

ELEC1011 Communications and Control

(3/11) Spectra of AM Signals

Trigonometric identities (CEP 2.3.4)

$$\cos(A) \cos(B) = \frac{1}{2} \cos(A - B) + \frac{1}{2} \cos(A + B) \quad (1)$$

So, multiplying a sinusoid having a frequency f_1 by another having a frequency f_2 moves it to the difference $|f_1 - f_2|$ and sum $|f_1 + f_2|$ of the frequencies.

$$\cos(2\pi f_1 t) \cos(2\pi f_2 t) = \frac{1}{2} \cos(2\pi |f_1 - f_2| t) + \frac{1}{2} \cos(2\pi |f_1 + f_2| t) \quad (2)$$

In the case where $f = f_1 = f_2$, we have

$$\begin{aligned} \cos^2(2\pi f t) &= \frac{1}{2} \cos(2\pi(0)t) + \frac{1}{2} \cos(2\pi(2f)t) \\ &= \frac{1}{2} + \frac{1}{2} \cos(4\pi f t) \end{aligned} \quad (3)$$

Fourier theory (CEP 2.4.1)

Fourier theory states that any arbitrary message signal can be thought of as a sum of sinusoids having different frequencies.

Each frequency is associated with a particular amplitude in the range 0 to ∞ .

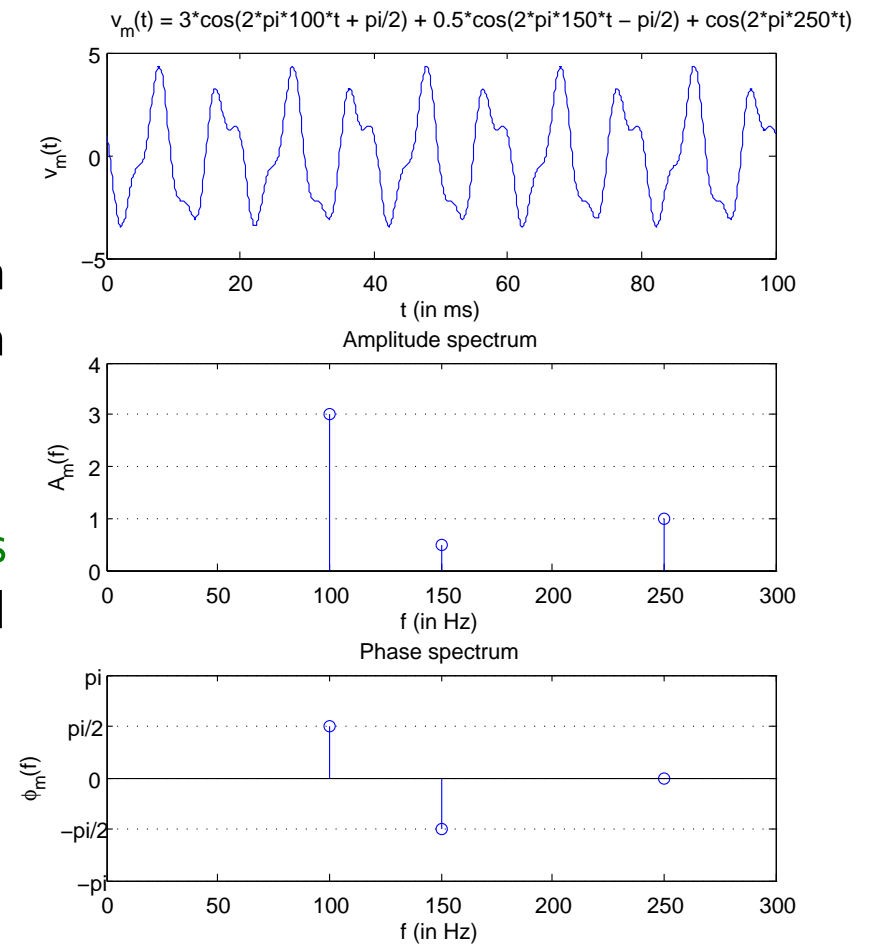
Also, each frequency is associated with a particular phase in the range $-\pi$ to π radians (or -180° to 180°).

$$v_m(t) = 3 \cos(2\pi 100t + \pi/2) + 0.5 \cos(2\pi 150t - \pi/2) + \cos(2\pi 250t)$$

- The frequency 100 Hz is associated with an amplitude of 3 and a phase of $\pi/2$ radians.
- The frequency 150 Hz is associated with an amplitude of 0.5 and a phase of $-\pi/2$ radians.
- The frequency 250 Hz is associated with an amplitude of 1 and a phase of 0 radians.
- All other frequencies are associated with an amplitude of 0.

Amplitude and phase spectra

- An **amplitude spectrum** shows which **frequencies** are associated with which **amplitudes**.
- A **phase spectrum** shows which **frequencies** (having non-zero amplitudes) are associated with which **phases**.



