# VIII. MOTION OF ELECTRONS IN ELECTRIC AND MAGNETIC FIELDS

*f06.01*

## INTRODUCTION

From previous experiments, you are familiar with the concept of electric and magnetic fields. The most important formulae related to this topic describe forces electric and magnetic fields exert on electric charge. In fact, these formulae are used to define fields themselves.

Electrons carry one unit of negative elementary charge. Since an electric or magnetic field exerts a force on electrons, it affects their motion. In spite of the fact that electric fields are everywhere around us, in every day life we do not directly observe the motion of electrons. However, such observations are possible with a device called a cathode-ray tube (CRT), which is just a technical name for a tube present in most of TV sets or computer monitors.

## PURPOSE

Verification of formulae for forces exerted by electric and magnetic fields on electric charge

## PRE-LAB ASSIGNMENTS

### A. Readings:

In an electric field $\vec{E}$, an electron experiences a force

$$\vec{F} = e\,\vec{E} \tag{1}$$

where $e$ is the electric charge of the electron. In a magnetic field $\vec{B}$, the electron is subject to a force

$$\vec{F} = e\,\vec{v} \times \vec{B} \tag{2}$$

where $\vec{v}$ is the velocity of the electron. A force causes acceleration according to the relation

$$\vec{a} = \frac{\vec{F}}{m} \tag{3}$$

where $m$ is the mass of the electron. Note from (1) and (2) that the electric field accelerates an electron in the direction parallel to the electric field, while the magnetic field accelerates an electron in the direction perpendicular to both the magnetic field and the velocity of the electron.

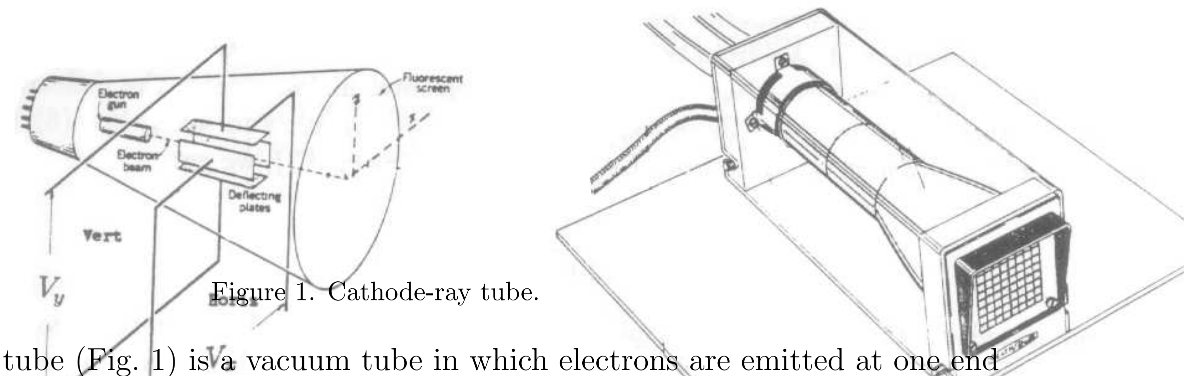


Figure 1. Cathode-ray tube.

A cathode-ray tube (Fig. 1) is a vacuum tube in which electrons are emitted at one end from a hot piece of metal (the cathode) and then accelerated by a strong electric field created in the electron gun. They travel across the tube, where they are also subject to deflecting electric fields created in between two pairs of parallel plates. Electron eventually hit the opposite end of the tube. This end is covered with a fluorescent material, and serves as a screen. Electrons that strike the screen cause the fluorescent material to glow, and thus the beam of electrons is visible as a spot on the screen.The actual position of the spot depends on the deflecting field inside the tube. Since electrons in the tube move at high speeds, their motion may be also affected by magnetic fields. By observing the position of the bright spot on the screen, we can measure the deflection of electrons by electric and magnetic fields.

