.

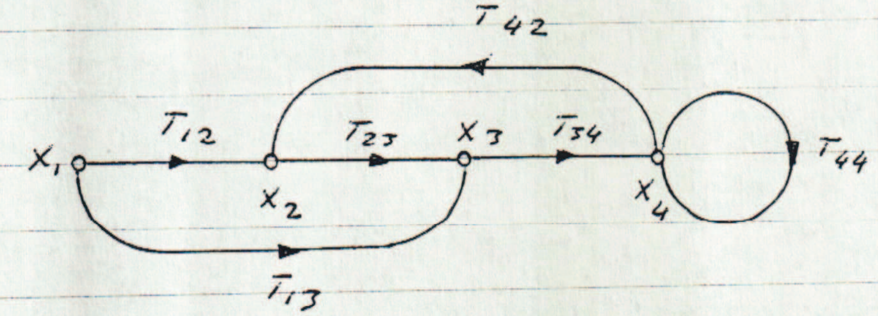
Ex ② given X_1 (Source)

$$X_2 = T_{12} X_1 + T_{42} X_4$$

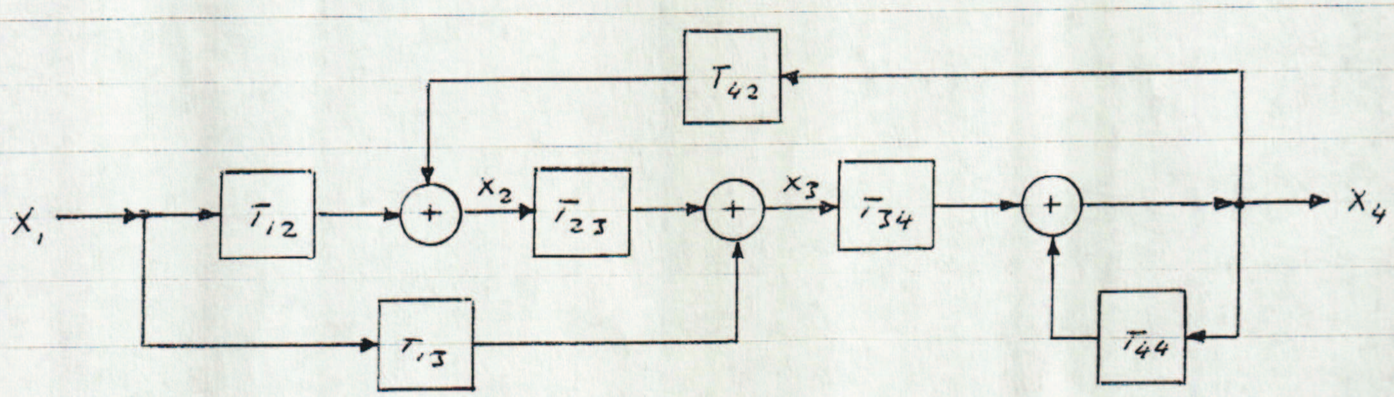
$$X_3 = T_{13} X_1 + T_{23} X_2$$

$$X_4 = T_{34} X_3 + T_{44} X_4$$

$\Downarrow$



Block Diagram of the above system:



SFG is considerably simpler since
- it omits the "blocks"
- does not distinguish between summing and pickoff points

Construction rules for SFG's

1) The variables of an equation system correspond to nodes; the coefficients are associated with the branches

2) Signals are transmitted only in the direction of the arrows.

3) Each signal transmitted along a branch is multiplied with the Transmittance of the branch.

4) Each (node) variable is equal to the algebraic sum of all incoming signals.

5) Each (node) variable is transmitted along each branch leaving the node.

