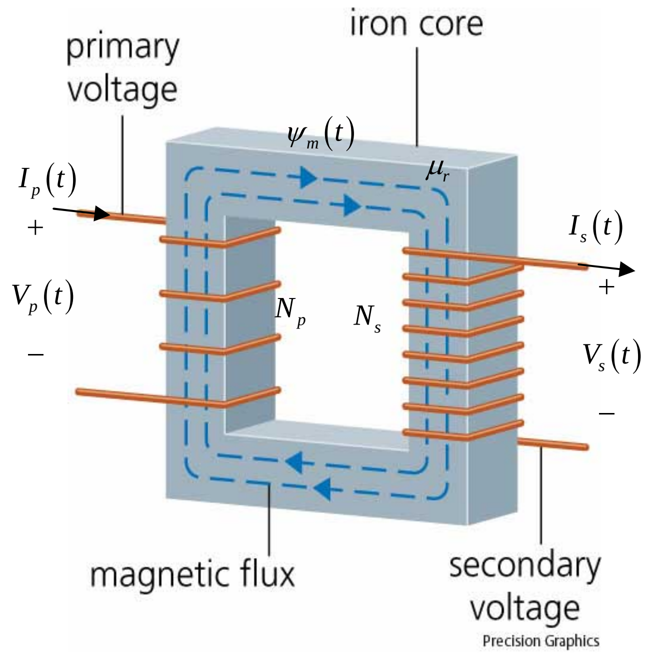
Transformers

- A transformer is an *N*-port device that transforms voltages, currents and impedances
  - Magnetically coupled
  - DC block (high-pass)
  - Parallel inductance (tuned-transformer)
- 2-port transformer:



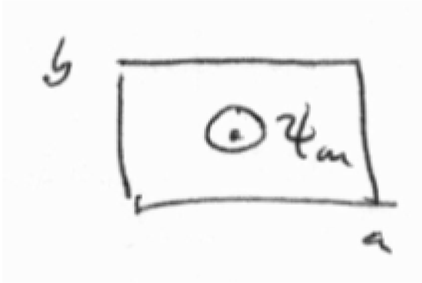
$I_p(t) =$ Primary winding current
$V_p(t) =$ Primary winding voltage (emf)
N_p = # of turns on the primary winding
μ_r = relative permeability of the core

$I_s(t) =$ Secondary winding current
$V_s(t) =$ Secondary winding voltage
N_s = # turns on the secondary winding
$\psi_m(t)$ = magnetic flux in the core

Magnetic Flux

- Total flux flowing through core:

$$\psi_m(t) = \iint_S \vec{B}(t)\cdot d\vec{s} = \iint_S \left[\vec{B}_p(t) + \vec{B}_s(t)\right]\cdot d\vec{s}$$
$$= \psi_{m,p}(t) + \psi_{m,s}(t)$$



- Flux is produced by the currents flowing through the windings
- Predicted by Ampere's law:

$$\psi_{m,j}(t) = N_j A_\ell I_j(t)$$

N_j = # of turns on the j-th winding,

$I_j(t)$ = current on the j-th winding,

A_ℓ = core constant (H/turn^2)

- The core constant is a function of the core permeability, the cross-sectional area, and the axial length.
  - Typically provided by the manufacturer
  - See Table D.2 of the text
- The core constant can be frequency dependent
  - Look for the rated frequency in the specs

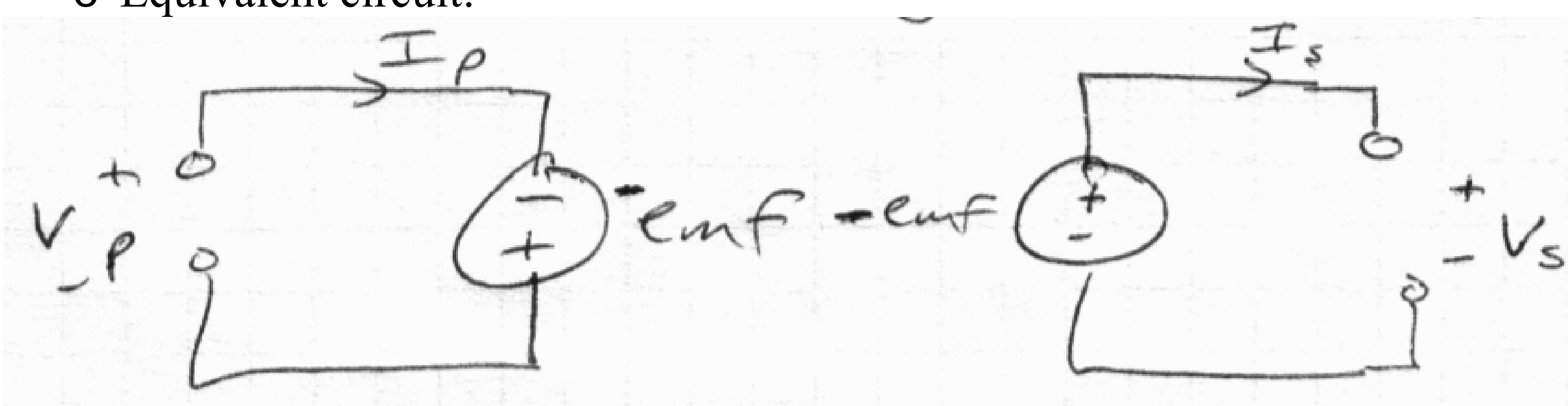
Faraday's Law:

- Faraday's law:

$$\oint_C \vec{E} \cdot d\vec{\ell} = -\frac{\partial}{\partial t} \iint_S \vec{B} \cdot d\vec{s}$$

$$emf \qquad = -N \frac{\partial}{\partial t} \psi_m$$

- The emf ("electro-motive force") is the "net push" felt by a charge on the contour C
  - An electrical potential energy
- The factor of *N* is because the flux is linking through *N* turns of the coil. Each turn experiences an emf of $\partial \psi_m / \partial t$.
- The emf is a measurable voltage
  - Equivalent circuit:



Sinusoidal Steady State Voltage

- Port voltages:

$$V_p = -\mathrm{emf_p} = j\omega N_p \psi_m$$

$$V_s = -\mathrm{emf_s} = j\omega N_s \psi_m$$

- Evaluating the quotient of these two equations:

$$\frac{V_p}{V_s} = \frac{N_p}{N_s}$$

- Turns Ratio:

$$V_p = \frac{N_p}{N_s} V_s; \qquad \frac{N_p}{N_s} = \text{ turns ratio}$$

- Observations:

$$\text{If } N_s > N_p, \text{ then } V_s > V_p \Rightarrow \text{"Step up transformer"}$$

$$\text{If } N_s < N_p, \text{ then } V_s < V_p \Rightarrow \text{"Step down transformer"}$$

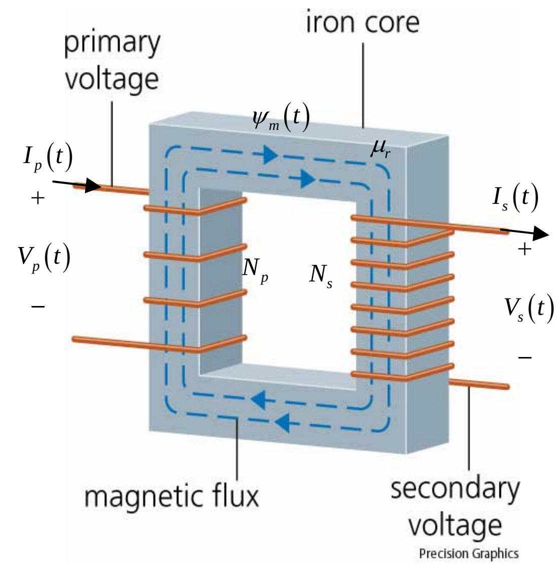
Transformer Currents

- Assume that the primary is driven by a voltage source (with internal resistance), and the secondary is terminated with a load.
- Recall:

$$\psi_m = \psi_{m,p} + \psi_{m,s}$$

- $\psi_{m,s}$ will be negative since the current is flowing *out* of the secondary winding
    - o Lenz's law: The current induced on the secondary is such that the secondary flux produced opposes the primary flux.
- The total flux in the core is:

$$\psi_m = N_p A_\ell I_p - N_s A_\ell I_s$$



- Port current relationship:

$$I_p = \frac{\psi_m}{N_p A_\ell} + \frac{N_s}{N_p} I_s . \qquad \because V_p = j\omega N_p \psi_m$$

$$I_p = \frac{V_p}{j\omega N_p^2 A_\ell} + \frac{N_s}{N_p} I_s$$

- However, $N_p^2 A_\ell$ is the inductance of the primary winding. Thus:

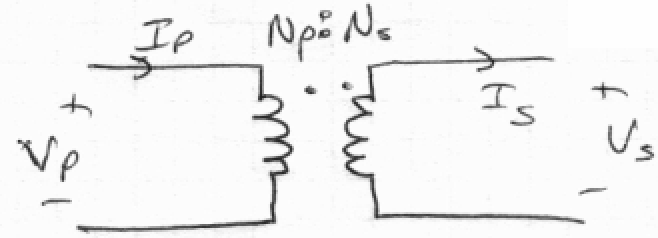
$$I_p = \frac{V_p}{j\omega L_p} + \frac{N_s}{N_p} I_s$$

$$I_p = \frac{V_p}{j\omega L_p} + \frac{N_s}{N_p} I_s$$

- $\frac{V_p}{j\omega L_p}$ is the magnetization current
  - Current is independent of the secondary winding.
  - This is the current of an N_p turn inductor wound on the core
- $\frac{N_s}{N_p} I_s$ is the transformer current due to a current on the secondary winding.
  - Current is transformed similar to the voltage – but with a reciprocal relationship
    - If $N_s > N_p,\ I_s < I_p \ \left(V_s > V_p\right)$ (step-up transformer)
    - If $N_p > N_s,\ I_p < I_s \ \left(V_p > V_s\right)$ (step-down transformer)

