UNIVERSITY OF SOUTHAMPTON ELEC1011W1

SEMESTER 2 EXAMINATION 2009/2010

COMMUNICATIONS AND CONTROL

Duration: 120 mins

Answer THREE *questions.*
Not more than TWO questions are to be answered from any section.
Use a SEPARATE *answer book for each section.*

An approximate marking scheme is indicated.
This exam contributes 75% and coursework contributes 25% of your total mark for this module.

University approved calculators MAY be used.

Section A

Question 1

(a) Derive the transfer function, T(p), for the system shown in Figure 1.
(7 marks)

(b) From the characteristic equation or otherwise show that the system has the following characteristics:

(i) A ***damped*** frequency of oscillation, $\omega_{\mathrm{D}} = 12\ \mathrm{rad\,s^{-1}}$

(ii) Poles at $-6 \pm 12\mathrm{j}$

(4 marks)

(c) Determine the system output, c(t), if

$$\mathrm{r(t)} = 0, \quad \mathrm{t} < 0$$
$$\mathrm{r(t)} = 5, \quad \mathrm{t} \geq 0$$

(10 marks)

(d) By considering $\mathrm{c}(\infty)$ and using T(p), calculate the steady state output of the system for a unit step input.
(4 marks)

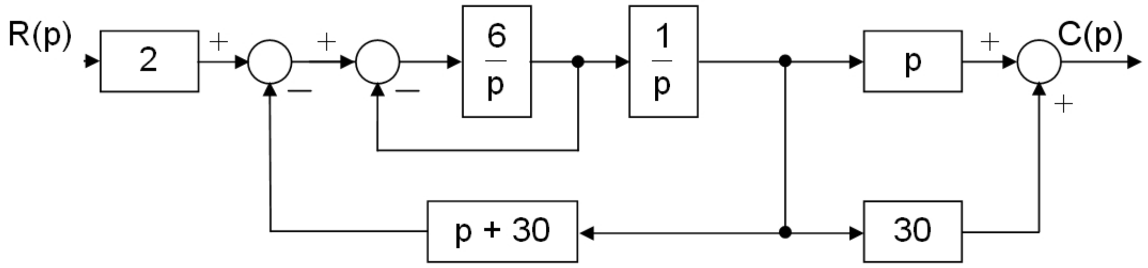

FIGURE 1

Question 2

(a) The switch, s, in the circuit shown in Figure 2 has been open for a long time. Determine the current that will flow through R_1 after the switch is closed. *(16 marks)*

(b) Calculate

(i) the elapsed time between the switch being closed and the current flowing through R_1 reaching its maximum value

(ii) the maximum current that flows through R_1

(iii) the current flowing 350 ms and 800 ms after the switch has closed.

Sketch the current waveform. *(9 marks)*

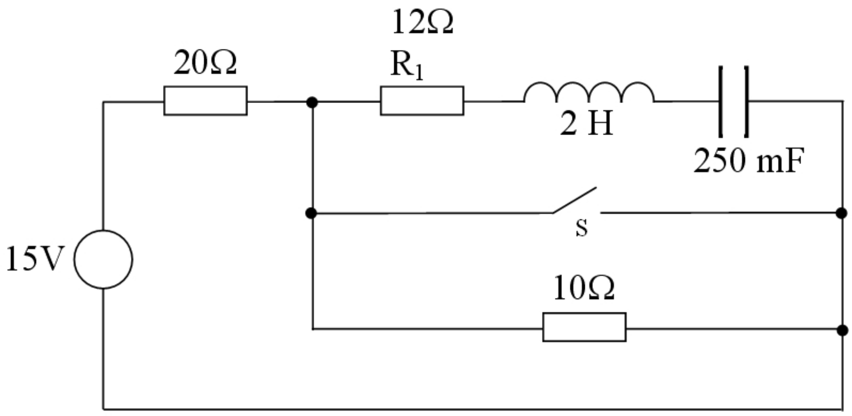

FIGURE 2

Question 3

A model of a dc motor coupled to a load via a gearbox is shown in Figure 3. Given that the back emf constant and torque constant are both equal to K

(a) From first principles, show that the relationship between the gear ratio, transmitted torque and angular velocity can be described as

$$\left|\frac{\mathrm{T_G}}{\mathrm{T_L}}\right| = \left|\frac{\dot{\theta}_\mathrm{L}}{\dot{\theta}_\mathrm{M}}\right| = \frac{\mathrm{N_1}}{\mathrm{N_2}}$$

Where $\mathrm{T_G}$ is the input torque of the gearbox. *(5 marks)*

(b) Hence determine an equation for the total torque generated, $\mathrm{T_M}$, in terms of $\dot{\theta}_\mathrm{M}$, $\mathrm{I_L}$, $\mathrm{I_M}$, $\mathrm{B_L}$, $\mathrm{B_M}$, $\mathrm{N_1}$ and $\mathrm{N_2}$ only. *(5 marks)*

(c) Using block diagram notation or otherwise determine the transfer function that relates the ***position*** of the load to the applied armature voltage $\mathrm{V_a}$.

