The MOSFET

Device Symbols

Whereas the JFET has a diode junction between the gate and the channel, the metal-oxide semiconductor FET or MOSFET differs primarily in that it has an oxide insulating layer separating the gate and the channel. The circuit symbols are shown in Fig. 1. Each device has gate (G), drain (D), and source (S) terminals. Four of the symbols show an additional terminal called the body (B) which is not normally used as an input or an output. It connects to the drain-source channel through a diode junction. In discrete MOSFETs, the body lead is connected internally to the source. When this is the case, it is omitted on the symbol as shown in four of the MOSFET symbols. In integrated-circuit MOSFETs, the body usually connects to a dc power supply rail which reverse biases the body-channel junction. In the latter case, the so-called "body effect" must be accounted for when analyzing the circuit.

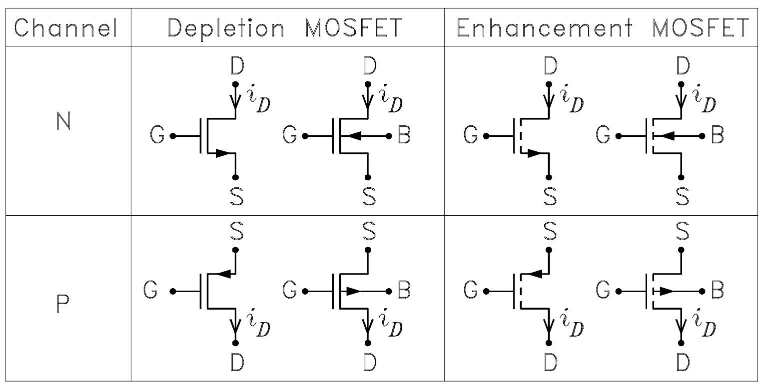

Figure 1: MOSFET symbols.

Device Equations

The discussion here applies to the n-channel MOSFET. The equations apply to the p-channel device if the subscripts for the voltage between any two of the device terminals are reversed, e.g. v_{GS} becomes v_{SG}. The n-channel MOSFET is biased in the active mode or saturation region for $v_{DS} \geq v_{GS} - v_{TH}$, where v_{TH} is the threshold voltage. This voltage is negative for the depletion-mode device and positive for the enhancement-mode device. It is a function of the body-source voltage and is given by

$$v_{TH} = V_{TO} + \gamma \left[\sqrt{\phi - v_{BS}} - \sqrt{\phi} \right] \tag{1}$$

where V_{TO} is the value of v_{TH} with $v_{BS} = 0$, γ is the body threshold parameter, ϕ is the surface potential, and v_{BS} is the body-source voltage. The drain current is given by

$$i_D = \frac{k'}{2} \frac{W}{L} \left(1 + \lambda v_{DS}\right) \left(v_{GS} - v_{TH}\right)^2 \tag{2}$$

where W is the channel width, L is the channel length, λ is the channel-length modulation parameter, and k' is given by

$$k' = \mu_0 C_{ox} = \mu \frac{\epsilon_{ox}}{t_{ox}}$$

In this equation, μ_0 is the average carrier mobility, C_{ox} is the gate oxide capacitance per unity area, ϵ_{ox} is the permittivity of the oxide layer, and t_{ox} is its thickness. It is convenient to define a transconductance coefficient K given by

$$K = \frac{k'}{2}\frac{W}{L}(1 + \lambda v_{DS}) = K_0 (1 + \lambda v_{DS}) \tag{3}$$

where K_0 is given by

$$K_0 = \frac{k'}{2}\frac{W}{L} \tag{4}$$

With these definitions, the drain current can be written

$$i_D = K (v_{GS} - v_{TH})^2 \tag{5}$$

Note that K plays the same role in the MOSFET drain current equation as β plays in the JFET drain current equation.

Some texts define $K = k' (W/L) (1 + \lambda v_{DS})$ so that i_D is written $i_D = (K/2) (v_{GS} - v_{TH})^2$. In this case, the numerical value of K is twice the value used here. To modify the equations given here to conform to this usage, replace K in any equation given here with $K/2$.