Amplitude spectrum of an arbitrary signal

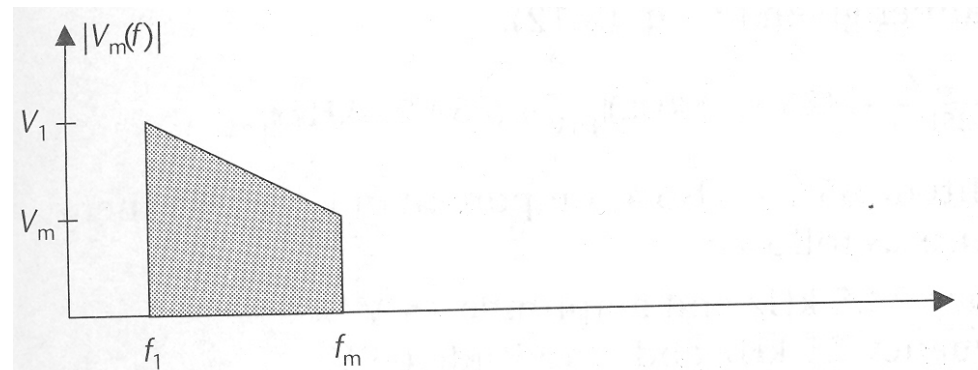


Figure 3.8 Symbolic representation of the amplitude spectrum of a message signal.

- In this signal, the **frequencies** in the range $f_1 - f_m$ are associated with non-zero **amplitudes**.
- It is a **baseband** signal because it contains **frequencies** that approach zero.
- The **bandwidth** of a baseband signal is equal to its maximum frequency f_{max} . In this case, $f_{max} = f_m$.

Amplitude modulation of a sinusoidal signal (CEP 3.3.1)

$$v_{am}(t) = (V_c + k_{am} \cdot v_m(t)) \cos(2\pi f_c t)$$

- Suppose that we use a modulation sensitivity of $k_{am} = 1$.
- Also, we use a **sinusoidal** message signal $v_m(t) = V_m \cos(2\pi f_m t)$, where $f_m \ll f_c$.
- In this case, the modulation factor is $m = \frac{V_m}{V_c}$, when $V_c \geq V_m$.

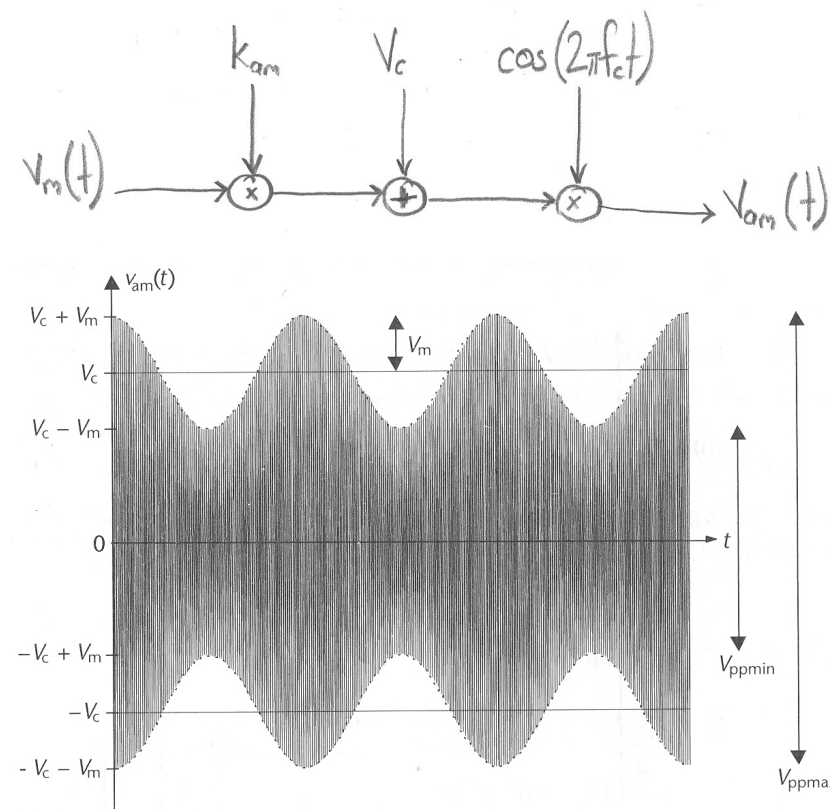


Figure 3.5 AM waveform for a sinusoidal message signal of frequency f_m and amplitude V_m . The plot is for carrier frequency $f_c = 100 f_m$, and carrier amplitude $V_c = 3V_m$.