Let us analyze the motion of electrons in the tube. The electrons are accelerated to velocity $v_z$ in the electron gun. If the accelerating electric potential is $V_a$ we can estimate $v_z$ from the conservation of energy, since the increase of the kinetic energy of the electron equals the change of potential energy in the electrostatic field, $e\,V_a$. Neglecting the initial speed of the electron, we have;

$$\frac{1}{2}mv_z^2 = eV_a \tag{4}$$

*ELECTRIC DEFLECTION*

The electrons move with the constant velocity $v_z$ until they reach the deflecting plates. Let us assume that the deflecting electric field is vertical. If the potential difference between the plates is $V_y$, the electric field in between the plates is approximately uniform and has a magnitude of:

$$E_y = \frac{V_y}{d} \tag{5}$$

where $d$ is the plate separation (see Fig. 2). This electric field will accelerate electrons in vertical directions with $a_y$ which can be obtained from formulae (1), (3) and (5):

$$a_y = \frac{F_y}{m} = \frac{e\,E_y}{m} = \frac{e\,V_y}{d\,m} \tag{6}$$

Thus, electrons acquire a vertical component of velocity $v_y = a_y\,t$, where $t$ is the time spent in the deflecting region. If the length of the plates is $l$, we can obtain $t$ from: $t = l/v_z$. Therefore,

$$v_y = \frac{e\,V_y\,l}{d\,m\,v_z} \tag{7}$$

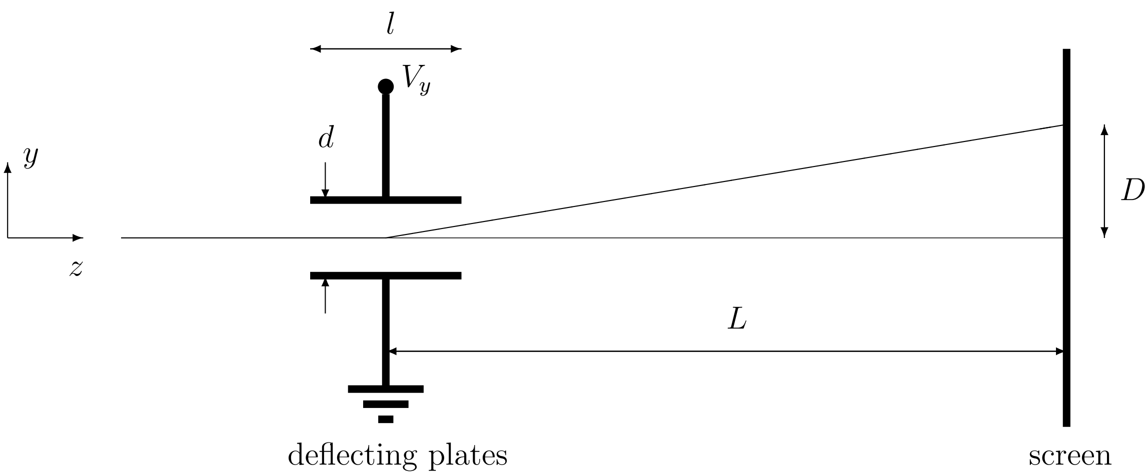


Figure 2. Deflection of electron beam in the tube.

The quantity measured in the experiment is the vertical beam displacement $D$ on the screen (see Fig. 2). Electrons travel with constant velocity between the plates and the screen. If $t'$ is the time in which electrons travel from the deflecting plates to the screen, then $D = v_y t'$. The time $t'$ can be found from $t' = L/v_z$, where $L$ is the horizontal distance from the deflecting plates to the screen. Combining these two equations $D = L v_y / v_z$, and using Eq. (7) we obtain $D = e\, V_y\, l\, L/(d\, m\, {v_z}^2)$. Since from Eq. (4) $e/(m\, {v_z}^2) = 1/(2V_a)$, we get

$$D = \frac{l\, L}{d\, 2} \frac{1}{V_a} \quad V_y \tag{8}$$

This formula tells us that the vertical displacement of the beam should be directly proportional to the deflection voltage $V_y$ (and also to the deflecting electric field $E_y$) and inversely proportional to the accelerating voltage $V_a$. You will verify this statement in the experiment.

*MAGNETIC DEFLECTION*

The CRT used in the Lab does not contain any electromagnets; therefore, the magnetic field $B$ will be created by external coils. The magnetic field will be approximately uniform, horizontal and perpendicular to the electron flight direction. The magnetic force acting on electrons will be, therefore, directed along the vertical axis $F_y = e v_z B$ (from Eq. (2)). Even though it is not rigorously correct, we can assume that the magnetic field acts on electrons along the distance $l'$ and that they travel a distance $L'$ to the screen. Thus, $v_y = a_y l'/v_z = F_y l'/(m\, v_z) = e\, l'\, B/m$. Following the same steps as in the previous section, it can be shown that

$$D = \frac{l'\, L'\, \sqrt{e}}{\sqrt{2\, m}} \frac{1}{\sqrt{V_a}} \quad B \tag{9}$$

The vertical displacement of the beam should be directly proportional to the deflecting magnetic field $B$ and inversely proportional to the square root of the accelerating voltage $V_a$. Again, you will verify this relation in the experiment. In this experiment, you will not know $B$ in absolute units but you will monitor the value of $B$ by measuring the voltage across the coils $V_{coil}$. This voltage is directly proportional to the current in the coils which, in turn, is directly proportional to the strength of the magnetic field created by the coils.

