# Class E DC/DC Converters with an Inductive Impedance Inverter 

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#### Abstract

The analysis, design equations, and experimental verification are presented for a Class $\mathbf{E}$ dc/dc switching-mode high-efficiency resonant dc/dc power converter with a transformer center-tapped rectifier. The analysis is performed at a constant current through the dcfeed choke and using the high- $Q_{1}$ assumption ( $Q_{1} \geq 5$ ). The converter may operate under safe conditions for load resistances ranging from a full load to an open circuit. This feature has been accomplished by using an inductive impedance inverter. The results of the analysis are then generalized for the entire family of Class $\mathbf{E}$ dc/dc converters. Experimental results show good agreement with the theoretical predictions. The measured total efficiency was 89 percent at 1 MHz with 15 W output power. A narrow-band $\mathbf{F M}$ regulation ( $\Delta f / f=12$ percent) of the de output voltage was achieved as the load was varied from a minimum resistance (full load) to an open circuit.


## I. Introduction

CLASS E switching-mode tuned power inverters [1][20] can operate efficiently in the megahertz frequency range. They are particularly suited for high-frequency operation because 1) the turn-on switching loss is zero, yielding high efficiency and 2 ) the transistor output capacitance is absorbed into the external shunt capacitance. Class E resonant dc/dc converters [21]-[32] were derived from Class $E$ inverters by adding rectifiers at the inverter output. The purpose of this paper is to 1 ) present an analysis and design rules for a Class E dc/dc converter with a transformer center-tapped rectifier and an inductive impedance inverter given in [25], [26], and 2) extend the results for other topologies of Class $\mathbf{E ~ d c} / \mathrm{dc}$ converters analyzed in [31].

Fig. 1(a) shows a circuit of the Class E dc/dc converter. It consists of a Class E inverter and a transformer centertapped full-wave rectifier. The Class E inverter is composed of a power MOSFET, an antiparallel diode D1, a radio frequency (RF) choke $L_{1}$, and a resonant circuit $C_{1-}$ $C_{2}-L_{2}-L_{3}$. The power MOSFET and the antiparallel diode $D 1$ form a bidirectional switch $S$ which turns on and off at the switching frequency $f=\omega / 2 \pi$. The inductor $L_{3}$ has two tasks: 1) it acts as a matching network and 2) as an inductive impedance inverter [27]. The shape of the current $i$ depends on the loaded quality factor $Q_{1}$ and the

[^0]

Fig. 1. Class E dc/dc converter. (a) Circuit. (b) Equivalent circuit. (c) Circuit of Class $E$ inverter with matching network. (d) Basic circuit of Class E inverter.
on switch duty cycle $D$ [17]. The current $i$ is approximately sinusoidal for $Q_{1} \geq 5$ and $D \geq 0.5$. Therefore, the output power of the inverter is approximately composed of only the fundamental component. The analysis of the inverter for optimum operation is given in [17] and for nonoptimum operation in [18] at any $Q_{1}$ and $D$. There are two modes of rectifier operation, the continuous mode and the discontinuous mode. Figs. 2 and 3 show the current and voltage waveforms for these modes. This paper is focused upon interfacing the inverter and the rectifier.

The converter analysis is based on the following assumptions.

1) The RF choke inductance $L_{1}$ is large enough so that the dc ripple current in the input current $I_{i}$ is negligible.
2) The loaded quality factor $Q_{1}$ of the resonant circuit is high ( $\left.Q_{1} \geq 5\right)$ so that the current $i$ is a sine wave and, therefore, the inverter output power is composed of the fundamental component only.
3) The converter components are ideal.


Fig. 2. Current and voltage waveforms in Class $\mathrm{E} \mathrm{dc} / \mathrm{dc}$ converter for continuous mode of operation.


Fig. 3. Current and voltage waveforms in Class E dc/dc converter for discontinuous mode of operation.