Mathematics

In this case, the AM signal is given by

$$\begin{aligned}v_{am}(t) &= (V_c + V_m \cos(2\pi f_m t)) \cos(2\pi f_c t) \\&= V_c \cos(2\pi f_c t) + V_m \cos(2\pi f_m t) \cos(2\pi f_c t)\end{aligned}$$

Using the trigonometric identity (2) gives

$$\begin{aligned}v_{am}(t) &= V_c \cos(2\pi f_c t) \\&\quad + \frac{V_m}{2} \cos[2\pi(f_c - f_m)t] \\&\quad + \frac{V_m}{2} \cos[2\pi(f_c + f_m)t]\end{aligned}$$

Frequency components

The AM signal contains three frequency components:

- the **carrier frequency** f_c with amplitude V_c ;
- the **Lower Side Frequency** (LSF) $f_c - f_m$ with amplitude $V_m/2$;
- the **Upper Side Frequency** (USF) $f_c + f_m$ with amplitude $V_m/2$.

There is no component at the message frequency f_m .

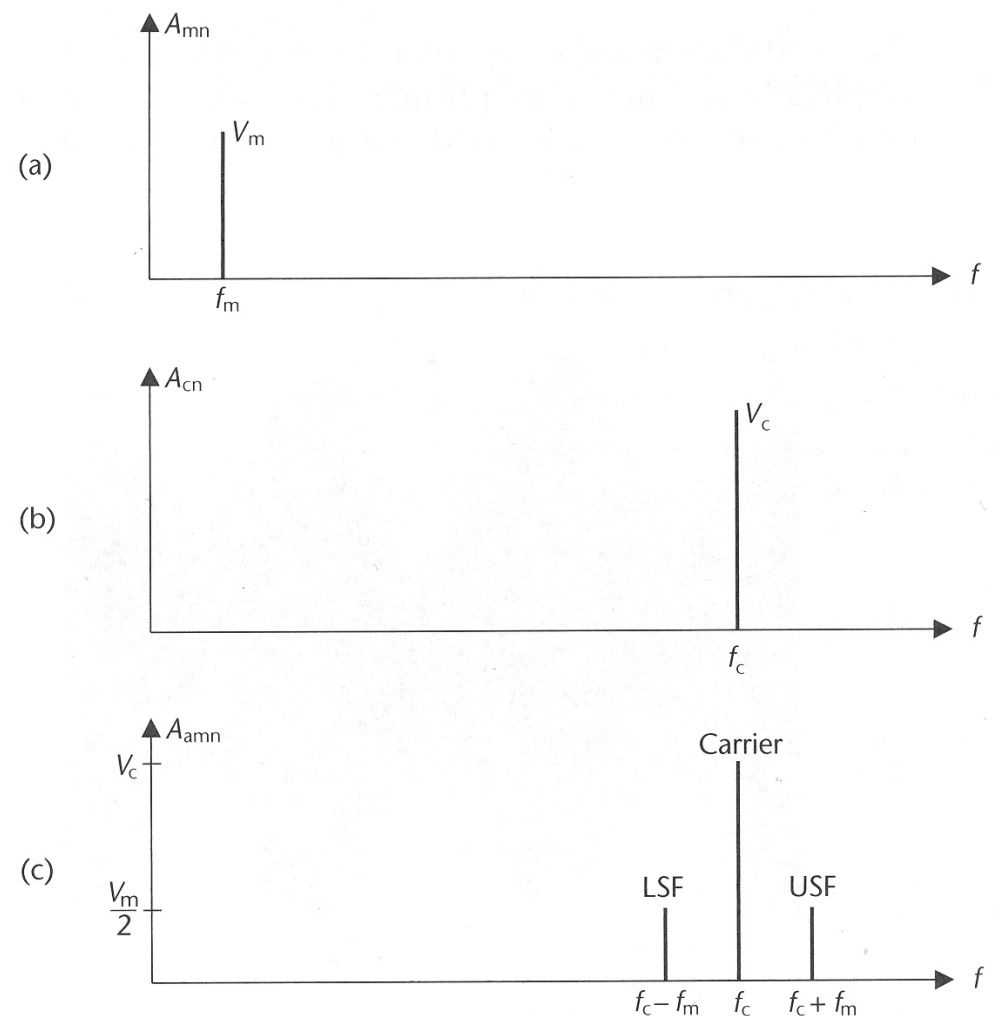


Figure 3.6 Single-sided amplitude spectrum of (a) sinusoidal modulating signal; (b) carrier signal; and (c) AM signal.