Transfer and Output Characteristics

The transfer characteristics are a plot of the drain current i_D as a function of the gate-to-source voltage v_{GS} with the drain-to-source voltage v_{DS} held constant. Fig. 2 shows the typical transfer characteristics for a zero body-to-source voltage. In this case, the threshold voltage is a constant, i.e. $v_{TH} = V_{TO}$. For $v_{GS} \leq V_{TO}$, the drain current is zero. For $v_{GS} > V_{TO}$, Eq. (5) shows that the drain current increases as the square of the gate-to-source voltage. The slope of the curve represents the small-signal transconductance g_m, which is defined in the following.

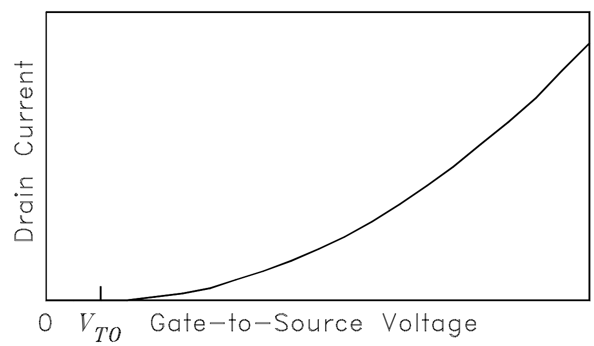

Figure 2: Drain current i_D versus gate-to-source voltage v_{GS} for constant drain-to-source voltage v_{DS}.

The output characteristics are a plot the drain current i_D as a function of the drain-to-source voltage v_{DS} with the gate-to-source voltage v_{GS} and the body-to-source voltage v_{BS} held constant. Fig. 3 shows the typical output characteristics for several values of gate-to-source voltage v_{GS}. The dashed line divides the triode region from the saturation or active region. In the saturation region, the slope of the curves represents the reciprocal of the small-signal drain-source resistance r_0, which is defined in the next section.

Small-Signal Models

There are two small-signal circuit models which are commonly used to analyze MOSFET circuits. These are the hybrid-π model and the T model. The two models are equivalent and give identical results. They are described below. In addition, a simplified small-model is derived which is called the source equivalent circuit. The models are first developed for the case of no body effect and then with the body effect. The former

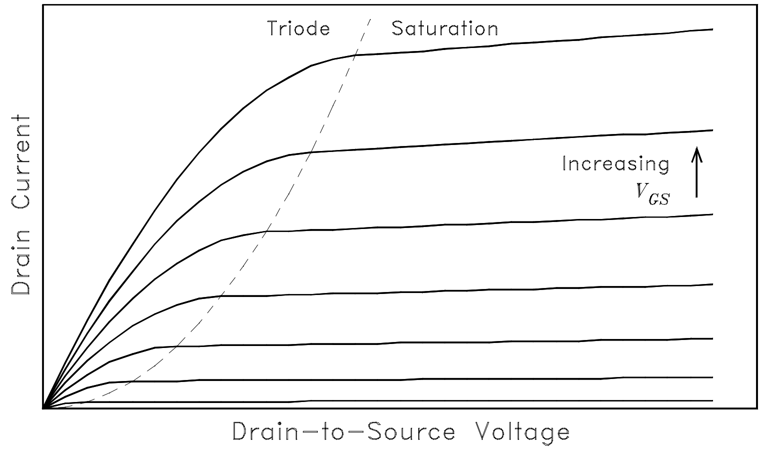

Figure 3: Drain current i_D versus drain-to-source voltage v_{DS} for constant gate-to-source voltage v_{GS}.

case assumes that the body-source voltage is zero, i.e. $v_{BS} = 0$. This is the case with discrete MOSFETs in which the source is connected physically to the body. It also applies to small-signal ac analyses for which the body and source leads are connected to the same or different dc voltages. In this case, the small-signal body-source voltage is zero, i.e. $v_{bs} = 0$, and there is no body effect.

No Body Effect

The small-signal models in this section assume that the body lead is connected to the source lead. The models also apply when the body and source leads are connected to different dc voltages so that the ac or signal voltage from body to source is zero.