Ex (3) (Homework)

Find the SFG of the following eq. system

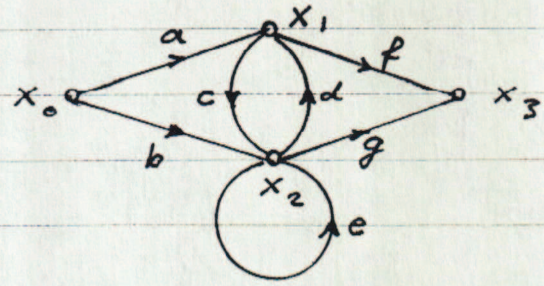
$$x_0 = x_0$$

$$x_1 = a \cdot x_0 \qquad + d \cdot x_2$$

$$x_2 = b \cdot x_0 + c \cdot x_1 + e \cdot x_2$$

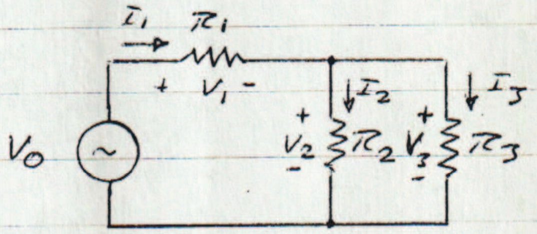
$$x_3 = \qquad f \cdot x_1 + g \cdot x_2$$

solution:



Ex ④ (Homework)

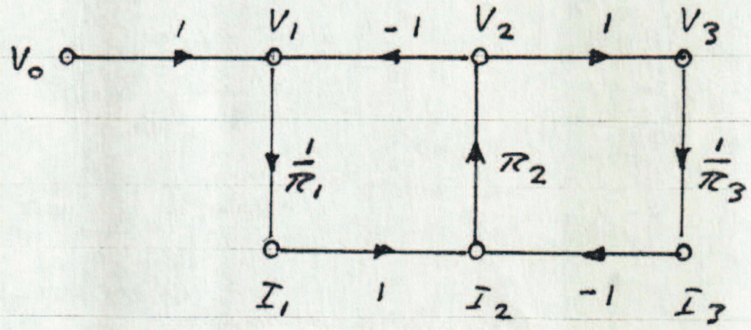
Sketch the SFG of the following NW



$\Rightarrow$ eq.

$$I_1 = \frac{1}{R_1} \cdot V_1$$
$$V_1 = V_0 - V_2$$
$$V_2 = I_2 \cdot R_2$$
$$I_2 = I_1 - I_3$$
$$I_3 = \frac{1}{R_3} V_3$$
$$V_3 = V_2$$

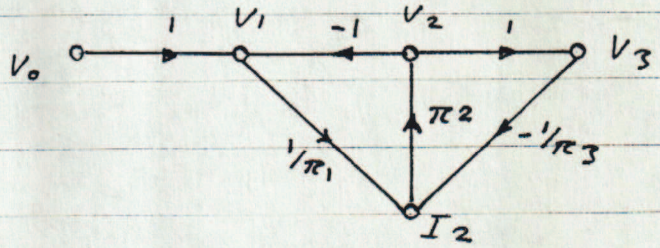
Solution:



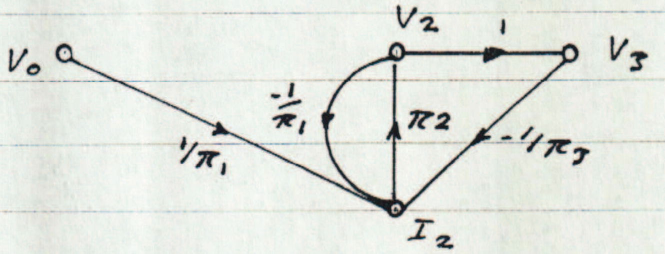
Question: How can we calculate transfer function V_3/V_0 ?

Approach: We reduce the SFG to its simplest form comprising only two nodes (V_0 and V_3).

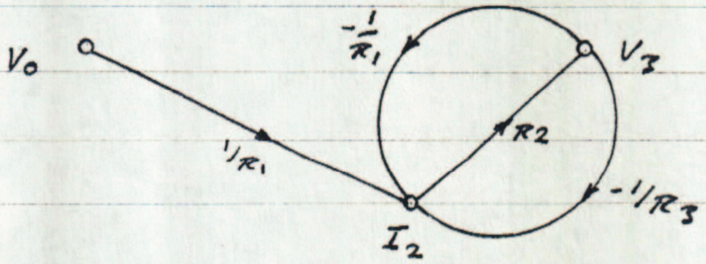
Step 1 Absorb nodes I_1 and I_3



Step 2 Absorb node V_1



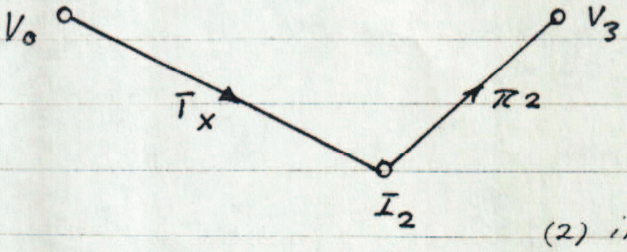
Step 3 Absorb node V_2



(1) $I_2 = V_0 \frac{1}{\pi_1} - V_3\left(\frac{1}{\pi_1}+\frac{1}{\pi_3}\right)$