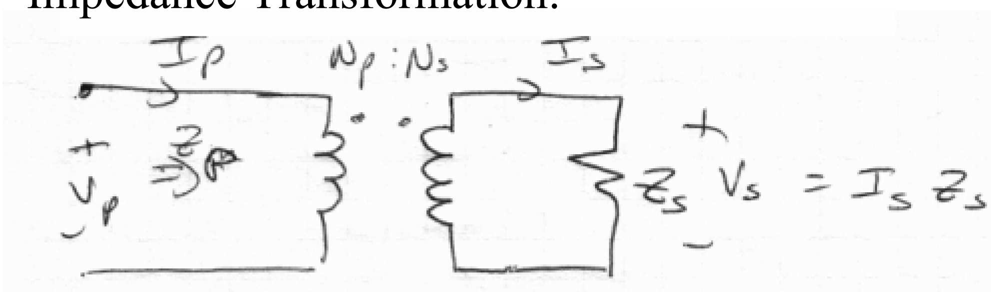
The Ideal Transformer

- Assume that $\dfrac{V_p}{j\omega L_p} << \dfrac{N_s}{N_p} I_s$
  - true if ω or L_p is sufficiently large
- Then: $I_p \approx \dfrac{N_s}{N_p} I_s$
- Ideal Transformer makes this assumption:



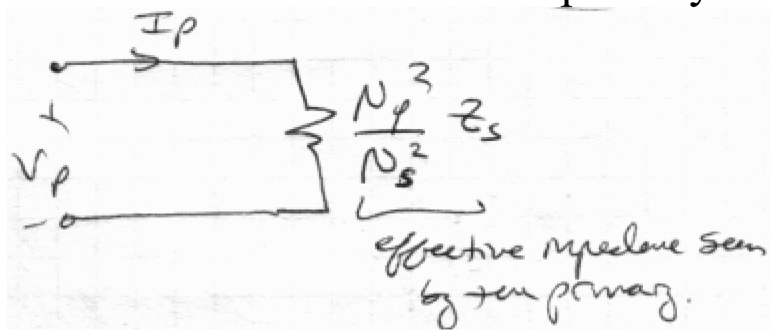
$$\boxed{V_s = \frac{N_s}{N_p} V_p \quad , \quad I_s = \frac{N_p}{N_s} I_p}$$

- Impedance Transformation:



$$Z_p = \frac{V_p}{I_p} = \frac{\left(N_p / N_s\right) V_s}{\left(N_s / N_p\right) I_s} = \frac{N_p^2}{N_s^2} Z_s$$

- An equivalent circuit can be drawn relative to the primary winding:



- The transformer effectively "transforms" the load impedance by the turns-ratio squared.

- Power of the ideal transformer:

$$P_p(t) = V_p(t) I_p(t) = \text{ instantaenous power}$$

$$P_s(t) = V_s(t) I_s(t)$$

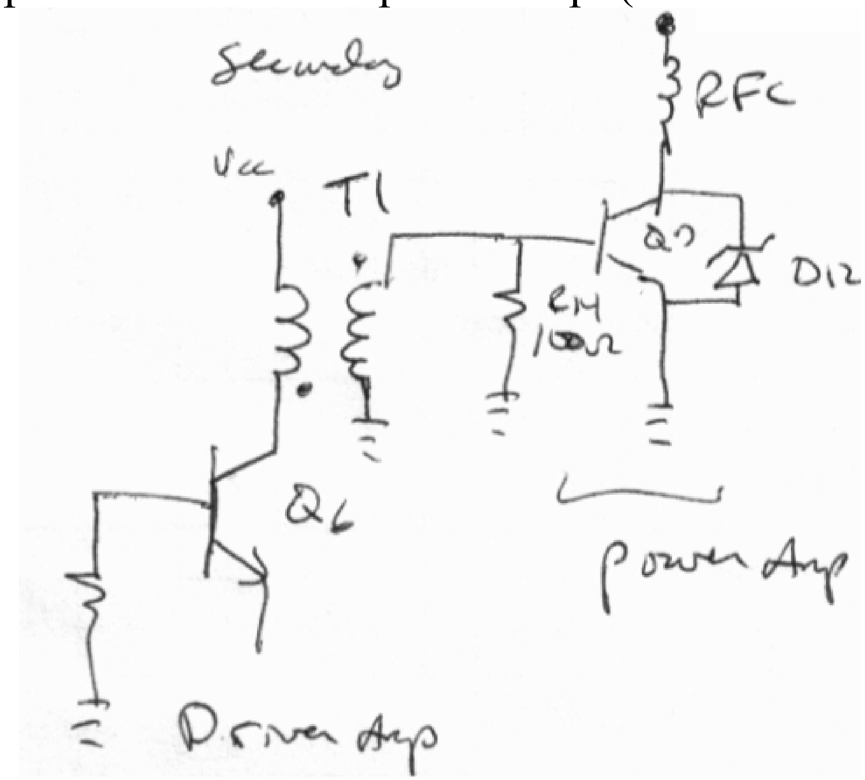
$$= \frac{N_s}{N_p} V_p(t) \frac{N_p}{N_s} I_p(t)$$

$$= V_p(t) I_p(t)$$

$$\therefore P_s(t) = P_p(t)$$

Transformers of the NORCAL40A

- Transformer "T1" (see ckt diagram on the inside of the front cover of text):
    - Couples the output of the driver amplifier to the input of the power amp
        - Decouples the DC bias networks
        - Doubles the efficiency of the driver amp, since the DC bias current is not dissipated by the effective RF collector load (R14).
        - Steps up the current in the power amp (turns ratio 14:4)