(10 marks)

(d) Show that the motor and load have an undamped natural frequency of oscillation equal to

$$\sqrt{\frac{\mathrm{R_a}\left(\mathrm{B_M} + \frac{\mathrm{N_1^2}}{\mathrm{N_2^2}}\mathrm{B_L}\right) + \mathrm{K}^2}{\mathrm{L_a}\left(\mathrm{I_M} + \frac{\mathrm{N_1^2}}{\mathrm{N_2^2}}\mathrm{I_L}\right)}}$$

(5 marks)

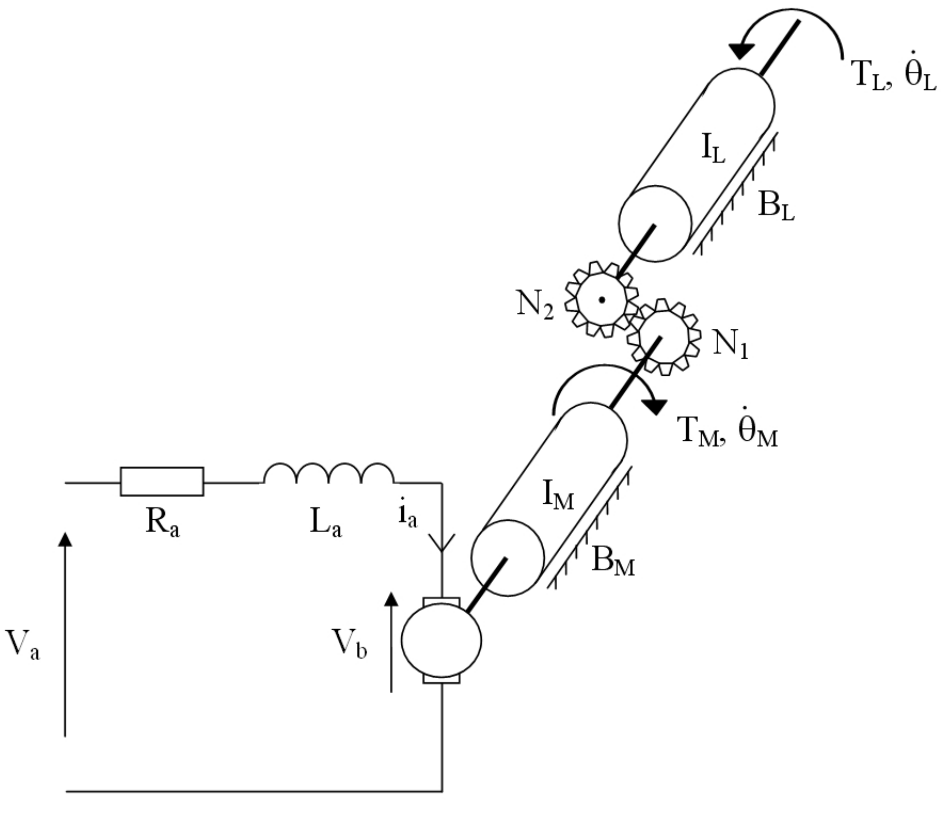

Figure 3

Section B

Question 4

Figure 4 shows a segment of the signal $x(t)$ that is continuously generated by a particular stationary information source. A source coding scheme has been devised to digitise the output of the source efficiently, producing a desirable quality at a minimal bit rate. This process begins by sampling the signal $x(t)$ at the Nyquist rate of $f_s = 10$ Hz. The resultant samples are quantised using $N = 8$ Lloyd-Max quantisation levels $\{\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_N\}$ shown in Figure 5. Some quantisation levels get selected to represent a greater portion of the samples than others, as specified by the occurrence probabilities $\{p_1, p_2, \ldots, p_N\}$ shown in Figure 5. Therefore, the quantisation levels are represented using the Huffman codewords $\{\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_N\}$ shown in Figure 5.

