

This is RMS in its most general form:

$$RMS = \sqrt{\frac{1}{T} \int_0^T f(t)^2 dt} \quad \text{Equation 1}$$

Think about what it means. The part in the integral is the **area underneath the square of the function**. The part underneath the square root takes this area and divides it by the period which gives you the average (mean) area of $f(t)^2$ per unit time. Then you take the square root of everything. The result is you get the **Root of the Mean of the Squared of the values (RMS)**.

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BONUS SECTION

Let's talk a bit about $f(t)^2$ is used instead of $f(t)$. In a world where all functions were entirely positive (or negative), we would probably be just using $f(t)$. But in our world, we have functions that are centered perfectly equally around zero like your typical AC sinusoid (or any pure AC periodic signal). If you take the area of one period of these symmetrical functions, you end up adding an equal amount of positive area and negative area so you always get zero area. This makes this definition not very useful for a huge class of functions. We could use $|f(t)|$ instead to solve this problem, but this makes it hard to calculate for very complex mathematical functions since you have to examine the function separately every time it crosses zero. So instead, we use $f(t)^2$. It makes everything positive and the entire equation is treated the same way mathematically. But the square part skews our result, so to help fix that we take the square root of everything.

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Now, Equation 1 is the most general form. But for functions with discontinuities (parts that have infinite slope like a sawtooth wave or square waves), it is only possible to solve the above integral by breaking up the integral into continuous segments.

$$RMS = \sqrt{\frac{\int_0^{t_1} f_1(t)^2 dt + \int_{t_1}^{t_2} f_1(t)^2 dt + \int_{t_2}^{t_3} f_1(t)^2 dt + \dots + \int_{t_{n-1}}^{t_n} f_1(t)^2 dt}{T}}$$

Now for a square wave in particular each "square" in the wave is a continuous segment, because they have an infinite slope on either side. So after a bunch of math (or just plain knowledge that the area beneath a rectangular area is length*width, we can write can replace :

$$\int_{t_1}^{t_2} f_1(t)^2 dt \text{ with } f_1^2(t_2 - t_1) \text{ since over that segment the function has a constant value. So}$$

we get:

$$RMS = \sqrt{\frac{f_0^2(t_1 - t_0) + f_1^2(t_2 - t_1) + f_2^2(t_3 - t_2) + \dots + f_n^2(t_n - t_{n-1})}{T}} = \sqrt{\frac{f_0^2(t_1 - t_0) + f_1^2(t_2 - t_1) + f_2^2(t_3 - t_2) + \dots + f_n^2(t_n - t_{n-1})}{(t_1 - t_0) + (t_2 - t_1) + (t_3 - t_2) + \dots + (t_n - t_{n-1})}}$$

$$= \sqrt{\frac{f_0^2 \Delta t_0 + f_1^2 \Delta t_1 + f_2^2 \Delta t_2 + \dots + f_n^2 \Delta t_n}{\Delta t_0 + \Delta t_1 + \Delta t_2 + \dots + \Delta t_n}}$$

Now, when you were told that the RMS was

$$RMS = \frac{f_{peak}}{\sqrt{2}}$$

this was only for pure sinusoids. If you take equation 1 and make $f(t) = \sin(t)$ and calculate it out, you get that result.