Transformers & Tuned Transformers

- Transformers T2 and T3 are "tuned transformers" (studied next)

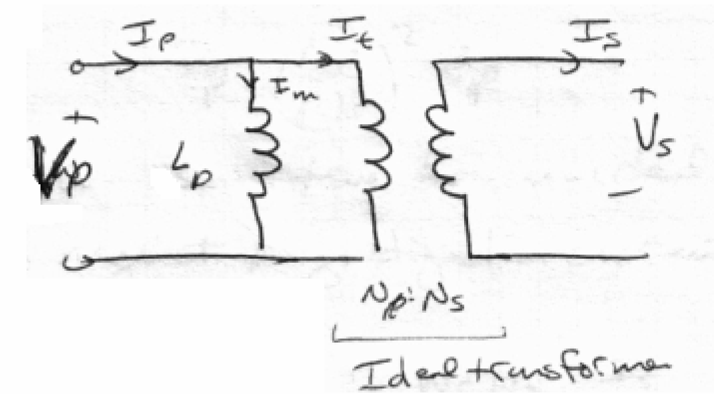
- T2
  - provides the inductance of a parallel resonator of the RF filter.
  - It also tranforms the impedance to match the 50 ohm filter to the 1500 ohm input of the RF Mixer.

- T3
  - Parallel LC (C6 combined with the parallel capacitance of the output of the RF mixer) is resonant at the IF frequency, such that the reactance cancels.
  - Used to transform the impedance to match the output of the RF mixer (3 kΩ) to the input of the IF filter (200 Ω)

$$Z_s = \left(\frac{N_s}{N_p}\right)^2 Z_p = \left(\frac{6}{23}\right)^2 3000 = 204.2\Omega$$

Tuned Transformer

- Equivalent circuit of the transformer (including magnetization and transformer currents):

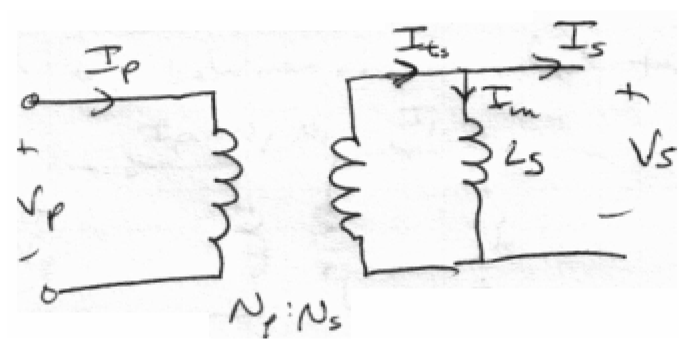


$$I_p = I_m + I_t = \frac{V_p}{j\omega L_p} + \frac{N_s}{N_p} I_s = \frac{V_p}{j\omega A_\ell N_p^2} + \frac{N_s}{N_p} I_s$$

or,

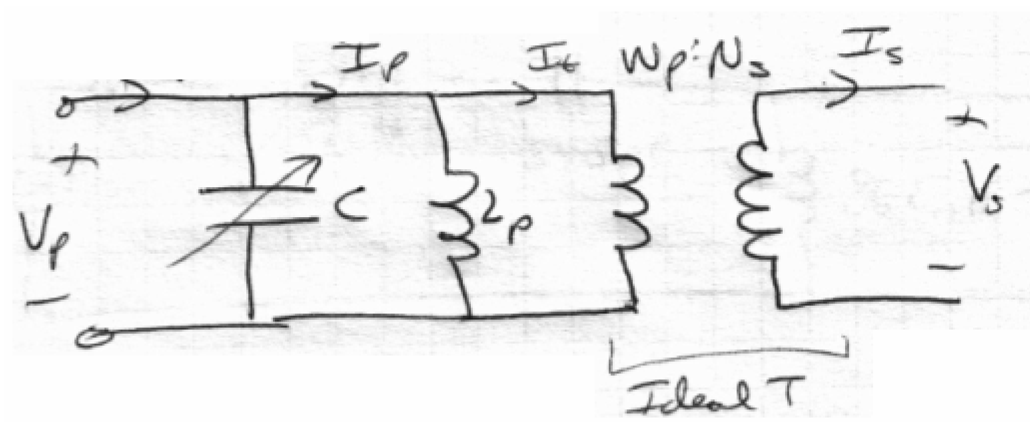
$$I_s = \frac{N_p}{N_s} I_p - \frac{N_p}{N_s}\frac{V_p}{j\omega A_\ell N_p^2} = \frac{N_p}{N_s} I_p - \frac{1}{N_s}\frac{V_p}{j\omega A_\ell N_p} = \frac{N_p}{N_s} I_p - \frac{1}{N_s}\frac{V_s\left(N_p / N_s\right)}{j\omega A_\ell N_p}$$

$$= \frac{N_p}{N_s} I_p - \frac{V_s}{j\omega A_\ell N_s^2} = \frac{N_p}{N_s} I_p - \frac{V_s}{j\omega L_s} = I_{t_s} - I_{m_s}$$