Power (CEP 2.5)

When a time-varying voltage $v(t)$ is applied to a resistance R , the **instantaneous power** dissipated is given by

$$p(t) = \frac{|v(t)|^2}{R}$$

The **normalised instantaneous power** is the amount that would be dissipated if the voltage was applied to a $R = 1 \Omega$ load.

$$p(t) = |v(t)|^2$$

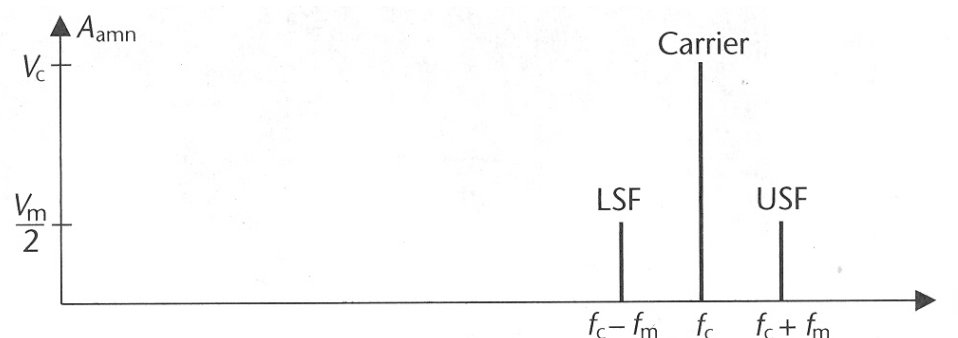
The **normalised average power** is the average of the normalised instantaneous power over one period T of the signal $v(t)$.

$$P = \frac{1}{T} \int_{-T/2}^{T/2} |v(t)|^2 dt$$

The normalised average power of a **sinusoidal** signal $v(t) = V \cos(2\pi ft)$ is given by

$$P = V^2/2$$

Power in our AM signal (CEP 3.3.3)



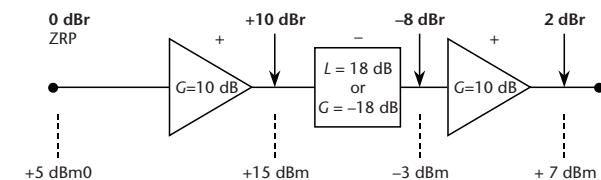
- Power in carrier $P_c = \frac{V_c^2}{2}$
- Power in side frequencies $P_{LSF} = P_{USF} = \frac{(V_m/2)^2}{2} = \frac{V_m^2}{8} = \frac{(mV_c)^2}{8} = \frac{m^2 P_c}{4}$
- Total power in side frequencies $P_{SF} = P_{LSF} + P_{USF} = \frac{m^2 P_c}{2}$
- Total power in AM signal $P = P_c + P_{SF} = P_c + \frac{m^2 P_c}{2} = P_c \left(1 + \frac{m^2}{2}\right)$
- Fraction of power in side frequencies $P_{SF}/P = \frac{m^2 P_c/2}{P_c(1+m^2/2)} = \frac{m^2}{2+m^2}$
- As the modulation index goes from 0 to 1, the fraction of power in the side frequencies goes from 0 to 1/3.
- At least 2/3 of the transmit power is in the carrier.

Decibels (CEP 2.6)

- A **power gain** of an amplifier is the ratio of the output power to the input power P_o/P_i .
- This power gain can be expressed in **decibels** (dB) as $G = 10 \log_{10}(P_o/P_i)$ dB.
- An **amplitude gain** V_o/V_i is related to the corresponding power gain according to $P_o/P_i = V_o^2/V_i^2$.
- Therefore, an amplitude gain can be expressed in dB as $G = 10 \log_{10}(V_o^2/V_i^2) = 20 \log_{10}(V_o/V_i)$ dB.

Figure 2.37

- A power P can be expressed in dBW by comparing it with 1 Watt $10 \log_{10}(P/1)$ dBW.
- A power P can be expressed in dBm by comparing it with 1 mW $10 \log_{10}(P/10^{-3})$ dBm.
- Gains expressed in these logarithmic units are **additive**, rather than multiplicative.



Taken from *Communication Engineering Principles*, © Iflok Otung, published 2001 by Palgrave

Frequency components

Consider the case where the message signal contains two sinusoids

$$v_m = V_m \cos(2\pi f_m t) + V_1 \cos(2\pi f_1 t)$$

In this case we obtain

$$\begin{aligned} v_{am}(t) &= V_c \cos(2\pi f_c t) \\ &+ \frac{V_m}{2} \cos[2\pi(f_c - f_m)t] \\ &+ \frac{V_m}{2} \cos[2\pi(f_c + f_m)t] \\ &+ \frac{V_1}{2} \cos[2\pi(f_c - f_1)t] \\ &+ \frac{V_1}{2} \cos[2\pi(f_c + f_1)t] \end{aligned}$$

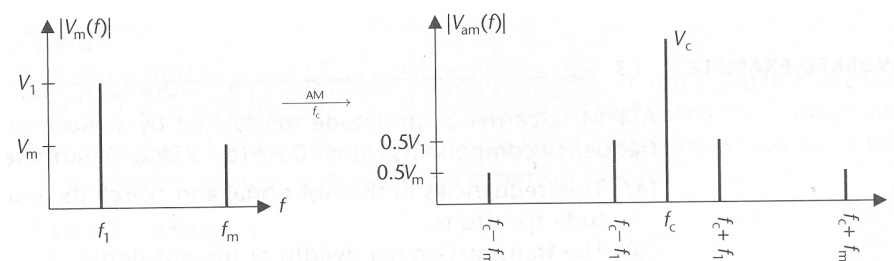


Figure 3.9 Translation of the maximum and minimum frequency components of a message signal in AM.