Hybrid-π Model

Consider the case where the body-source voltage is zero, i.e. $v_{BS} = 0$. In this case, the threshold voltage in Eq. 1 is a constant and given by $v_{TH} = V_{TO}$. Let the drain current and each voltage be written as the sum of a dc component and a small-signal ac component as follows:

$$i_D = I_D + i_d \tag{6}$$

$$v_{GS} = V_{GS} + v_{gs} \tag{7}$$

$$v_{DS} = V_{DS} + v_{ds} \tag{8}$$

If the ac components are sufficiently small, we can write

$$i_d = \frac{\partial I_D}{\partial V_{GS}} v_{gs} + \frac{\partial I_D}{\partial V_{DS}} v_{ds} \tag{9}$$

where the derivatives are evaluated at the dc bias values. Let us define

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = K\left(V_{GS} - V_{TH}\right) = 2\sqrt{K I_D} \tag{10}$$

$$r_0 = \left[\frac{\partial I_D}{\partial V_{DS}}\right]^{-1} = \left[\frac{k'}{2}\frac{W}{L}\lambda\left(V_{GS} - V_{TH}\right)^2\right]^{-1} = \frac{1/\lambda + V_{DS}}{I_D} \tag{11}$$

It follows that the small-signal drain current can be written

$$i_d = i'_d + \frac{v_{ds}}{r_0} \tag{12}$$

where

$$i'_d = g_m v_{gs} \tag{13}$$

The small-signal circuit which models these equations is given in Fig. 4. This is called the hybrid-π model.

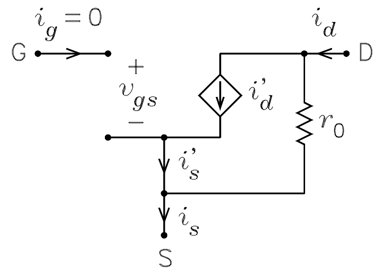

Figure 4: π model of the MOSFET.

T Model

The T model of the MOSFET is shown in Fig. 5. The resistor r_0 is given by Eq. (11). The resistor r_s is given by

$$r_s = \frac{1}{g_m} \tag{14}$$

where g_m is the transconductance defined in Eq. (10). The currents are given by

$$i_d = i'_s + \frac{v_{ds}}{r_0} \tag{15}$$

$$i'_s = \frac{v_{gs}}{r_s} = g_m v_{gs} \tag{16}$$

The currents in the T model are the same as for the hybrid-π model. Therefore, the two models are equivalent. Note that the gate and body currents in Fig. 5 are zero because the controlled source supplies the current that flows through r_s.

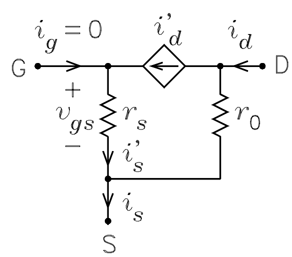

Figure 5:

A Source Equivalnet Circuit

Figure 6 shows the MOSFET T model described above with a Thévenin source in series with the gate. We wish to solve for the equivalent circuit in which the source i'_d is replaced by a single source which connects from the drain node to ground having the value $i'_d = i'_s$. We call this the source equivalent circuit. Looking up into the branch labeled i'_s, we can write $v_s = v_{tg} - i'_s r_s$ With $v_{tg} = 0$, the resistance r_s seen looking up into the branch labeled i'_s is given by

$$r_s = \frac{1}{g_m} \tag{17}$$

The source equivalent circuit is shown in Fig.7. Note that there is no R_{tg} in the circuit because there is no current through R_{tg} in the original circuit. Compared to the corresponding circuit for the BJT, the MOSFET circuit replaces v_{tb} with v_{tg} and r'_e with r_s. Because the gate current is zero, set $\alpha = 1$ and $\beta = \infty$ in converting any BJT formulas to corresponding MOSFET formulas.