## II. Analysis

The analysis of the converter for the continuous mode of operation is presented below. Under the high- $Q_{1}$ assumption ( $Q_{1} \geq 5$ ), the current $i$ in the $L_{2}-C_{2}$ branch is approximately a cosine wave [17] given by

$$
\begin{equation*}
i=I_{m} \cos \omega t \tag{1}
\end{equation*}
$$

where $I_{m}$ is the amplitude of $i$. Neglecting the voltage drop across the rectifier diodes $D 2$ and $D 3$, the voltage $v$ at the input of the transformer is a square wave with the
magnitudes $n V_{o}$ and $-n V_{o}$, i.e.,

$$
v=\left\{\begin{align*}
n V_{o}, & \text { for }-\frac{\pi}{2}-\phi<\omega t \leq \frac{\pi}{2}-\phi  \tag{2}\\
-n V_{o}, & \text { for } \frac{\pi}{2}-\phi<\omega t \leq \frac{3 \pi}{2}-\phi
\end{align*}\right.
$$

The transformer/rectifier circuit can be replaced with a square-wave voltage source as shown in Fig. 1(b). The fundamental component of the voltage $v$ is given by

$$
\begin{equation*}
v_{1}=V_{1 m} \cos (\omega t+\phi) \tag{3}
\end{equation*}
$$

where

$$
\begin{equation*}
V_{1 m}=\frac{4 n V_{o}}{\pi} \tag{4}
\end{equation*}
$$

is the amplitude of $v_{1}$ and $\phi$ is the initial phase of $v_{1}$. The rms value of $v_{1}$ is

$$
\begin{equation*}
V_{1}=\frac{V_{1 m}}{\sqrt{2}}=\frac{4 n V_{o}}{\pi \sqrt{2}} . \tag{5}
\end{equation*}
$$

The dc-to-ac voltage transfer function of the rectifier is

$$
\begin{equation*}
M_{R}=\frac{V_{o}}{V_{1}}=\frac{\pi \sqrt{2}}{4 n}=\frac{\pi}{2 \sqrt{2} n} \simeq \frac{1.11}{n} \tag{6}
\end{equation*}
$$

Since the voltage $v$ across the inductance $L_{3}$ is a square wave, the current $i_{L 3}$ is a symmetrical triangle wave. The slope of $i_{L 3}$ is $i_{L 3}\left(\omega t_{2}\right) /(\pi / 2)$, where $\omega t_{2}=\pi / 2-\phi$. From (1), $i_{L 3}\left(\omega t_{2}\right)=i\left(\omega t_{2}\right)=I_{m} \cos (\pi / 2-\phi)=I_{m}$ $\sin \phi$. Thus the current $i_{L 3}$ is

$$
\begin{equation*}
i_{L 3}=\frac{2 I_{m} \sin \phi}{\pi}(\omega t+\phi) . \tag{7}
\end{equation*}
$$

From (1) and (7), the input current of the transformer is

$$
\begin{equation*}
i_{R}=i-i_{L 3}=I_{m} \cos \omega t-\frac{2 I_{m} \sin \phi}{\pi}(\omega t+\phi) \tag{8}
\end{equation*}
$$

Thus, the current $i_{D}$ at the input of the $C_{f}-R_{L}$ circuit is

$$
\begin{align*}
i_{D} & =i_{D 2}+i_{D 3}=n\left|i_{R}\right| \\
& =\left|n I_{m} \cos \omega t-\frac{2 n I_{m} \sin \phi}{\pi}(\omega t+\phi)\right| . \tag{9}
\end{align*}
$$

Hence, the dc output current is

$$
\begin{equation*}
I_{o}=\frac{1}{\pi} \int_{-\pi / 2-\phi}^{\pi / 2-\phi} i_{D} d(\omega t)=\frac{2 n I_{m} \cos \phi}{\pi} \tag{10}
\end{equation*}
$$

from which (7) becomes

$$
\begin{equation*}
i_{L 3}=\frac{I_{o} \tan \phi}{n}(\omega t+\phi) \tag{11}
\end{equation*}
$$