(2) $V_3 = I_2 \pi_2$

Step 4 Eliminate the Feedback Loops



(1') $I_2 = V_0 \cdot T_x$

(2') $V_3 = I_2 \cdot \pi_2$

(2) in (1) $\Rightarrow I_2 = V_0 \frac{1}{\pi_1} \frac{1}{\left(1+\pi_2/\pi_1+\pi_2/\pi_3\right)}$

(1) = (1') $\Rightarrow T_x = \frac{1/\pi_1}{\left(1+\pi_2/\pi_1+\pi_2/\pi_3\right)}$

Step 5 Absorb node I_2

$$V_0 \xrightarrow{\dfrac{R2/R1}{1+R2/R1+R2/R3}} V_3$$

$$\Rightarrow \quad \frac{V_3}{V_0} = \frac{R2/R1}{1+R2/R1+R2/R3} = \frac{R2\,R3}{R1\,R3+R2\,R3+R1\,R2}$$

SFG transformation Rules

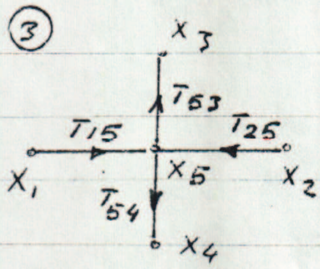
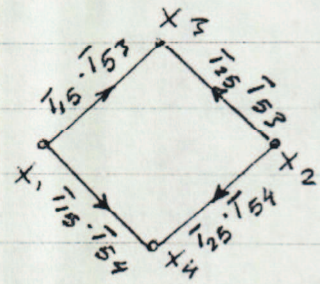
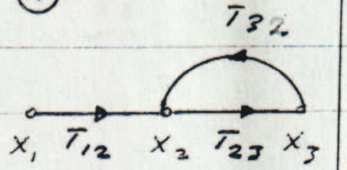
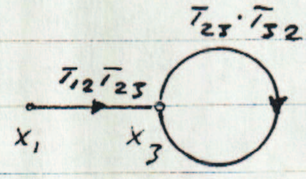
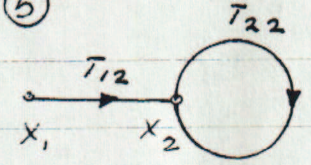
Original graph	eq. syst	mod. eq.	mod. graph
① $X_1 \xrightarrow{T_{12}} X_2 \xrightarrow{T_{23}} X_3$	X_1: Source $X_2 = T_{12} \cdot X_1$ $X_3 = T_{23} \cdot X_2$	X_1: Source $X_3 = T_{12} \cdot T_{23} \cdot X_1$	$X_1 \xrightarrow{T_{12} \cdot T_{23}} X_3$
② $X_1 \overset{T_{12}'}{\underset{T_{12}''}{\rightrightarrows}} X_2$	X_1: Source $X_2 = T_{12}' X_1 + T_{12}'' X_1$	X_1: Source $X_2 = (T_{12}' + T_{12}'') X_1$	$X_1 \xrightarrow{T_{12}' + T_{12}''} X_2$
③ 	X_1: Source X_2: Source $X_3 = T_{53}\, X_5$ $X_4 = T_{54}\, X_5$ $X_5 = T_{15}\, X_1 + T_{25}\, X_2$	X_1: Source X_2: Source $X_3 = T_{15} \cdot T_{53} \cdot X_1 + T_{25} \cdot T_{53} \cdot X_2$ $X_4 = T_{15} \cdot T_{54} \cdot X_1 + T_{25} \cdot T_{54} \cdot X_2$	

table continued

4 	X_1 : Source $X_2 = T_{12} X_1 + T_{32} X_3$ $X_3 = T_{23} \cdot X_2$	X_1 : Source $X_3 = T_{12} \cdot T_{23} X_1$ $+ T_{23} \cdot T_{32} X_3$	
5 	X_1 : Source $X_2 = T_{12} X_1 + T_{22} X_2$	X_1 : Source $X_2 = \frac{T_{12}}{1 - T_{22}} X_1$	$X_1 \xrightarrow{\frac{T_{12}}{1-T_{22}}} X_2$

back to Ex 4

The rules we applied in our 5 steps to simplify the SFG were:

Step 1 Rule 1

Step 2 Rule 3

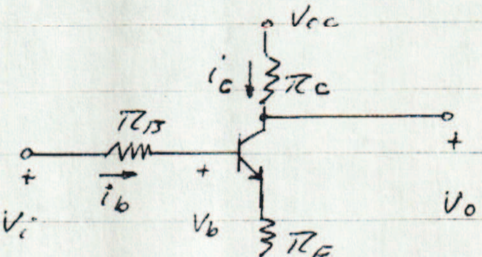
Step 3 Rule 3

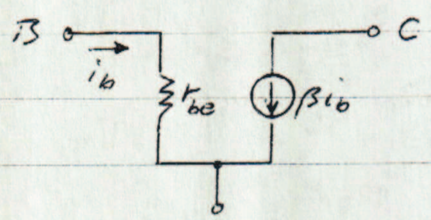
Step 4 Rule 2 + 4 + 5

Step 5 Rule 1

Ex 5 Homework

Common Emitter Amplifier: Replace Tr. by

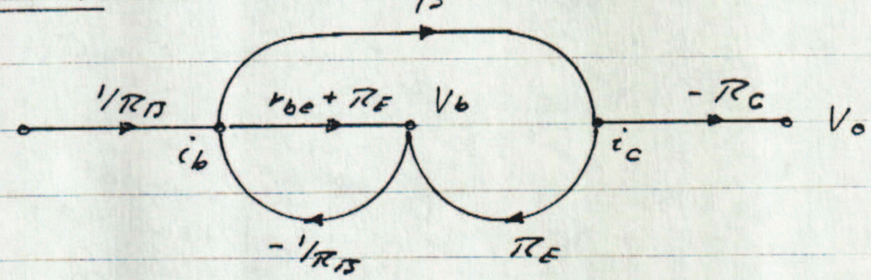




a) Sketch the SFG of the given circuit comprising the following (node) variables: V_i, i_b, V_b, i_c, V_o

b) Calculate the transfer function V_o/V_i

<u>Solution</u>



equations:

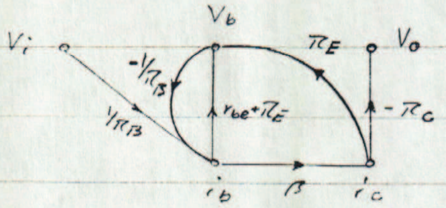
$i_b = (V_i - V_b)\frac{1}{R_B}$

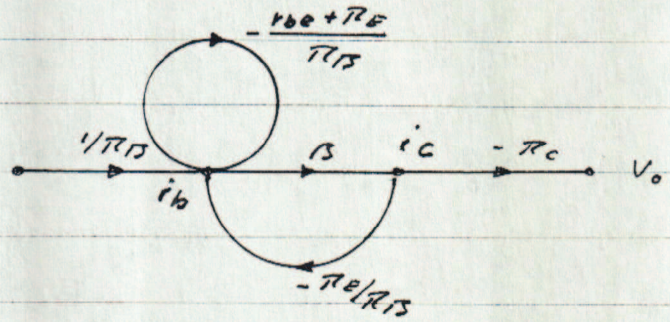
$V_b = i_b r_{be} + (i_b + i_c) R_E$

$i_c = \beta i_b$

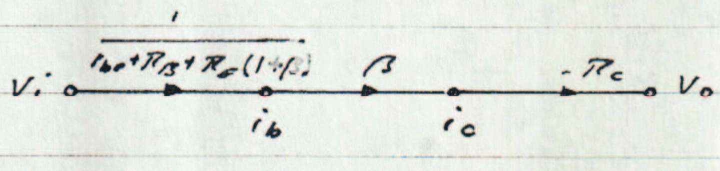
$V_o = -R_C i_c$

<u>Step 1</u> absorb V_b (Rule ③)