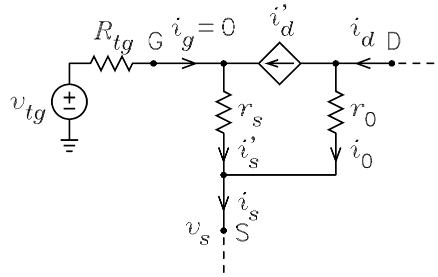

Figure 6:

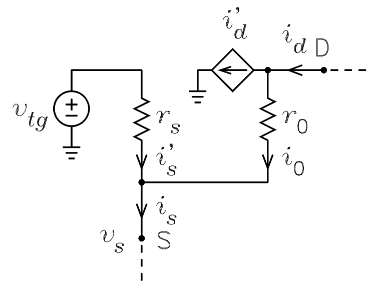

Figure 7:

With Body Effect

The small-signal models in this section assume that the body lead is connected to ac signal ground. In integrated circuit design, this ac signal ground is typically a dc power supply rail. In this case, any ac signal voltage on the source lead causes an ac signal voltage between the body and source. The effect of this voltage is called the body effect.

Hybrid-π Model

Let the drain current and each voltage be written as the sum of a dc component and a small-signal ac component as follows:

$$i_D = I_D + i_d \tag{18}$$

$$v_{GS} = V_{GS} + v_{gs} \tag{19}$$

$$v_{BS} = V_{BS} + v_{bs} \tag{20}$$

$$v_{DS} = V_{DS} + v_{ds} \tag{21}$$

If the ac components are sufficiently small, we can write

$$i_d = \frac{\partial I_D}{\partial V_{GS}} v_{gs} + \frac{\partial I_D}{\partial V_{BS}} v_{bs} + \frac{\partial I_D}{\partial V_{DS}} v_{ds} \tag{22}$$

where the derivatives are evaluated at the dc bias values. Let us define

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = K\left(V_{GS} - V_{TH}\right) = 2\sqrt{KI_D} \tag{23}$$

$$g_{mb} = \frac{\partial I_D}{\partial V_{BS}} = \frac{\gamma\sqrt{KI_D}}{\sqrt{\phi - V_{BS}}} = \chi g_m \tag{24}$$

$$\chi = \frac{\gamma}{2\sqrt{\phi - V_{BS}}} \tag{25}$$

$$r_0 = \left[\frac{\partial I_D}{\partial V_{DS}}\right]^{-1} = \left[\frac{k'}{2}\frac{W}{L}\lambda\left(V_{GS}-V_{TH}\right)^2\right]^{-1} = \frac{V_{DS}+1/\lambda}{I_D} \tag{26}$$

The small-signal drain current can thus be written

$$i_d = i_{dg} + i_{db} + \frac{v_{ds}}{r_0} \tag{27}$$

where

$$i_{dg} = g_m v_{gs} \tag{28}$$

$$i_{db} = g_{mb} v_{bs} \tag{29}$$

The small-signal circuit which models these equations is given in Fig. 8. This is called the hybrid-π model. If the body (B) lead is connected to the source, then $v_{bs} = 0$ and the circuit becomes that given in Fig. 4.

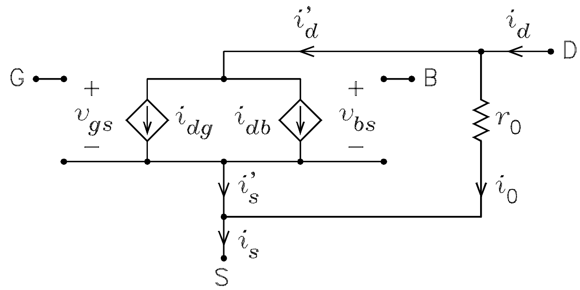

Figure 8: Hybrid-π model of the MOSFET.