**B. Exercises:**

Please answer the questions on Report Sheet VIII–1.

**REPORT SHEET VIII–1**

Date______________________ Name______________________

Instructor______________________

PRE-LAB EXERCISES

**Exercise 1.**
Electrons in a cathode-ray tube are accelerated along the length of the tube (in the z-direction) by a potential difference $V_a = 300$ Volts. What is the speed of the electrons, compared to the speed of light $c$?
**Hints:**
a) Use conservation of energy (Eq. (4)).
b) electron mass $m = 9.1 \times 10^{-31}$ kg can also be expressed as $m = 5 \times 10^5$ $e$Volts/$c^2$.

What is the speed of electrons in $m/s$? ($c = 3 \times 10^8$ $m/s$)

**Exercise 2.**
In addition to the accelerating voltage above, electrons are also subject to a deflection voltage $V_y = 30$ V. What will be the vertical deflection on the screen $D$, if the deflecting plates of length $l = 1''$ are separated by a distance $d = 0.2''$, and the distance from the plates to the screen is $L = 5''$.
**Hint:** Use Eq. (8).

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LABORATORY ASSIGNMENTS

## Caution – High Voltage

The cathode-ray tube is under a voltage of 500 V, which may be dangerous to your life. Do not touch the connections on the rear end of the cathode-ray tube. The power supply wiring should be done by the instructor. Do not change any connection except those to the battery, the 30V power supply, the voltmeter, the deflecting plates and the electromagnet.

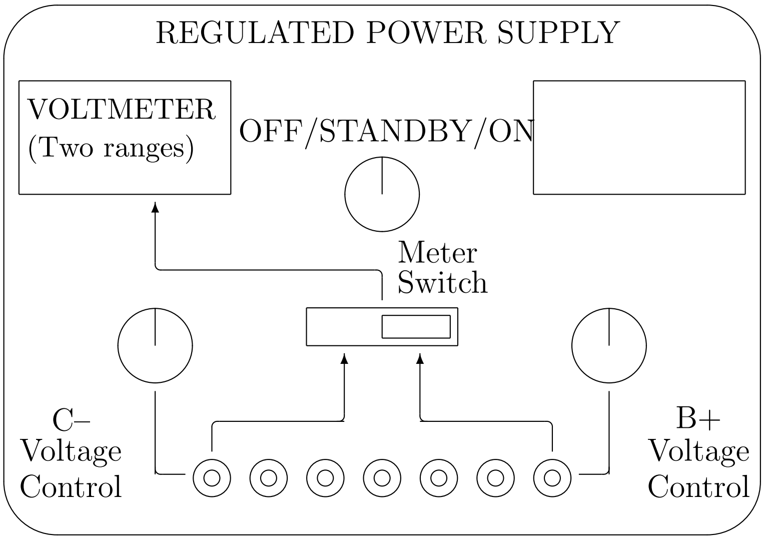



Figure 3. Regulated Power Supply for the CRT.

**Materials Needed:**

- Cathode-ray tube
- Regulated power supply for the tube
- 4.5V Battery
- 30V Power Supply
- Voltmeter
- small permanent magnets
- (**B**) Two solenoidal coils
- (**B**) Rheostat
- (**B**) Compass
- Cables

## Experiment A: *Electron motion in an electric field*

### Procedures

**A-1.** The apparatus should be wired up by your instructor before you come to the laboratory. You may check connections against Fig. 5.

Leave the 30V power supply and voltmeter off and disconnected for now. Check that the 4.5 V battery is connected to the system (see Fig. 5).

Turn the regulated power supply from the OFF to the STANDBY position and leave it at this position for one minute. You should see a red glow near the rear end of the cathode-ray tube.