Substituting (2) and (11) into $v=X_{L 3} d i_{L 3} / d(\omega t)$, one obtains

$$
\begin{equation*}
n V_{o}=X_{L 3} \frac{d i_{L 3}}{d(\omega t)}=\frac{I_{o} X_{L 3} \tan \phi}{n} \tag{12}
\end{equation*}
$$

where $X_{L 3}=\omega L_{3}$. Since $R_{L}=V_{o} / I_{o}$, (12) reduces to

$$
\begin{equation*}
\tan \phi=\frac{n^{2} R_{L}}{X_{L 3}} \tag{13}
\end{equation*}
$$

The voltage $v$ at the input of the inductance $L_{3}$ is a square wave. It contains odd harmonics. However, since the current $i$ contains only the fundamental component, the power at the input of $L_{3}$ also contains only the fundamental component $P_{1}$. This power is converted into the dc output power $P_{o}$. From (4) and (10),

$$
\begin{equation*}
P_{1}=\frac{I_{m} V_{1 m}}{2} \cos \phi=I_{o} V_{o}=P_{o} \tag{14}
\end{equation*}
$$

The current $i_{L 3}$ is an odd periodic function with respect to $\omega t=-\phi$. It contains only odd sine terms in its Fourier series representation. For example, the fundamental component of $i_{L 3}$ is $i_{L 31 m}=I_{L 31 m} \sin (\omega t+\phi)$, where $I_{L 31 m}=$ $8 I_{m} \sin \phi / \pi^{2}$. The current $i_{R}$ exhibits half-wave symmetry and, therefore, it contains odd cosine and odd sine terms. Since the voltage $v$ contains only odd cosine terms, the input power of the transformer also contains only odd components. All these components are converted into the dc output power $P_{o}$. The analysis based on an infinite number of components is difficult to perform. Therefore, an easier approach is presented below.

Fig. 1(c) shows an $L_{i}-R_{i}$ model of the input impedance of the circuit composed of the inductor $L_{3}$, transformer, and rectifier. This model is determined at the fundamental frequency $f$. The $L_{i}-R_{i}$ model is created in such a way that the power $P_{1}$ given by (14) is delivered to $R_{i}$ at the fundamental frequency $f$ exclusively. The currents through $R_{i}$ and $X_{i}=\omega L_{i}$ are

$$
\begin{align*}
i_{R i} & =I_{R i m} \cos (\omega t+\phi)  \tag{15}\\
i_{L i} & =I_{L i m} \sin (\omega t+\phi) \tag{16}
\end{align*}
$$

Fig. 4 depicts the phasor diagram of the fundamental components $I_{\text {Rim }}, I_{L i m}, I_{m}$, and $V_{1 m}$, which leads to

$$
\begin{align*}
I_{\text {Rim }} & =I_{m} \cos \phi  \tag{17}\\
I_{\text {Lim }} & =I_{m} \sin \phi . \tag{18}
\end{align*}
$$

Note that $I_{L 31 m}$ is less than $I_{L i m}$ by a factor of $8 / \pi^{2}$. Substitution of (17) into (10) gives

$$
\begin{equation*}
I_{o}=\frac{2 n I_{R i m}}{\pi} \tag{19}
\end{equation*}
$$

This is a well-known expression for the dc output current in a full-wave rectifier without filter capacitor. Using (4), (19), and $R_{L}=V_{o} / I_{o}$, one obtains

$$
\begin{equation*}
R_{i}=\frac{V_{1 m}}{I_{R i m}}=\frac{8 n^{2}}{\pi^{2}} R_{L}=0.81 n^{2} R_{L} \tag{20}
\end{equation*}
$$

The same expressions can be derived as follows. The total input power of the transformer is $P_{R i}=V_{1 m}^{2} /\left(2 R_{i}\right)=$ $8 n^{2} V_{o}^{2} /\left(\pi^{2} R_{i}\right)$ and the dc output power is $P_{o}=V_{o}^{2} / R_{L}$. Since $P_{o}=P_{R i}$, one obtains (20). From Fig. 4, (20), $I_{\text {Rim }}$