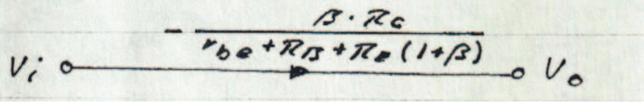




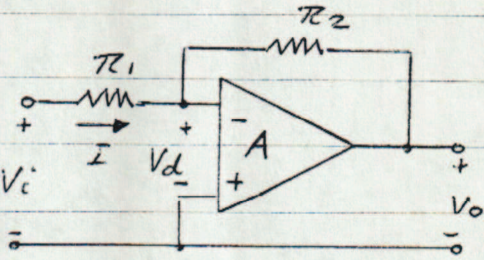
<u>Step 2</u> eliminate loops (Rule ④ + ⑤)



<u>Step 3</u> absorb nodes i_b and i_c (Rule ①)



Ex ⑥ Inverting Amplifier with Opamp



Opamp:

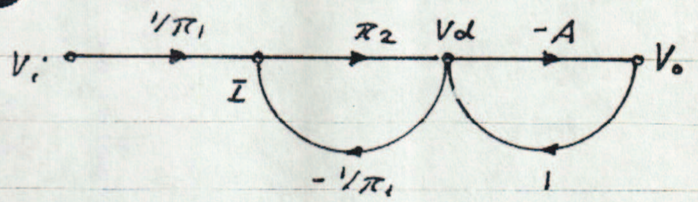
$V_o = -A \cdot V_d$

$R_i = \infty$ (Input Res.)

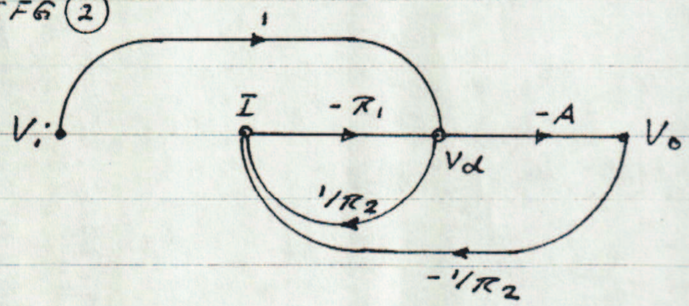
eq. syst. ①	eq. syst. ②
$I = (V_i - V_d)\frac{1}{R_1}$	$I = (V_d - V_o)\frac{1}{R_2}$
$V_d = V_o + I \cdot R_2$	$V_d = V_i - I \cdot R_1$
$V_o = -A \cdot V_d$	$V_o = -A \cdot V_d$

⇩ ⇩

SFG ①



SFG ②

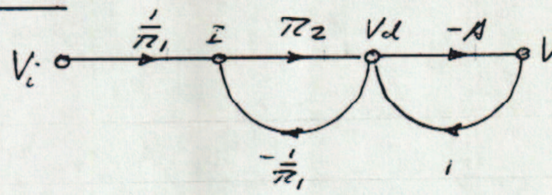


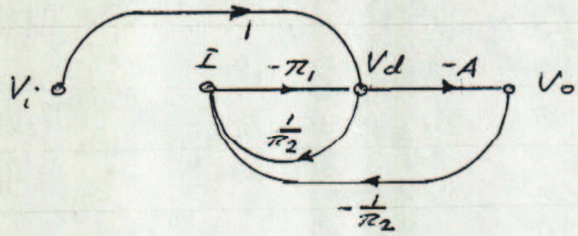
<u>Conclusion:</u>

The SFG representation of a physical system is <u>not unique</u>, since there are different ways of writing the equations that describe a system.