- Observations
  - The primary inductance maps into the secondary inductance through the transformer
  - The secondary current drops due to the magnetization current.
  - If the reactance of the shunt inductance is sufficiently small, it significantly loads the transformer

- The reactance can be "tuned" out with a shunt capacitance at a resonant frequency.
  - Example:

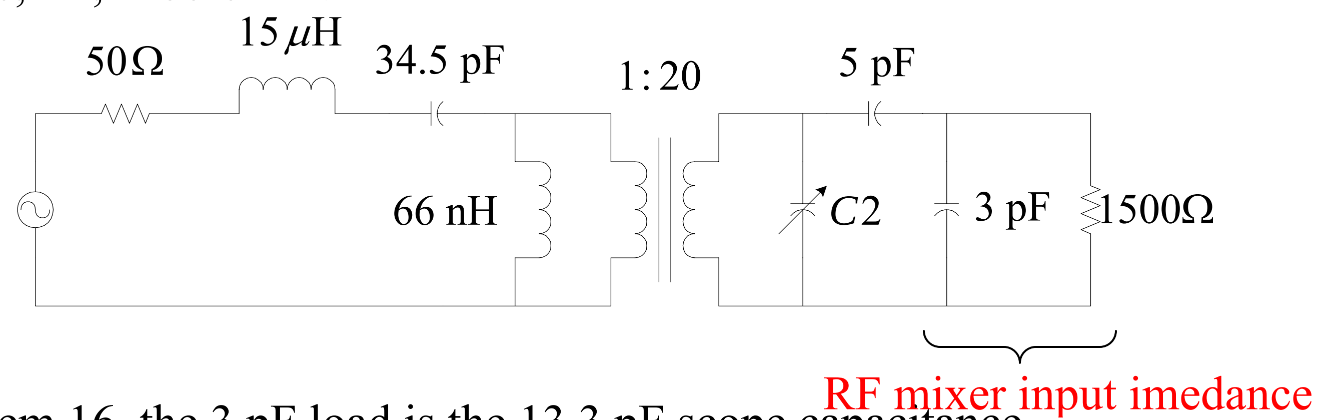


Adjust C such that:

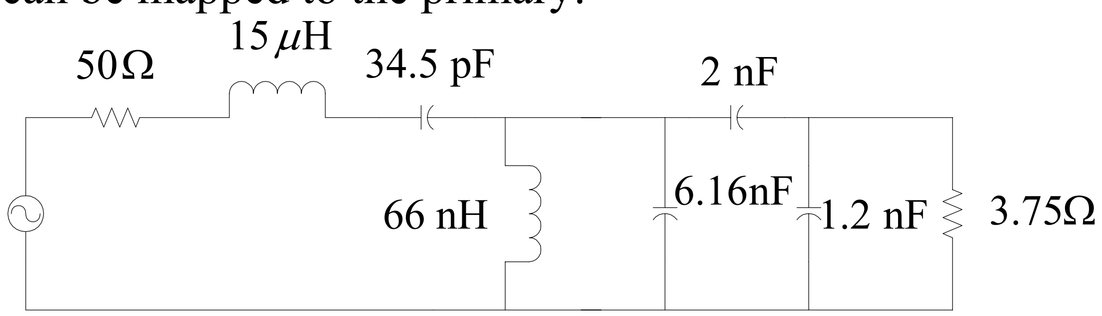
$$j\omega_o C = \frac{-1}{j\omega_o L_p} \Rightarrow \omega_o = \frac{1}{\sqrt{L_p C}}, \text{ or, } \quad C = \frac{1}{L_p \omega_o^2} = \frac{1}{\left(2\pi f_o\right)^2 L_p}$$

The network then behaves as an ideal transformer at f_o. (T3)

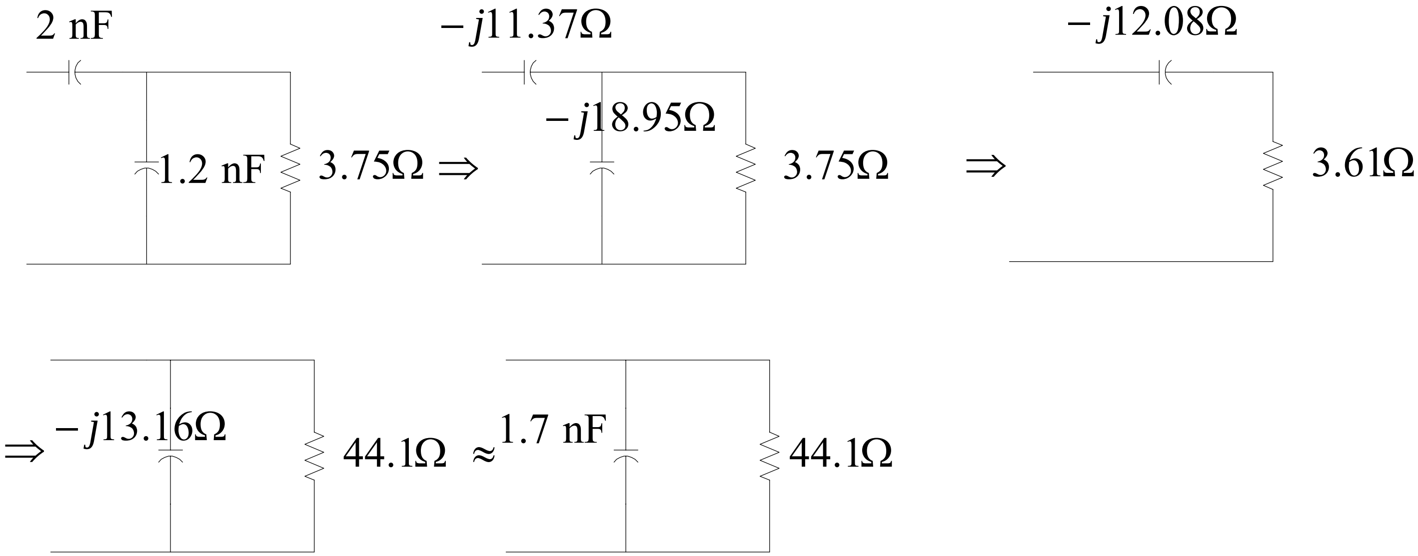
- The tuned transformer can also be used as a parallel resonator branch of a band-pass filter
- The parallel susceptances will cancel at the center frequency of the filter.
- L_p and C are chosen according to the BPF design (from prototype).
- Example, T2, Problem 16