Amplitude modulation of an arbitrary signal (CEP 3.3.2)

The message signal is transformed into an **lower sideband** and a **upper sideband** having half the amplitude.

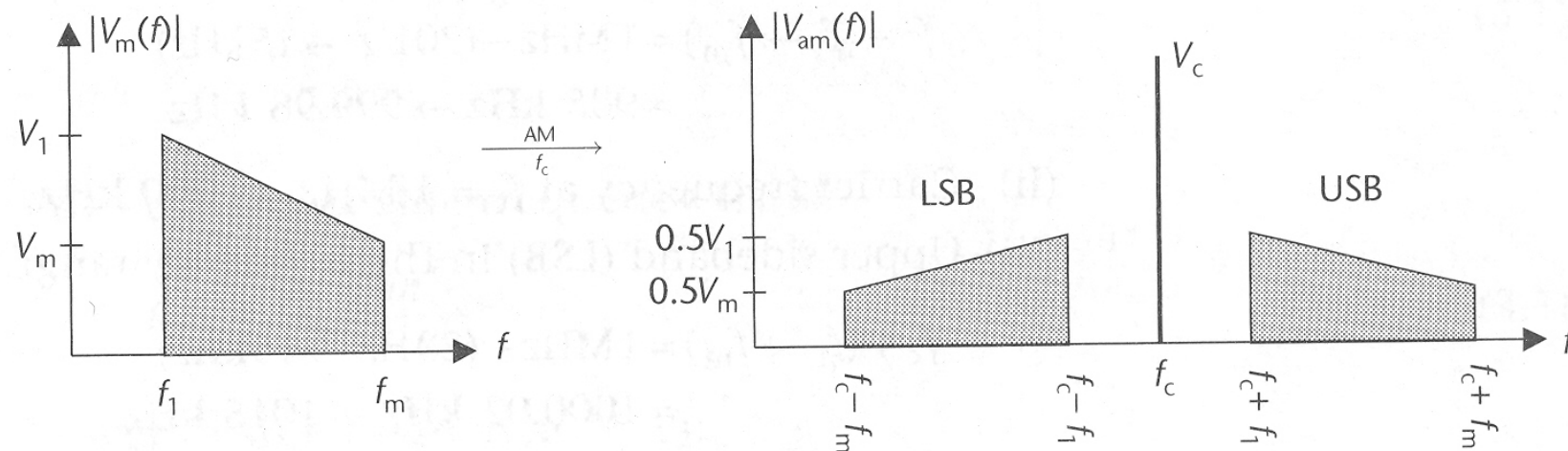
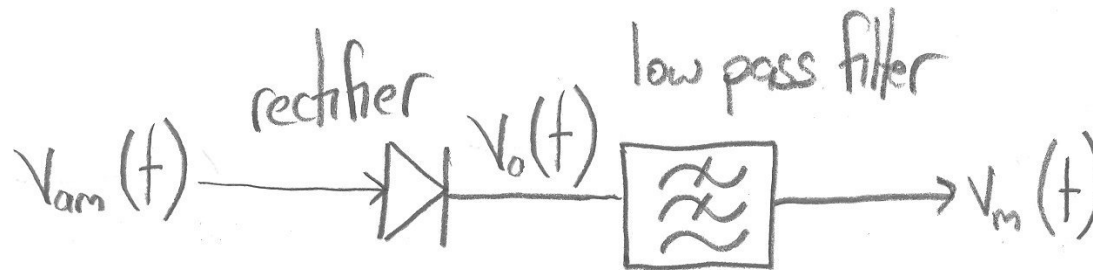


Figure 3.10 Production of lower and upper sidebands in AM.