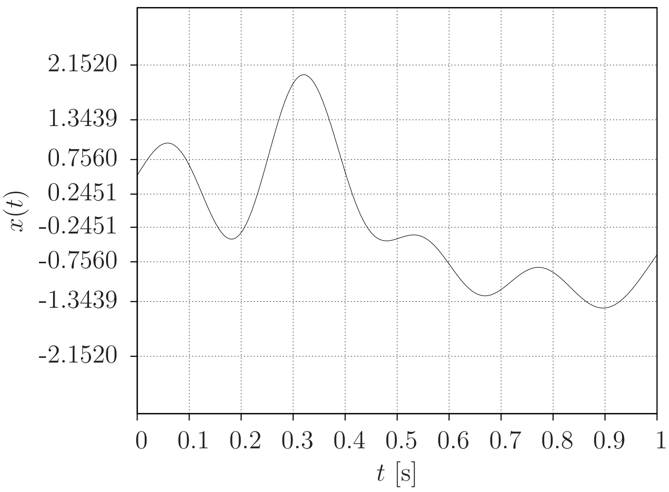

FIGURE 4: A one second segment taken from the signal $x(t)$ that is continuously generated by a particular stationary information source.

Index i	Quantisation level $\bar{x}_i$	Probability of occurrence p_i	Huffman codeword $\mathbf{c}_i$
1	2.1520	0.0407	01110
2	1.3439	0.1069	111
3	0.7560	0.1615	010
4	0.2451	0.1909	00
5	-0.2451	0.1909	10
6	-0.7560	0.1615	110
7	-1.3439	0.1069	0110
8	-2.1520	0.0407	01111

FIGURE 5: The quantisation levels used in the source coding scheme, together with their probabilities of occurrence and allocated binary codewords.

(a) Briefly describe the advantages and disadvantages of digitising an analogue signal before modulating it onto a channel.

(8 marks)

(b) Explain why the parameters of the sampling, quantisation and Huffman coding elements of the source coding scheme cannot be redesigned to digitise the signal $x(t)$ any more efficiently.

(6 marks)

(c) State the sequence of quantisation levels that results from the sampling and quantisation of the one second segment of $x(t)$ shown in Figure 4. Determine the binary sequence that results from the Huffman coding of these quantisation levels. Also, determine the sequence of quantisation levels that are represented by the binary sequence `11001101011000010`. What is the duration of the signal segment that is represented by this sequence of quantisation levels?

(4 marks)

(d) Determine the entropy $H = \sum_{i=1}^{N} p_i \log_2(1/p_i)$ of the quantisation levels and the corresponding Huffman coding efficiency R. Determine the average rate R_b at which the source encoder generates bits and the lowest Quarternary ($M = 4$) Phase Shift Keying (QPSK) symbol rate R_s that would allow the transmission of the signal $x(t)$ to keep pace with its generation, on average. Compare the associated QPSK signal bandwidth to the bandwidth that would be required if analogue Single SideBand Suppressed Carrier (SSBSC) modulation was used to convey the signal $x(t)$.

(7 marks)

Question 5

The generator matrix $\mathbf{G}$ and the parity check matrix $\mathbf{H}$ of the $R_c = 4/7$ Hamming channel code are given by

$$\mathbf{G} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{H} = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

(a) Sketch a block diagram showing how a channel encoder and decoder interact with the other components of a transmitter and receiver. Briefly explain the role of each component.

(7 marks)

(b) Shannon's channel capacity law is given as $C = B\log_2(1 + S/N)$. In a coded Binary Frequency Shift Keying (BFSK) transmitter, the bandwidth required is given by $B = 3R_s/2$, where $R_s = R_b/R_c$ is the channel symbol rate, R_b is the information bit rate and R_c is the channel coding rate. For the case where the $R_c = 4/7$ Hamming code is employed, determine the particular Signal to Noise Ratio (SNR) S/N at which $R_b = C$ and express this in decibels. Explain the relevance of this particular SNR. Determine the corresponding SNR S/N for the case where the Hamming code is omitted, giving $R_c = 1$. Hence, explain the advantage of channel coding. What is the corresponding disadvantage of channel coding?