T Model

The T model of the MOSFET is shown in Fig. 9. The resistor r_0 is given by Eq. (26). The resistors r_s and r_{sb} are given by

$$r_s = \frac{1}{g_m} \tag{30}$$

$$r_{sb} = \frac{1}{g_{mb}} = \frac{1}{\chi g_m} = \frac{r_s}{\chi} \tag{31}$$

where g_m and g_{mb} are the transconductances defined in Eqs. (23) and (24). The currents are given by

$$i_d = i_{sg} + i_{sb} + \frac{v_{ds}}{r_0} \tag{32}$$

$$i_{sg} = \frac{v_{gs}}{r_s} = g_m v_{gs} \tag{33}$$

$$i_{sb} = \frac{v_{bs}}{r_{sb}} = g_{mb} v_{bs} \tag{34}$$

The currents are the same as for the hybrid-π model. Therefore, the two models are equivalent. Note that the gate and body currents are zero because the two controlled sources supply the currents that flow through r_s and r_{sb}.

Source Equivalent Circuit

Figure 10 shows the MOSFET T model with a Thévenin source in series with the gate and the body connected to signal ground. We wish to solve for the equivalent circuit in which the sources i_{sg} and i_{sb} are replaced by a single source which connects from the drain node to ground having the value $i'_d = i'_s$. We call this

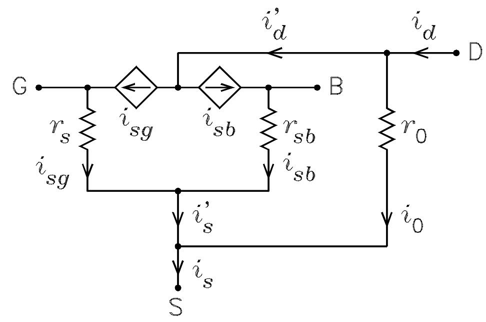

Figure 9: T model of the MOSFET.

the source equivalent circuit. The first step is to look up into the branch labeled i_s' and form a Thévenin equivalent circuit. With $i_s' = 0$, we can use voltage division to write

$$v_{s(oc)} = v_{tg} \frac{r_{sb}}{r_s + r_{sb}} = v_{tg} \frac{r_s/\chi}{r_s + r_s/\chi} = \frac{v_{tg}}{1+\chi} \tag{35}$$

With $v_{tg} = 0$, the resistance r_s' seen looking up into the branch labeled i_s' is

$$r_s' = r_s \| r_{sb} = \frac{r_s}{1+\chi} = \frac{1}{(1+\chi)\, g_m} \tag{36}$$

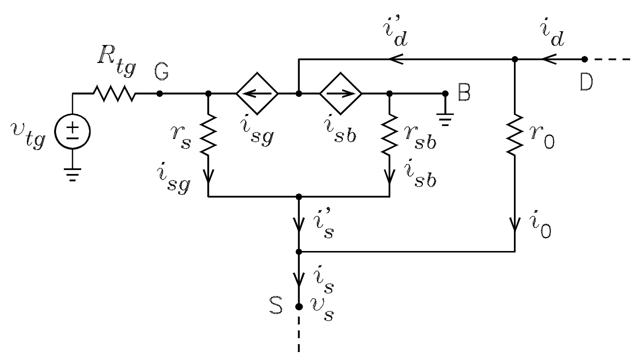

Figure 10: T model with Thévenin source connected to the gate and the body connected to signal ground.

The source equivalent circuit is shown in Fig.11. Compared to the corresponding circuit without the body effect, the circuit replaces v_{tg} with $v_{tg}/(1+\chi)$ and r_s with $r_s' = r_s/(1+\chi)$. To convert the source equivalent circuit with the body effect to one without the body effect, simply set $\chi = 0$.

The r_0 Approximations

No Body Effect

The r_0 approximations approximate r_0 as an open circuit except when calculating the resistance seen looking into the drain. Fig. 7 shows the source equivalent circuit for calculating $r_{id} = v_t/i_d$. The resistor r_s is given by $r_s = 1/g_m$. We can write

$$i_d = i_0 + i_s = i_0 \left(1 - \frac{R_{ts}}{r_s + R_{ts}}\right) = \frac{v_t}{r_0 + r_s \| R_{ts}} \left(1 - \frac{R_{ts}}{r_e + R_{ts}}\right) \tag{37}$$

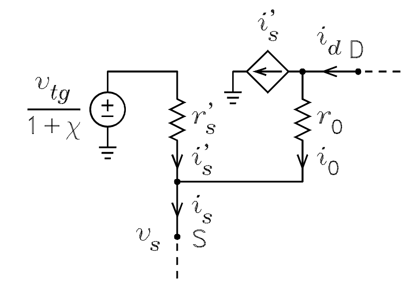

Figure 11: Source equivalent circuit.