**A-2.** Turn the regulated power supply to the ON position. The total accelerating voltage $V_a$ is the sum of two voltages, the C$-$ voltage (controlled by the left knob) and the B+ voltage (controlled by the right knob); see Fig. 3. The voltmeter displaying the C$-$ or B+ voltage is placed on the left of the power supply, and has two ranges: 0–400 V for B+ (the upper scale) and 0–130 V for C$-$ (the lower scale). The range switch for the meter is at the center of the power supply. Set this switch to the position on the right. Vary the B+ control knob on the power supply and observe the voltage change on the the dual-range voltmeter. Set the B+ voltage to approximately 350 V.

Set the switch to the position on the left and using the left knob, vary the C$-$ voltage. Set the C$-$ voltage to the value that gives a maximally sharp spot on the screen (approx. 125 V). Record the value of $V_a$ (i.e. sum of B+ and C$-$ voltages) in the table in Report Sheet VIII–2.

In the absence of a deflection voltage, the beam spot should be near the center of the screen. Its position can be changed by a careful adjustment of the position of a small magnet placed on top of the tube (do not change the magnet's position unless necessary). If you cannot get a sharp spot, or cannot get it near the center, ask the instructor for help.

**A-3.** Connect the 30V power supply to the vertical deflection plates. **Important:** The B+ output on the regulated power supply must be connected (in addition to its CRT connection) to the negative output of the 30V power supply for all experiments. Failure to make this connection may result in big increase in the beam spot size.

Leave the horizontal deflection plates and the coils unconnected. Connect also the digital voltmeter to read the deflection voltage (note that Fig. 5 shows the voltmeter connected to the coils, which will be the case in experiment B).

Using the knob on the 30V power supply vary the deflection voltage and observe the motion of the beam across the screen. If the beam moves diagonally you must have connected both vertical and horizontal deflection plates to the power supply – disconnect the horizontal plates. If the beam moves horizontally swap the 30V power supply connection to the other deflection plate leads.

The spot will move from zero only in one direction (e.g. in +Y direction). To deflect the spot in the opposite direction (e.g. in −Y direction), change the polarity of the connection to the 30V power supply.

**A-4.** In this step you will measure and plot the displacement $D$ versus the applied deflection voltage $V_y$ for two different settings of the accelerating voltage $V_a$.
Because the measurements of the deflecting voltage can be made much more precisely than the position on the screen, you should measure the deflection voltages corresponding to a few definite positions of the spot, for instance when the center of the spot crosses the division marks on the screen. For each measurement make a point on the graph included in Report Sheet VIII–2. Measure deflective potential at least three division marks down and at least three division marks up of the center. Cover the largest range of beam deflections allowed by the division marks and the voltage range. Plot the data as they are taken without tabulating them, except for the largest deflections up ($D_u$) and down ($D_d$). Record $D$ and $V_y$ for these extreme deflections in the table in Report Sheet VIII–2 next to the value of $V_a$ for this set of measurements.

Reduce the value of the voltage B+ by approximately 100 V and adjust the voltage C− to obtain a sharp spot. Record the new value of $V_a$. Repeat the previous measurements of $D$ versus the deflection voltage, recording them on the same graph. Also put coordinates of the largest deviations into the table together with the new value of $V_a$.

If the theory discussed above is correct, you should obtain two different straight lines, each corresponding to a different value of the accelerating voltage $V_a$. Calculate the slopes of those lines from the data in your table:

$$Slope = \frac{|D_u| + |D_d|}{|V_u| + |V_d|}$$

From Eq. (8) the slope of the $V_y$ dependence of $D$ is equal to $l\,L/(d\,2)\quad 1/V_a$, thus it should be inversely proportional to $V_a$. Verify this expectation by calculating the product of $V_a$ and of the measured slope (store your result in the $6^{th}$ column in the table). These products should be independent of $V_a$ if Eq. (8) is right.

Assuming $d = 0.2''$, $L = 5''$, $l = 1''$ calculate the slopes predicted by Eq. (8) and compare them with what you actually measured (Report Sheet VIII–2).