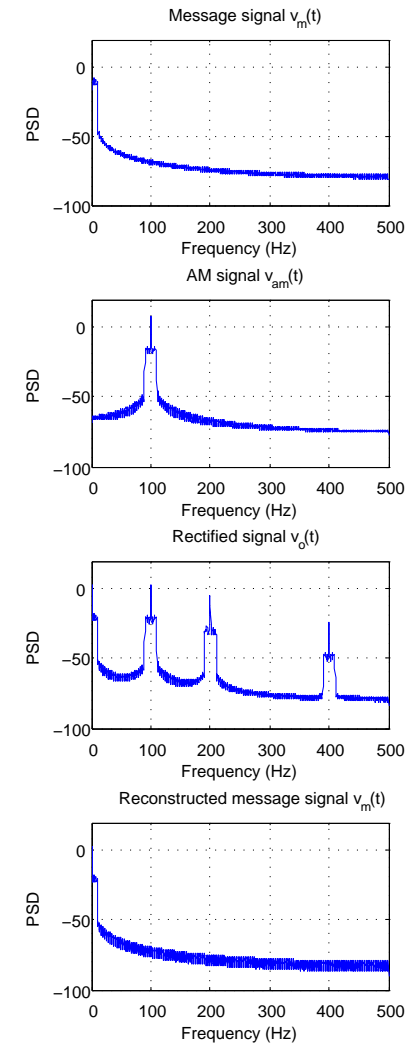
The AM signal occupies frequencies in the range $(f_c - f_m) - (f_c + f_m)$ and has a **bandwidth** of $2f_m$.

The **bandwidth** of an AM signal is double that of the baseband message signal.

Spectrum of demodulation (CEP 3.5.1)



- If the AM signal is not overmodulated, **envelope detection** can be used.
- This is **non-coherent** because it does not require exact knowledge of the carrier frequency or phase.
- The **rectifier** replicates the signal to the baseband (and other higher frequencies).
- The **Low Pass Filter** (LPF) removes the high frequency replicas of the signal.
- The LPF cut-off frequency should be between the maximum frequency in the signal f_{max} and the carrier frequency f_c . The lower it is, the more channel-induced noise that can be removed.



Non-coherent vs coherent detection (CEP 3.6)

- The sidebands contain all of the message information. The carrier contains none.
- However, in order to avoid overmodulation and allow **non-coherent** envelope detection, the carrier must use at least $2/3$ of the transmit power.
- If we want a more power-efficient AM scheme, we need to overmodulate the carrier and use **coherent** detection.
- However, this requires a more complicated receiver, which can obtain exact knowledge of the carrier **frequency** and **phase**.

Coherent detection (CEP 3.5.2)

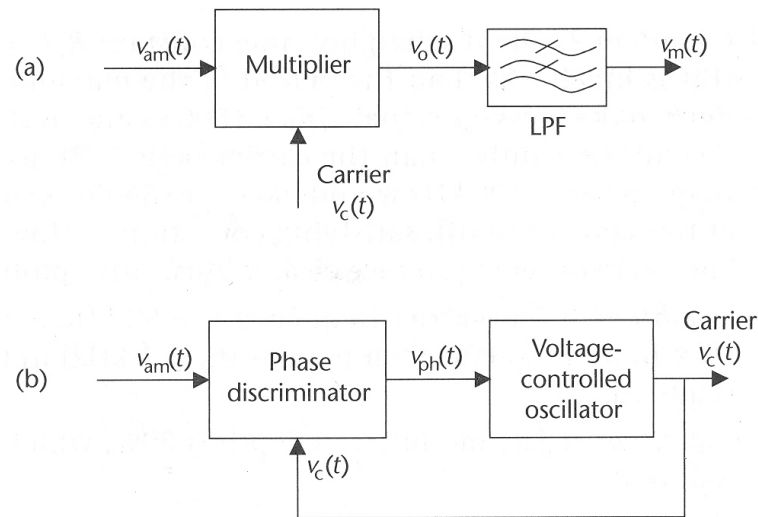


Figure 3.20 (a) Coherent AM demodulator; (b) phase-locked loop (PLL).

- The **VCO** produces a sinusoid $v_c(t)$ having the frequency f_c when its input $v_{ph}(t)$ is zero.
- This frequency is increased or decreased when the input is positive or negative, respectively.
- The output of the **phase discriminator** $v_{ph}(t)$ will be zero when $v_c(t)$ and $v_{am}(t)$ are in phase, otherwise it will be positive or negative as appropriate.
- The negative feedback loop keeps $v_c(t)$ and $v_{am}(t)$ in phase.