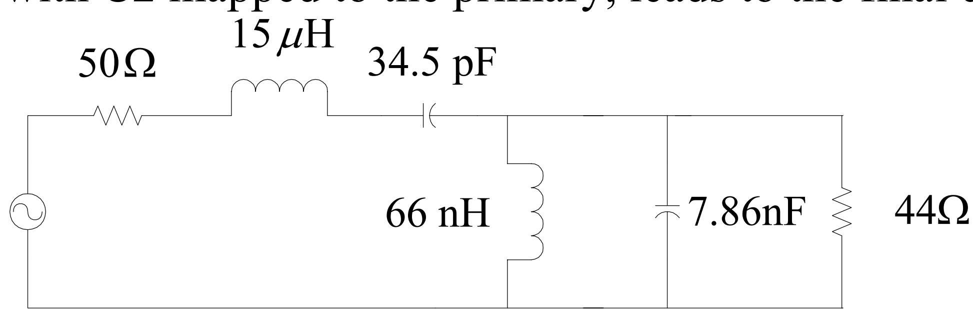
- In problem 16, the 3 pF load is the 13.3 pF scope capacitance
  - Here, we assume the actual load, and let $C_2 = 15.4$ pF.
- The circuit can be mapped to the primary:



- The load with the blocking capacitor is transformed via a parallel-to-series conversion, then a series to parallel conversion as:



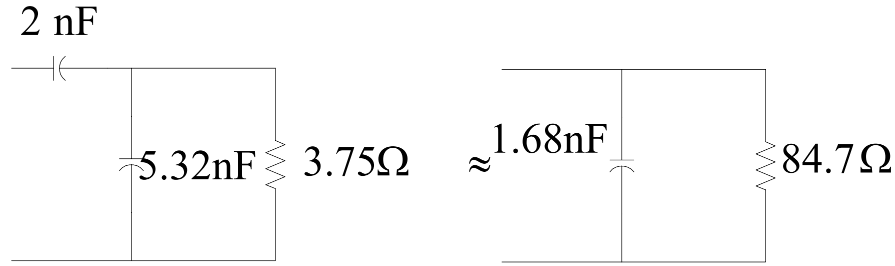
- Combining this with C2 mapped to the primary, leads to the final circuit:



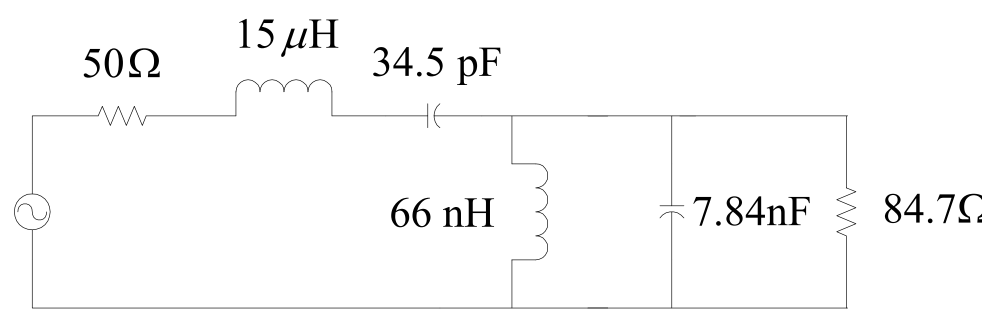
- This is close to the Butterworth design ("BandPassFilters.pdf", pg. 5)
  - Parallel resonator slightly out of tune. $Z_{in} = 43.8 - j1.6\Omega$ @ 7 MHz