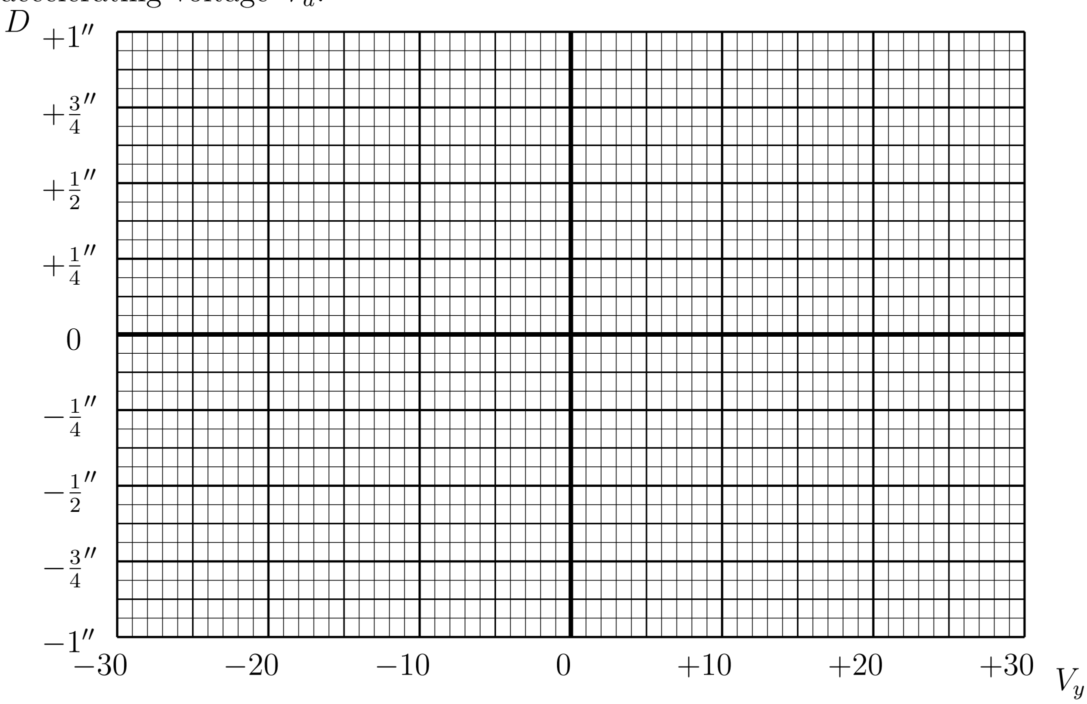
## REPORT SHEET VIII–2

Date________________________ Name________________________

Instructor________________________ Partner(s)________________________

**A-4**

Use the "graph paper" below to make graphs displaying the displacement $D$ versus the deflection voltage (with the deflection voltage on the x-axis) for the two different values of the accelerating voltage $V_a$.




Try to draw a straight line through each set of your measurements (use a ruler). Is the relation between $D$ and $V_y$ linear?

☐ yes ☐ no

| | Deflection up | | Deflection down | | Measured | | Expected |
|---|---|---|---|---|---|---|---|
| $V_a$ | $D_u$ | $V_u$ | $D_d$ | $V_d$ | Slope | Slope$\times V_a$ | Slope |
| (V) | (in) | (V) | (in) | (V) | (in/V) | (in) | (in/V) |
| | | | | | | | |
| | | | | | | | |

Do your results confirm the expected dependence on $V_a$?

☐ yes ☐ no

Do the expected slopes roughly agree with the observations?

☐ yes ☐ no

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## **Experiment B:** *Electron motion in magnetic field*

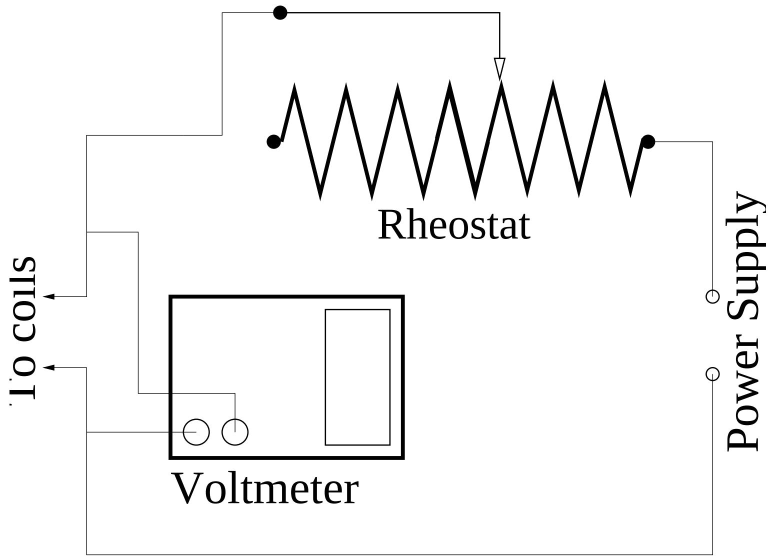


Figure 4. Connection of the power supply to the coils.