Coherent detection (CEP 3.5.2)

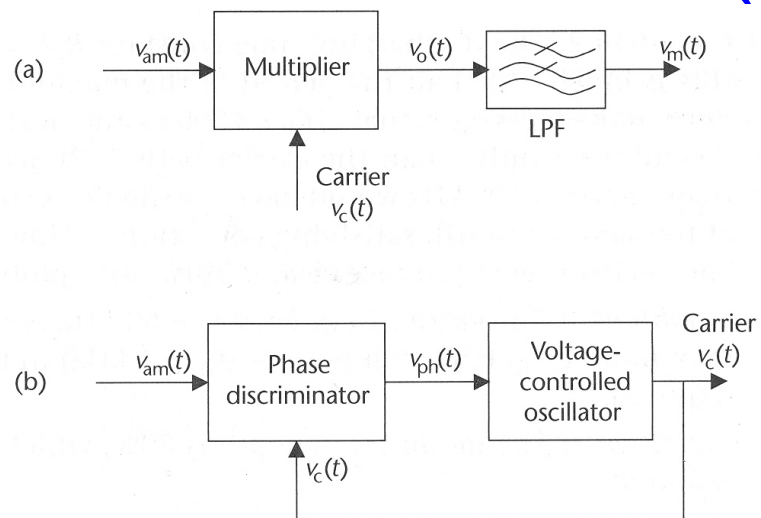
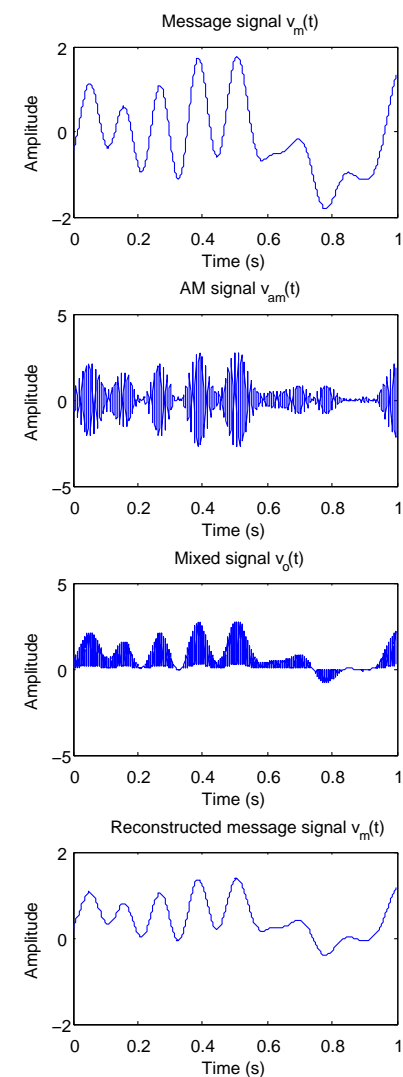


Figure 3.20 (a) Coherent AM demodulator; (b) phase-locked loop (PLL).

$$\begin{aligned}
 v_{am}(t) &= (V_c + k_{am} \cdot v_m(t)) \cos(2\pi f_c t) \\
 v_c(t) &= \cos(2\pi f_c t) \\
 v_o(t) &= v_{am}(t) \cdot v_c(t) \\
 &= (V_c + k_{am} \cdot v_m(t)) \cos^2(2\pi f_c t)
 \end{aligned}$$



Coherent detection (CEP 3.5.2)

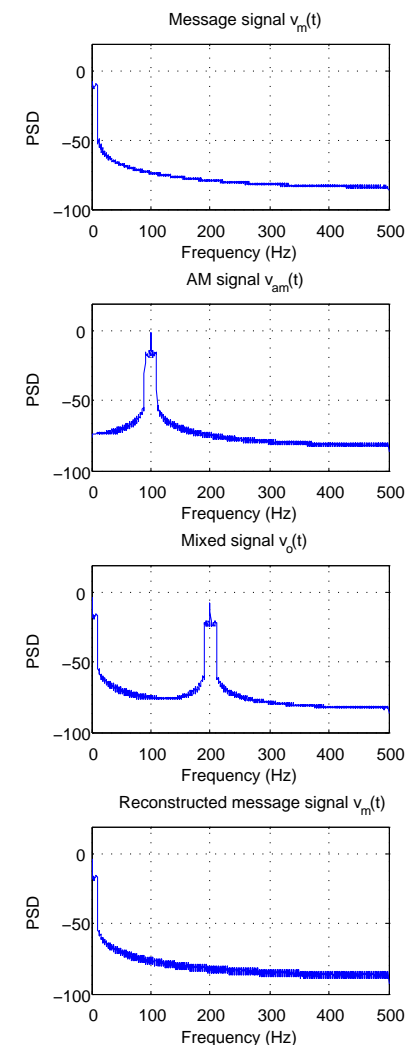
$$v_o(t) = (V_c + k_{am} \cdot v_m(t)) \cos^2(2\pi f_c t)$$

Using the trigonometric identity (3) we get

$$\begin{aligned} v_o(t) &= (V_c + k_{am} \cdot v_m(t)) \left(\frac{1}{2} + \frac{1}{2} \cos(4\pi f_c t) \right) \\ &= \frac{1}{2}(V_c + k_{am} \cdot v_m(t)) + \frac{1}{2}(V_c + k_{am} \cdot v_m(t)) \cos(4\pi f_c t) \end{aligned}$$

