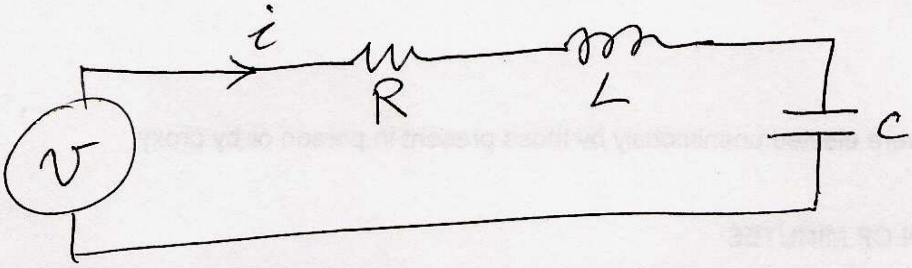

$$iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = V\cos(\omega t + \phi) \qquad (1)$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = -\frac{V\omega}{L}\sin(\omega t + \phi) \qquad (2)$$

For the overdamped case.

$$i_c = e^{\alpha t}\left(A e^{\beta t} + B e^{-\beta t}\right) \text{ where}$$

$$\alpha = -\frac{R}{2L},\ \beta = \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

For the particular solution there are 2 methods.

1. use $i_p = a\cos(\omega t + \phi) + b\sin(\omega t + \phi)$.

or

2. $i_p = K e^{j(\omega t + \phi)}$

This is easier and it works because $\sin(\omega t + \phi)$ is the "imaginary" part of $e^{j(\omega t + \phi)}$

Remember $e^{jx} = \cos x + j\sin x$.

So I'll use this method.

$$i_p = K e^{j(\omega t+\phi)} \quad \text{—— } 3$$

$$\frac{di_p}{dt} = jK\omega e^{j(\omega t+\phi)}$$

$$\frac{d^2 i_p}{dt^2} = -K\omega^2 e^{j(\omega t+\phi)}$$

Substitute in 2 .

$$-K\omega^2 e^{j(\omega t+\phi)} + \frac{R}{2} jK\omega e^{j(\omega t+\phi)} + \frac{K}{LC} e^{j(\omega t+\phi)}$$

$$= -\frac{V\omega}{L} e^{j(\omega t+\phi)}$$

$$-K\omega^2 + j\frac{R\omega K}{L} + \frac{K}{LC} = -\frac{V\omega}{L}$$

$$K\left(\frac{1}{LC} - \omega^2 + j\frac{R\omega}{L}\right) = -\frac{V\omega}{L}$$

$$K = -\frac{V\omega}{L} \frac{1}{\frac{1}{LC} - \omega^2 + j\omega\frac{R}{L}}$$

Substitute in 3 and replace $e^{j(\omega t+\phi)}$ with $\cos(\omega t+\phi) + j\sin(\omega t + \phi)$

$$i_p = -\frac{V\omega}{L}\left\{\frac{\cos(\omega t+\phi) + j\sin(\omega t+\phi)}{\frac{1}{LC} - \omega^2 + j\omega\frac{R}{L}}\right\}$$

Now find the imaginary part

$$= \operatorname{Im}\left(-\frac{V\omega}{L}\right)\frac{\left(\frac{1}{LC}-\omega^2-j\omega\frac{R}{L}\right)\left(\cos(\omega t+\phi)+j\sin(\omega t+\theta)\right)}{\left(\frac{1}{LC}-\omega^2\right)^2+\left(\omega\frac{R}{L}\right)^2}$$

$$= -\frac{V\omega}{L}\;\frac{-\omega\frac{R}{L}\cos(\omega t+\phi)+\left(\frac{1}{LC}-\omega^2\right)\sin(\omega t+\theta)}{\left(\frac{1}{LC}-\omega^2\right)^2+\left(\frac{\omega R}{L}\right)^2}$$

$$= -V\omega C\left\{\frac{(1-\omega^2LC)\sin(\omega t+\theta)-\omega RC\cos(\omega t+\phi)}{(1-\omega^2LC)^2+(\omega RC)^2}\right\}$$

$$= V\omega C\left\{\frac{\omega RC\cos(\omega t+\phi)-(1-\omega^2LC)\sin(\omega t+\phi)}{(1-\omega^2LC)^2+(\omega RC)^2}\right\}$$

Now use the identity

$$a\sin x+b\cos x = G\sin(x+\theta)$$

$$\text{where } G=\sqrt{a^2+b^2},\quad \theta=\arctan\frac{b}{a}$$

ie. $i = i_c + i_p$

then add i_c and set $i=0$ at $t=0$.

and from (1) $\frac{di}{dt}=\frac{V}{L}\cos(\phi)$ at $t=0$.

Thus find constants A and B.