Loading by Scope Probe

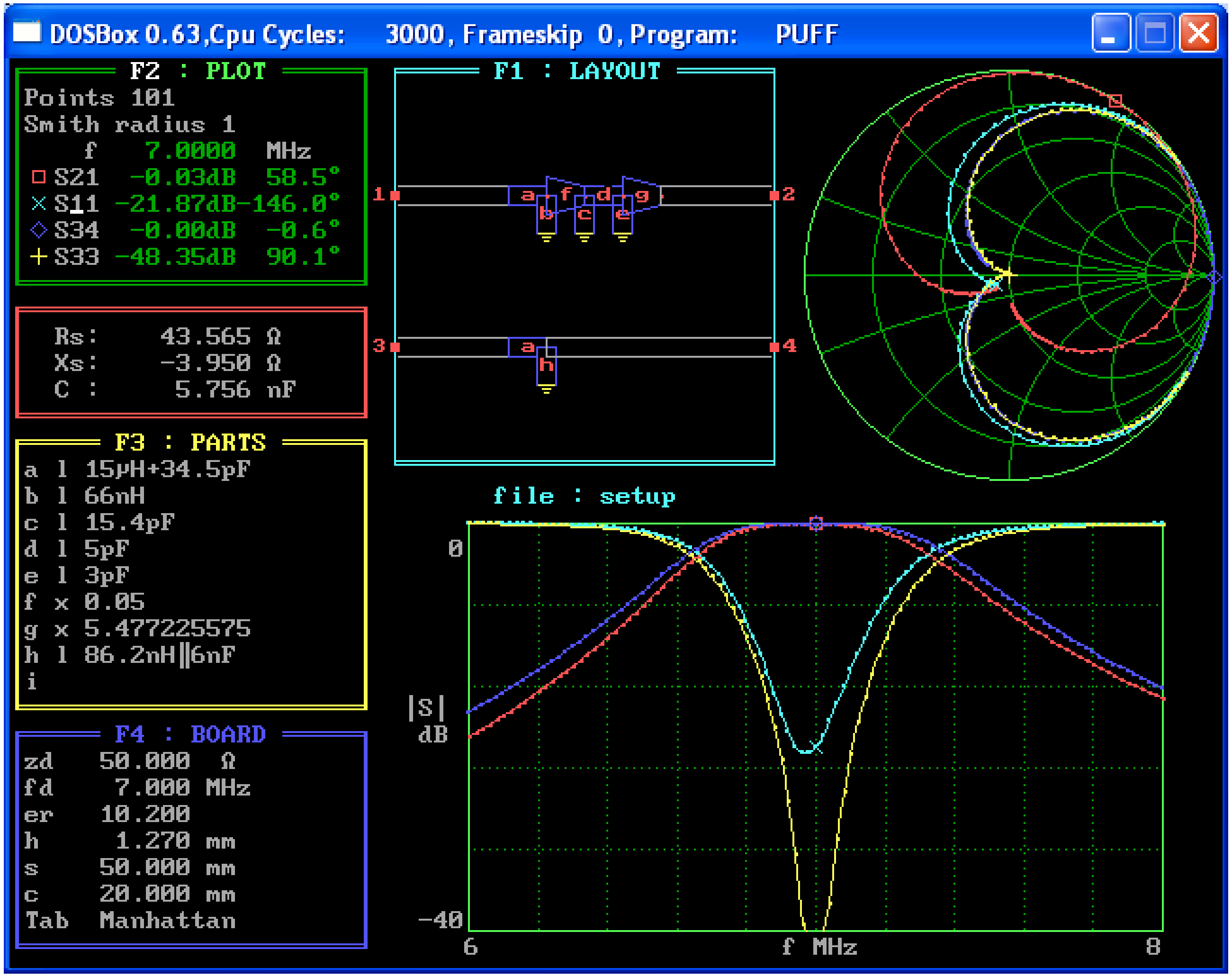
- In P16, scope probe capacitively loads the ckt (Cprobe = 13.3 pF)