The LPF removes the high frequency component, leaving

$$v'_m(t) = \frac{1}{2}(V_c + k_{am} \cdot v_m(t))$$



Tuned receiver (CEP 3.5.3 and 3.5.3.1)

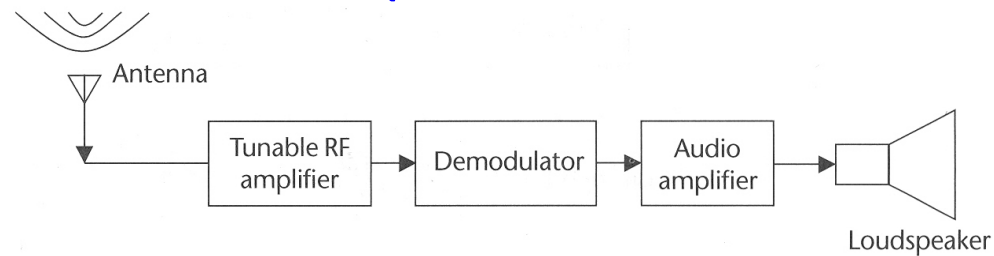


Figure 3.21 Tunable radio frequency (TRF) receiver.

- The **tunable RF amplifier** rejects all signals except for the desired one.
- However, a very complex design is required to achieve the **high-selectivity** necessary to reject the unwanted adjacent signals owing to their **high frequencies**.

Superheterodyne receiver (CEP 3.5.3.2)

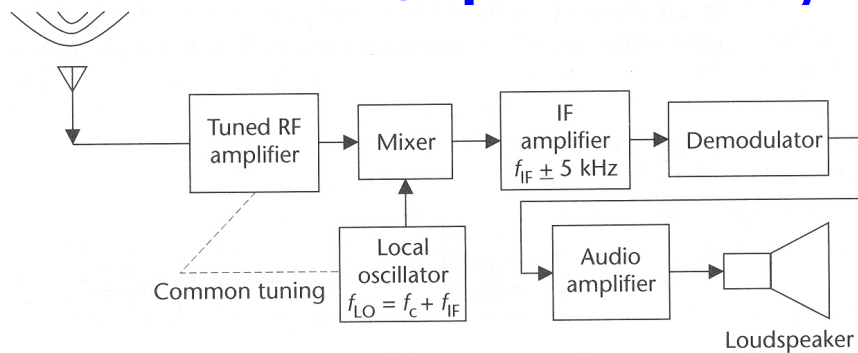
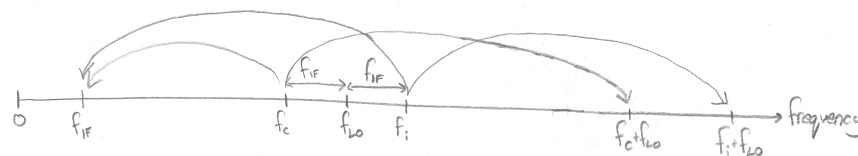


Figure 3.22 The superheterodyne AM receiver.



- The **mixer** multiplies the AM signal by a sinusoid having a frequency $f_{LO} = f_c + f_{IF}$, where $f_{IF} \ll f_c$.
- According to trigonometric identity (2), the **mixer** moves the AM signal from being centred at f_c to the difference $f_{LO} - f_c = f_{IF}$ and sum $f_{LO} + f_c = 2f_c + f_{IF}$ of the frequencies f_{LO} and f_c .

- The **high-selectivity IF amplifier** removes the unwanted signals at **low frequencies** adjacent to f_{IF} , as well as the high frequency replica centred at the sum frequency $2f_c + f_{IF}$.
- However, the **mixer** will also move an unwanted image signal centred at $f_i = f_{LO} + f_{IF} = f_c + 2f_{IF}$ to f_{IF} . The **low-selectivity tuned RF amplifier** is therefore required to remove the **high-frequency** image signal before mixing takes place.
- A low complexity design results because **high-selectivity** is only used at **low frequencies**. At **high frequencies**, **low-selectivity** is used.

Exercise

1. Sketch the amplitude spectrum of the message signal $v_m(t) = 3 \cos(20\pi t + \pi/4) + 2 \sin(60\pi t) - \cos(100\pi t)$.
2. Sketch the amplitude spectrum of the signal $v_{am}(t)$ that results when $v_m(t)$ is AM modulated onto a 1 kHz carrier using a DC offset of $V_c = 1$ and a modulation sensitivity of $k_{am} = 0.5$.
3. Calculate the fraction of the transmit power that is in the sidebands.
4. What kind of demodulation is required to recover the message signal $v_m(t)$ and why? Sketch the amplitude spectrum of the signal $v_o(t)$ obtained after the first step of the demodulator.
5. Sketch the phase spectra of the signals $v_m(t)$, $v_{am}(t)$ and $v_o(t)$.