Fig. 4. Phasor diagram of fundamental components.

$$
\begin{align*}
=V_{1 m} / R_{i}, \text { and } I_{L i m} & =V_{1 m} / X_{i}, \\
\tan \phi & =\frac{I_{L i m}}{I_{R i m}}=\frac{R_{i}}{X_{i}}=\frac{8 n^{2} R_{L}}{\pi^{2} X_{i}} . \tag{21}
\end{align*}
$$

Comparing (13) and (21) results in

$$
\begin{equation*}
X_{i}=\frac{8}{\pi^{2}} X_{L 3} . \tag{22}
\end{equation*}
$$

Thus, the relationships among $R_{i}, X_{i}, R_{L}$, and $X_{L 3}$ are given by (20), (21), and (22). The circuit of Fig. 1(c) is a Class E inverter with a matching load network. At the fundamental frequency $f$, the $L_{i}-R_{i}$ parallel combination can be converted into an $L_{S^{-}} R$ series combination resulting in the basic topology of the Class E inverter, shown in Fig. 1(d).

If the dc output voltage is low (e.g., $V_{o}=5 \mathrm{~V}$ ), the voltage drop $V_{F}$ across the conducting rectifier diodes can be taken into account [31]. In this case, one obtains $v=$ $\pm n\left(V_{o}+V_{F}\right), V_{1 m}=4 n V_{o}\left(1+V_{F} / V_{o}\right) / \pi, M_{R}=$ $\pi /\left[2 \sqrt{2} n\left(1+V_{F} / V_{o}\right) \simeq 1.11 /\left[n\left(1+V_{F} / V_{o}\right)\right], R_{i}=\right.$ $8 n^{2} R_{L}\left(1+V_{F} / V_{o}\right) / \pi^{2} \simeq 0.81 n^{2} R_{L}\left(1+V_{F} / V_{o}\right)$, and the rectifier efficiency $\eta_{R}=1 /\left(1+V_{F} / V_{o}\right)$.
The boundary between the continuous and discontinuous modes of operation occurs when $d i_{L 3} / d(\omega t)=$ $d i / d(\omega t)$ at $\omega t_{1}=-\pi / 2-\phi_{\mathrm{cr}}$, where $\phi_{\mathrm{cr}}$ is the critical value of $\phi$ at the boundary. Hence, from (1) and (7),

$$
\begin{equation*}
\tan \phi_{\mathrm{cr}}=\frac{\pi}{2} \tag{23}
\end{equation*}
$$

from which $\phi_{\mathrm{cr}}=57.52^{\circ}$. For $\phi<\phi_{\mathrm{cr}}$, the rectifier acts in the continuous mode of operation, shown in Fig. 2. From (20), (21), and (22), this mode occurs for

$$
\begin{equation*}
R_{i}<\frac{\pi}{2} X_{i}=\frac{4}{\pi} X_{L 3} \tag{24}
\end{equation*}
$$

or equivalently,

$$
\begin{equation*}
R_{L}<\frac{\pi}{2 n^{2}} X_{L 3} \tag{25}
\end{equation*}
$$

Substitution of (10) into (9) results in $i_{D}$ in terms of $I_{o}$ :

$$
\begin{equation*}
i_{D}=I_{o}\left|\frac{\pi \cos \omega t}{2 \cos \phi}-(\tan \phi)(\omega t+\phi)\right| \tag{26}
\end{equation*}
$$

The peak forward current $I_{D M}$ of the rectifier diodes $i_{D 2}$ and $i_{D 3}$ is determined by $d i_{D} / d(\omega t)=0$. Hence, from (26), the maximum value of $i_{D}$ occurs at

$$
\begin{equation*}
\omega t_{\max }=-\arcsin \left(\frac{2 \sin \phi}{\pi}\right) \tag{27}
\end{equation*}
$$