### Procedures

**B-1.** The two coils should be positioned and wired as shown in the Fig. 5. Switch off the 30V power supply, disconnect it from the deflective plates and connect it to the coils. There should still be a connection between the B+ output of the regulated power supply and the negative output of the 30V power supply. To reduce voltage supplied to the coils include rheostat in one leg of the connection as illustrated in Fig. 4. The voltmeter should be connected to measure voltage supplied to the coils as shown in Fig. 4.

Switch on the 30V power supply and turn the control knob all the way to the right. Move the slider on the rheostat until the voltmeter reads approximately 3V.

Use a small compass to check the direction of the magnetic field produced by each coils. Tap the compass since needle can sometimes get stuck. Make sure, that the polarity of the two electromagnets is the same.

You should observe a vertical deflection of the spot caused by the magnetic field.

Measure and plot, on the graph provided on Report Sheet VIII–3, the deflection $D$ versus the voltage drop across the coils (which is proportional to the magnetic field). Use the method suggested in A-4 to obtain the data points. Switch the 30V power supply connection to reverse the current flow in the coils. The spot should now be deflected in the opposite direction. Make measurements of $D$ versus the voltage drop also for this configuration. As in experiment A, change the B+ voltage by 100 V, adjust C− and repeat the measurements for the second value of the accelerating voltage.

For each value of the accelerating voltage $V_a$ (sum of B+ and C− voltages) calculate the slope. Store your results in the table. For the $6^{th}$ column in the table, calculate the product of $\sqrt{V_a}$ and of the measured slope. These products should be independent of $V_a$ if Eq. (9) is right. Do your results confirm the expected dependence on $V_a$?

**B-2.** ( *Absolutely Mandatory*)

Switch off the 30V power supply. Disconnect one lead from the battery. Reduce the B+ and C− voltages to zero. Switch off the big power supply.

Show your table to the instructor before turning in your report.