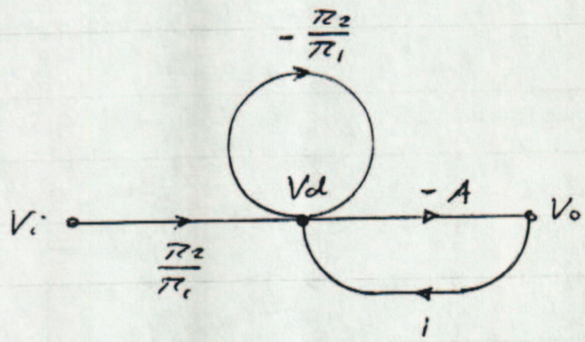
Homework: Proof identity of SFG ① with SFG ②

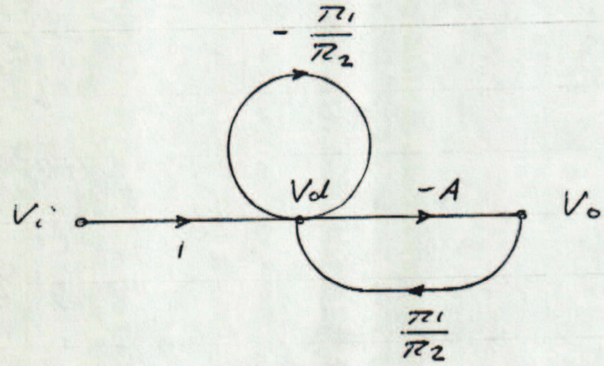
Solution



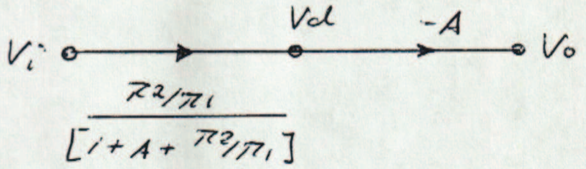


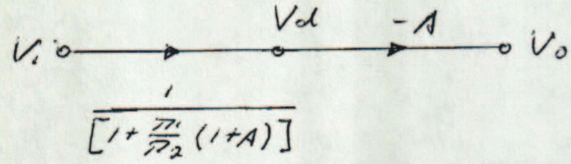
absorb node I





eliminate loops





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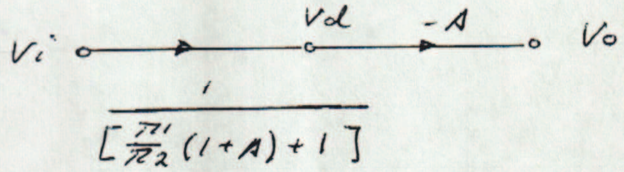

$$\boxed{V_o = -V_i \frac{R_2}{R_1(1+1/A) + R_2/A} = -V_i \frac{R_2}{R_1} \frac{1}{(1+\frac{1}{A}[1+\frac{R_2}{R_1}])}}$$

Example

① $X_0 = X_0$

② $X_1 = aX_0 + dX_2$

③ $X_2 = bX_0 + cX_1 + eX_2$

④ $X_3 = fX_1 + gX_2$

Find $\dfrac{X_3}{X_0}$

algebraic solution:

$$\frac{1}{d}X_1 - \frac{a}{d}X_0 = \frac{c}{1-e}X_1 + \frac{b}{1-e}X_0 \quad \text{(From 2 and 3)}$$

$$X_1 = \frac{\left(b/_{1-e} + a/_d\right)}{\left(1/_d - c/_{1-e}\right)} X_0$$

$$X_3 = fX_1 + g\left(\frac{c}{1-e}X_1 + \frac{b}{1-e}X_0\right) \quad \text{(From 3 and 4)}$$

$$\boxed{X_3 = f\left(\frac{\frac{b}{1-e} + \frac{a}{d}}{\frac{1}{d} - \frac{c}{1-e}}\right)X_0 + \frac{gc}{1-e}\left(\frac{\frac{b}{1-e} + \frac{a}{d}}{\frac{1}{d} - \frac{c}{1-e}}\right)X_0 + \frac{b}{1-e}X_0}$$

Typical Solution

Simplify

$$\boxed{\frac{X_3}{X_0} = \frac{fdb + fa - fea + \frac{dgcb}{1-e} + agc}{1 - e - dc} + \frac{gb}{1-e}}$$

Most people would ~~stop~~ stop here,

but $\boxed{\dfrac{gb}{1-e} = \dfrac{\frac{gb}{1-e} - \frac{egb}{1-e} - \frac{dcgb}{1-e}}{1 - e - dc}}$,

Thus, and $\dfrac{gb - egb}{1-e} = gb$

X_3 $fdb + fa + fea + agc + gb$ | best answer