It follows that r_{id} is given by

$$r_{id} = \frac{v_t}{i_d} = \frac{r_0 + r_s \| R_{ts}}{1 - R_{ts}/\left(r_s + R_{ts}\right)} = r_0\left(1 + \frac{R_{ts}}{r_s}\right) + R_{ts} \tag{38}$$

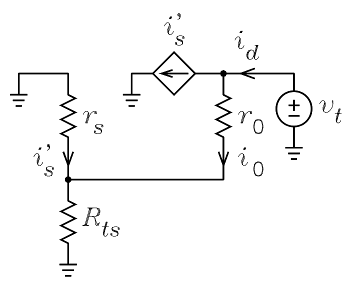

Figure 12: Circuit for calculating the resistance r_{id} seen looking into the drain.

The r_0 approximations for the source equivalent circuit, the hybrid π model, and the T model, respectively, are given Figs. 13 through 15. Because r_0 no longer connects to the source, there is only one source current and $i_s = i'_s$. If $r_0 = \infty$, then r_{ic} is an open circuit in each.

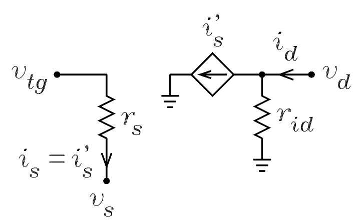

Figure 13: Source equivalnet circuit with r_0 approximations.

With Body Effect

The r_0 approximations approximate r_0 as an open circuit except when calculating the resistance seen looking into the drain. Fig. 16 shows the source equivalent circuit for calculating $r_{id} = v_t/i_d$. The resistor r'_s is given by Eq. (36). We can write

$$i_d = i_0 + i'_s = i_0\left(1 - \frac{R_{ts}}{r'_s + R_{ts}}\right) = \frac{v_t}{r_0 + r'_s \| R_{ts}}\left(1 - \frac{R_{ts}}{r'_e + R_{ts}}\right) \tag{39}$$

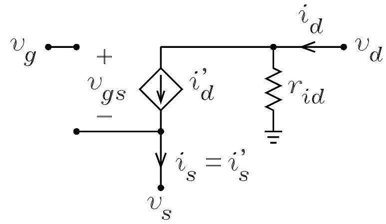

Figure 14: Hybrid π model with the r_0 approximations.

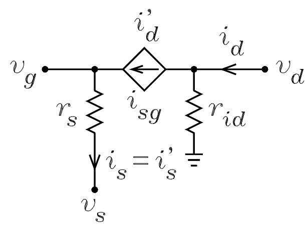

Figure 15: T model with the r_0 approximations.

It follows that r_{id} is given by

$$r_{id} = \frac{v_t}{i_d} = \frac{r_0 + r'_s \| R_{ts}}{1 - R_{ts}/\left(r'_s + R_{ts}\right)} = r_0\left(1 + \frac{R_{ts}}{r'_s}\right) + R_{ts} \tag{40}$$

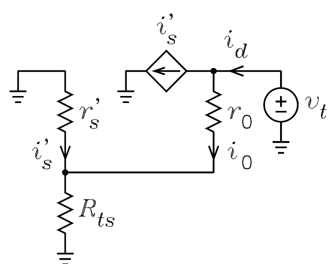

Figure 16: Circuit for calculating the resistance r_{id} seen looking into the collector.

The r_0 approximations for the source equivalent circuit, the hybrid π model, and the T model, respectively, are given Figs. 17 through 19. Because r_0 no longer connects to the source, there is only one source current and $i_s = i'_s$. If $r_0 = \infty$, then r_{ic} is an open circuit in each.