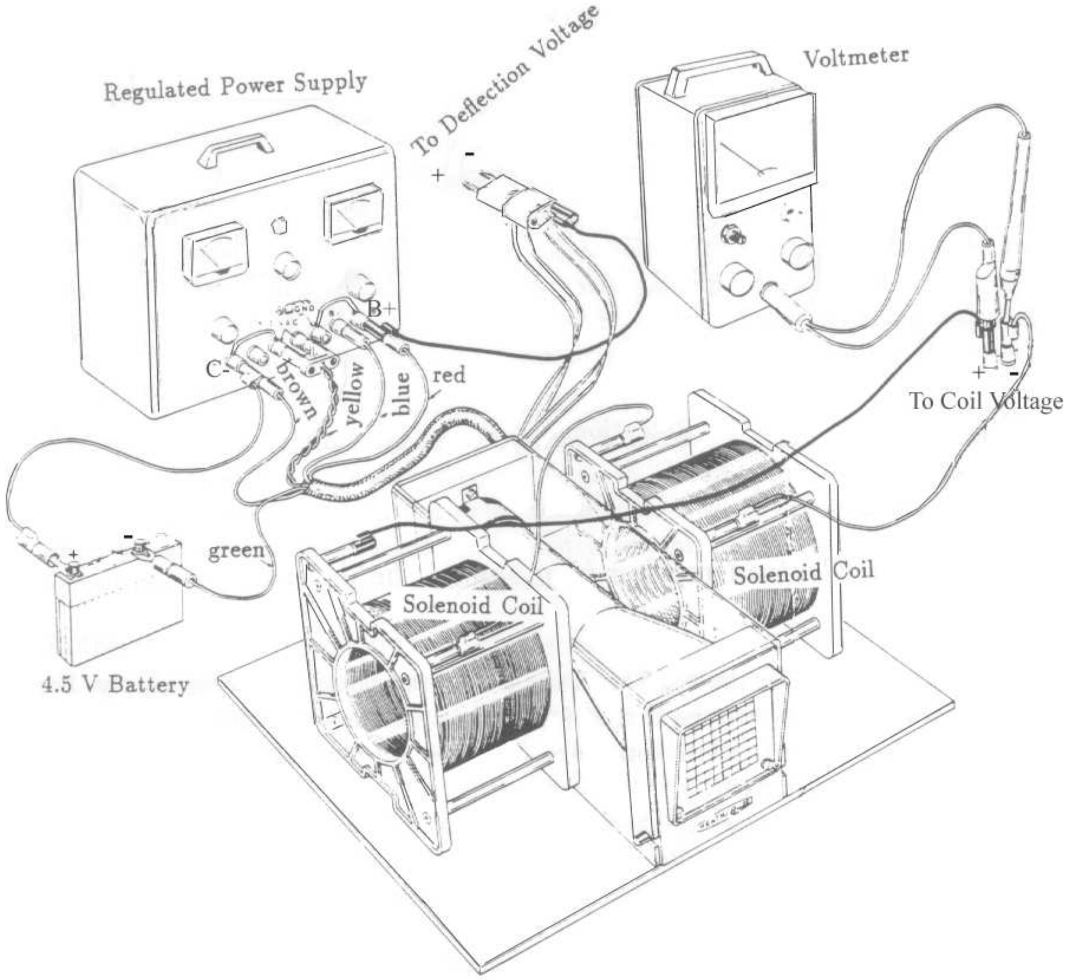


Figure 5. Drawing of experimental apparatus.

## REPORT SHEET VIII–3

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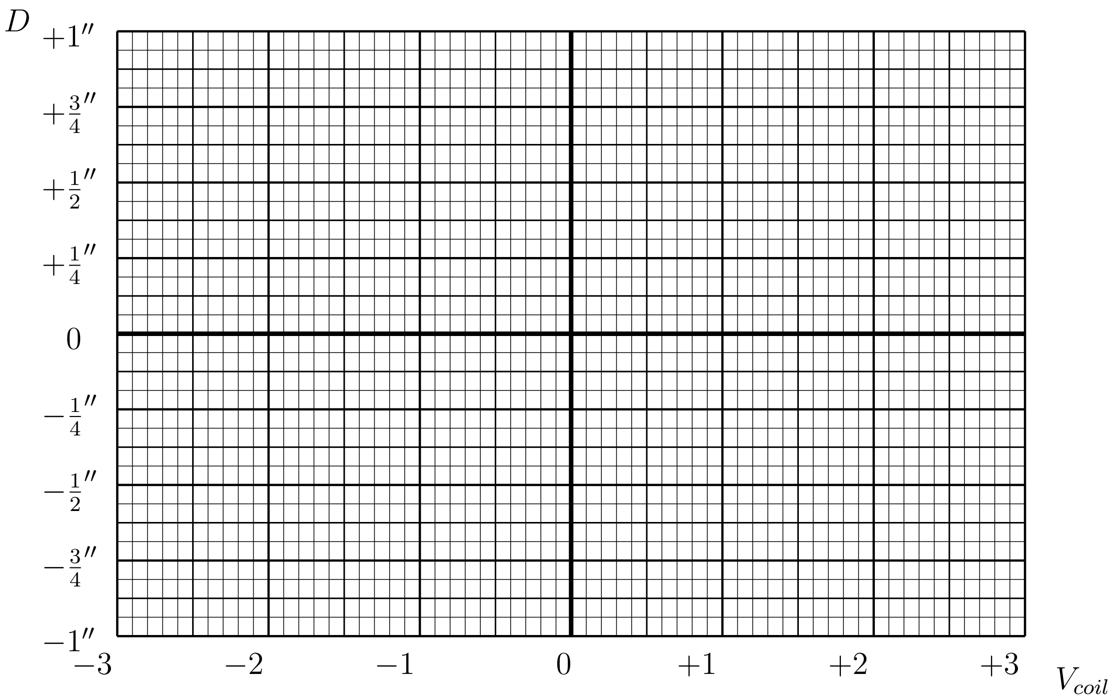
Instructor____________________ Partner(s)____________________

### B-1

Below make a graph showing $D$ versus the voltage through the coils.




Try to draw a straight line through each set of your measurements (use a ruler). Is the relation between $D$ and $V_{coil}$ linear?

☐ yes ☐ no

| | Deflection up | | Deflection down | | Measured | |
|---|---|---|---|---|---|---|
| $V_a$ (V) | $D_u$ (in) | $V_u$ (V) | $D_d$ (in) | $V_d$ (V) | Slope (in/V) | Slope$\times\sqrt{V_a}$ (in/$\sqrt{\text{V}}$) |
| | | | | | | |
| | | | | | | |

Do your results roughly confirm the expected dependence on $V_a$?

☐ yes ☐ no