(8 marks)

(c) For the case of the information bits $\mathbf{x}_1 = [\ 1\ \ 0\ \ 1\ \ 1\ ]^T$, determine the corresponding encoded bits $\mathbf{y}_1 = \mathbf{G}\mathbf{x}_1$. In the case where the encoded bits $\mathbf{y}_2 = [\ 1\ \ 0\ \ 0\ \ 1\ \ 1\ \ 0\ \ 0\ ]^T$ are transmitted, but the bits $\hat{\mathbf{y}}_2 = [\ 1\ \ 1\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ ]^T$ are received, show that the presence of errors can be detected using the syndrome $\mathbf{s}_2 = \mathbf{H}\hat{\mathbf{y}}_2$. Similarly, in the case where the encoded bits $\mathbf{y}_3 = [\,0\ \ 1\ \ 0\ \ 0\ \ 1\ \ 0\ \ 1\,]^T$ are transmitted, but the bits $\hat{\mathbf{y}}_3 = [\,1\ \ 1\ \ 0\ \ 1\ \ 0\ \ 0\ \ 1\,]^T$ are received, show that the presence of bit errors *cannot* be detected. Correct the single bit error in the received bits $\hat{\mathbf{y}}_4 = [\,1\ \ 0\ \ 0\ \ 1\ \ 0\ \ 1\ \ 0\,]^T$ and determine the corresponding information bits $\mathbf{x}_4$. Finally, use these results to infer the minimum Hamming distance d_{min} of the $R_{\mathrm{c}} = 4/7$ Hamming code. *(10 marks)*

Question 6

Figure 6a provides the schematic of a Double SideBand Suppressed Carrier (DSBSC) modulator and demodulator, where the latter employs the Low Pass Filter (LPF) of Figure 6b. Throughout this question, the message signal $x(t) = \cos(2000\pi t)$ and a carrier frequency of $f_c = 10$ kHz are employed in the schematic of Figure 6a. Similarly, the resistance $R = 1$ kΩ and capacitance $C = 31.83$ nF are employed in the LPF of Figure 6b. Finally, the trigonometric identity $\cos(a)\cos(b) = \frac{1}{2}\cos(a-b) + \frac{1}{2}\cos(a+b)$ is useful throughout this question.

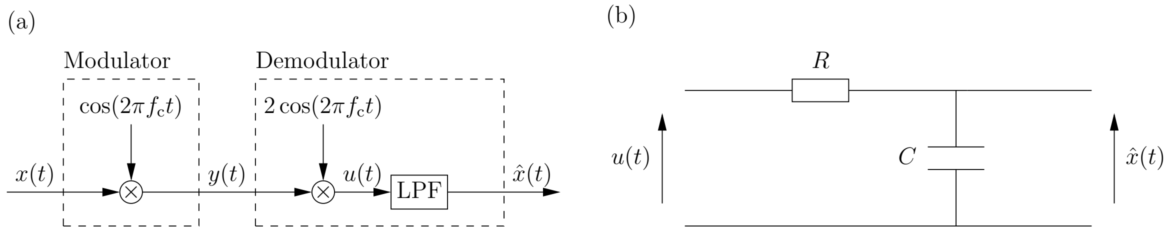

FIGURE 6: (a) Schematic of a DSBSC modulator and demodulator. (b) Schematic of the LPF employed in the DSBSC demodulator.

(a) Sketch and accurately annotate the amplitude spectrum of the DSBSC modulated signal $y(t)$ shown in Figure 6a. Also sketch and accurately annotate the equivalent amplitude spectra that would be obtained using undermodulated Amplitude Modulation (AM), as well as Single SideBand Suppressed Carrier (SSBSC) modulation. With reference to your sketches where appropriate, describe the advantages and disadvantages of DSBSC modulation relative to these alternative modulation techniques.

(10 marks)

(b) Derive an expression of the form $H(j\omega) = X + jY$ for the transfer function of the LPF shown in Figure 6b. Show that this corresponds to an amplitude response of $|H(j\omega)| = \frac{1}{\sqrt{1+\omega^2R^2C^2}}$ and a phase response of $\angle H(j\omega) = \tan^{-1}(-\omega RC)$. Finally, derive and solve an expression for the 3 dB bandwidth f_0 of the LPF shown in Figure 6b.

(10 marks)

(c) For the reconstructed signal $\hat{x}(t)$ shown in Figure 6a, derive an expression of the form

$$\hat{x}(t) = A_1 \cos(2\pi f_1 t + \phi_1) + A_2 \cos(2\pi f_2 t + \phi_2) + A_3 \cos(2\pi f_3 t + \phi_3).$$

(5 marks)

END OF PAPER