Small-Signal High-Frequency Models

Figures 20 and 21 show the hybrid-π and T models for the MOSFET with the gate-source capacitance c_{gs}, the source-body capacitance c_{sb}, the drain-body capacitance c_{db}, the drain-gate capacitance c_{dg}, and the gate-body capacitance c_{gb}added. These capacitors model charge storage in the device which affect its high-frequency performance. The first three capacitors are given by

$$c_{gs} = \frac{2}{3} W L C_{ox} \tag{41}$$

$$c_{sb} = \frac{c_{sb0}}{\left(1 + V_{SB}/\psi_0\right)^{1/2}} \tag{42}$$

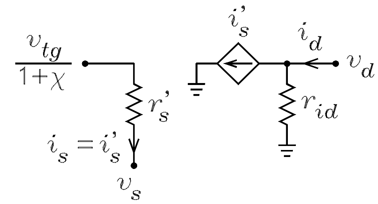

Figure 17: Source equivalent circuit with r_0 approximations.

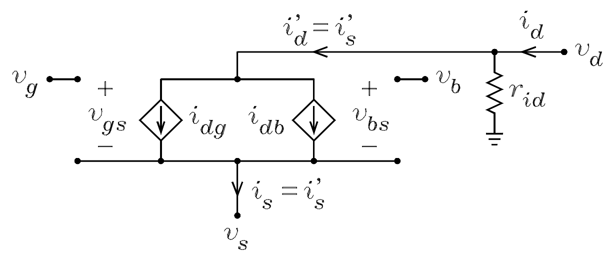

Figure 18: Hybrid π model with the r_0 approximations.

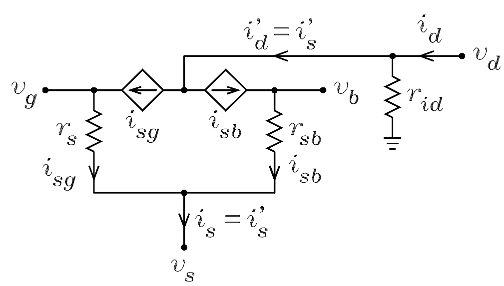

Figure 19: T model with the r_0 approximations.

$$c_{db} = \frac{c_{db0}}{\left(1 + V_{DB}/\psi_0\right)^{1/2}} \tag{43}$$

where V_{SB} and V_{DB} are dc bias voltages; c_{sb0} and c_{db0} are zero-bias values; and ψ_0 is the built-in potential. Capacitors c_{gd} and c_{gb} model parasitic capacitances. For IC devices, c_{gd} is typically in the range of 1 to 10 fF for small devices and c_{gb} is in the range of 0.04 to 0.15 fF per square micron of interconnect.

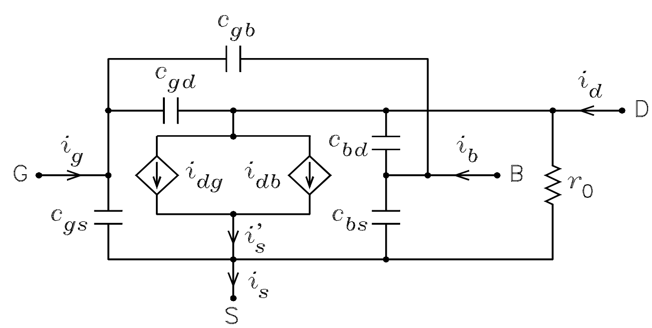

Figure 20: High-frequency hybrid-π model.

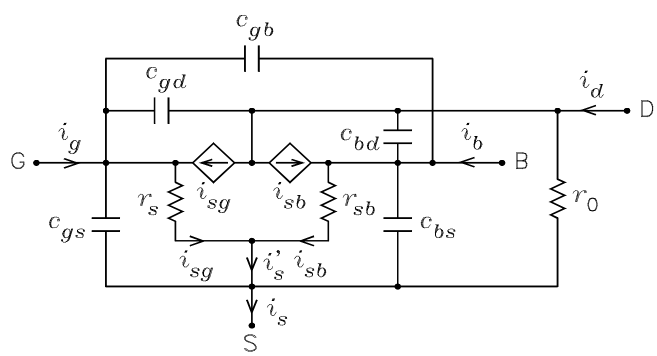

Figure 21: High-frequency T model.