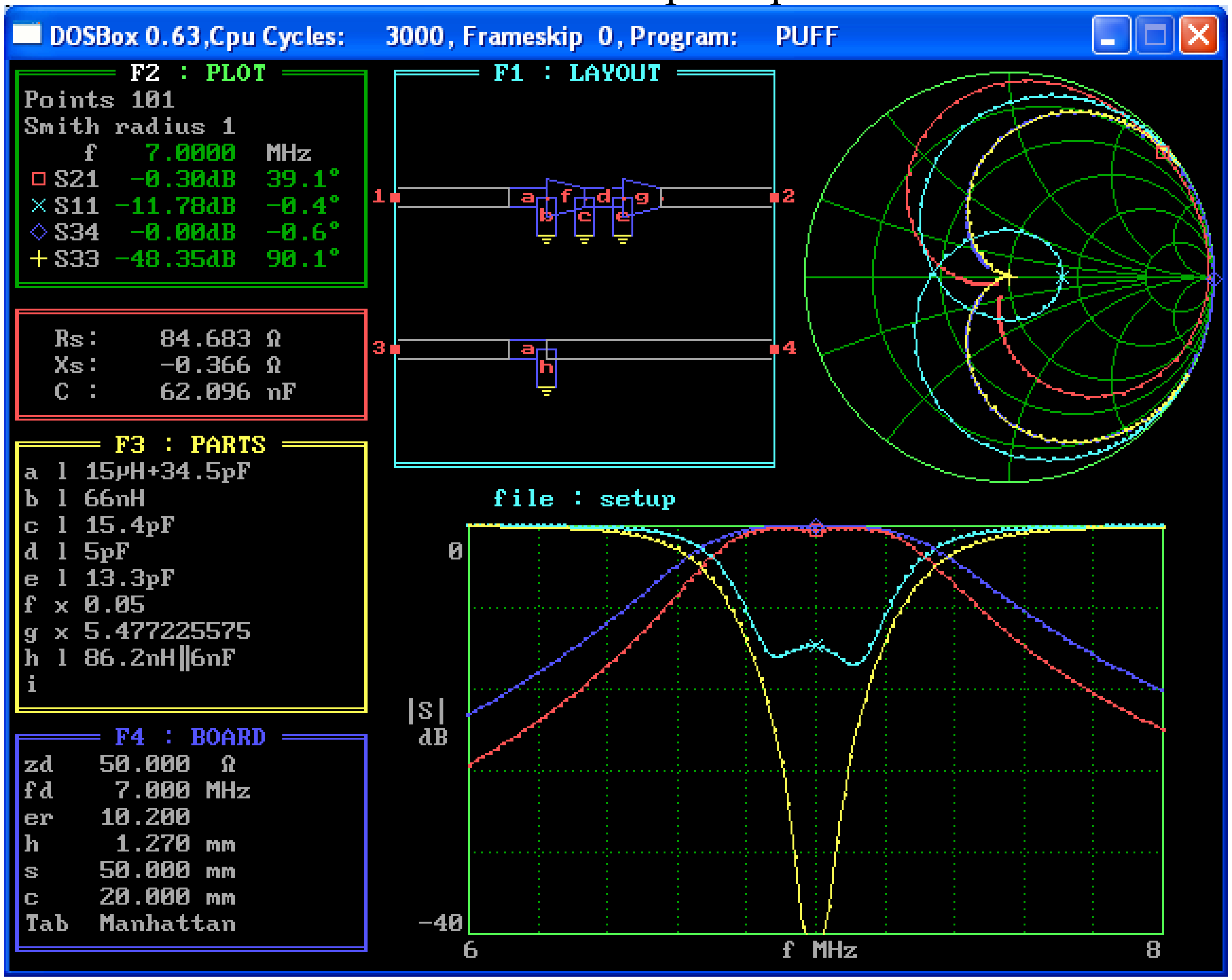
- Final circuit:



- Load has significantly increased.
  - $Z_{in} = 84.6 - j1.66\Omega$ @ 7 MHz
- The probe loads the circuit, degrading the performance from what it actually is. Though, it still approximates it, as seen from the following PUFF simulations

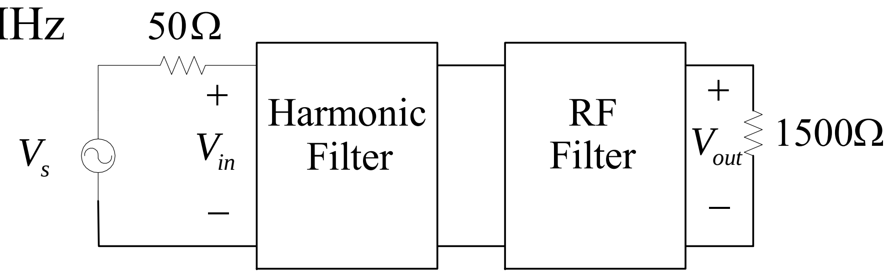

DOSBox 0.63,Cpu Cycles: 3000, Frameskip 0, Program: PUFF
F2 : PLOT
Points 101
Smith radius 1
f 7.0000 MHz
S21 -0.03dB 58.5°
S11 -21.87dB-146.0°
S34 -0.00dB -0.6°
S33 -48.35dB 90.1°
Rs: 43.565 Ω
Xs: -3.950 Ω
C : 5.756 nF
F3 : PARTS
a l 15μH+34.5pF
b l 66nH
c l 15.4pF
d l 5pF
e l 3pF
f x 0.05
g x 5.477225575
h l 86.2nH||6nF
i
F4 : BOARD
zd 50.000 Ω
fd 7.000 MHz
er 10.200
h 1.270 mm
s 50.000 mm
c 20.000 mm
Tab Manhattan
F1 : LAYOUT
file : setup
|S| dB
0
-40
6
f MHz
8

Loaded with the Scope Capacitance



Loss Factor

- The principal purpose of the combined RF and Harmonic Filter is to reject the image frequency of the RF mixer, as well as other "spurs" of the Gilbert cell mixer.
  - o Image frequency = 4.9 MHz – 2.1 MHz = 2.8 MHz
  - o Dominant *spur* = 5.6 MHz



- At resonance (7 MHz), there is insertion loss due to miss-matches and losses of the filters. We measure this via the loss factor at 7 MHz
- We will also measure the "Rejection" Loss at the IF and spurious frequencies.
- We measure the loss factor as: $L = 10\log\left(\frac{P^+}{P_{out}}\right)$
  - P_{out} = power delivered to the load = $\frac{1}{8}\frac{V_{out}^2}{1500}$
  - P^+ = the *available power* from the source = $\frac{1}{8}\frac{V_{in}^2}{50}$
  - o Note that the available power P^+ is the maximum power available from the source, and is the power that would be dissipated by a matched load.
  - o Also, note that $L \neq 20\log(V_{out}/V_{in})$!! Why not?