Substitution of this into (26) gives $I_{D M}$, e.g., $I_{D M} / I_{o}=$ $\pi / 2=1.57$ at $\phi=0^{\circ}, I_{D M} / I_{o}=1.665$ at $\phi=45^{\circ}$, and $I_{D M} / I_{o}=1.774$ at $\phi=57^{\circ}$. The peak reverse voltages of the rectifier diodes are

$$
\begin{equation*}
V_{D 2 M R}=V_{D 3 M R}=-2 V_{o} . \tag{28}
\end{equation*}
$$

For $\phi>\phi_{\mathrm{cr}}$, the rectifier operates in the discontinuous mode, illustrated in Fig. 3. When $i_{R}=i-i_{L 3}$ is positive, diode $D 3$ conducts. When $i_{R}=i-i_{L 3}$ is negative, diode $D 2$ conducts. However, there are time intervals when both diodes are off. Then (neglecting the parasitic capacitances of the two diodes), the $C_{1}-C_{2}-L_{2}-L_{3}$ resonant circuit is disconnected from the $C_{f}-R_{L}$ circuit and resonates like an unloaded tank. Consequently, the voltage $v$ is no longer a square wave and the current $i_{L 3}$ is no longer a triangle wave. The waveforms in the experimental circuit will be shown in Section VII.

## III. Impedance Inverter

The optimum operation of the basic Class E inverter of Fig. 1(d) occurs at the optimum load resistance $R=R_{\text {opt }}$ [1]-[19]. The lossless operation of the inverter can be obtained for $0 \leq R \leq R_{\max }=R_{\mathrm{opp}}$. In contrast, the converter load resistance $R_{L}$ is usually in the range $R_{L \text { min }} \leq R_{L}<$ $\infty$. According to (17), the rectifier input resistance $R_{i}$ is proportional to $R_{L}$. Hence, the range of $R_{i}$ is $R_{i \min } \leq R_{i}$ $<\infty$. Clearly, an impedance inverter is required between the Class $\mathbf{E}$ inverter and the rectifier to remove the impedance incompatibility.

The circuit of Fig. 1(c) is a Class E inverter with a $C_{1-}{ }^{-}$ $C_{2}-L_{2}-L_{i}$ matching load network in which an impedance inversion can be achieved [27]. Fig. 1(d) shows its equivalent circuit obtained by using the equivalent two-terminal networks method. The relationships among the component values at the operating frequency $f$ are as follows [27]:

$$
\begin{align*}
q_{B} & =\frac{R_{i}}{X_{i}}=\tan \phi  \tag{29}\\
R & =\frac{R_{i}}{1+q_{B}^{2}}=\frac{R_{i}}{1+\left(\frac{R_{i}}{X_{i}}\right)^{2}}  \tag{30}\\
X_{L S} & =\frac{X_{i}}{1+\frac{1}{q_{B}^{2}}}=\frac{X_{i}}{1+\left(\frac{X_{i}}{R_{i}}\right)^{2}}  \tag{31}\\
X_{L} & =\omega L=X_{L 2}+X_{L S} \tag{32}
\end{align*}
$$

Expressions (30) and (31) are plotted in Fig. 5.
From (30), $d R / d R_{i}=0$ at $R_{i}=X_{i}$, i.e., at $q_{B}=1$. Substitution of this into (30) yields the maximum value of $R$ :

$$
R_{\max }=\frac{X_{i}}{2} .
$$


(b)

Fig. 5. Relationships among $R, R_{i}, X_{i}$, and $X_{L S}$. (a) $R$ as a function of $R_{i}$. (b) $X_{L S}$ as a function of $R_{i}$.

For $R_{i}=X_{i}$, (31) becomes

$$
X_{L S}=\frac{X_{i}}{2}
$$

For $q_{B}^{2} \gg 1$, i.e., $R_{i} \gg X_{i}$,

$$
R \simeq \frac{X_{i}^{2}}{R_{i}}, \quad X_{L S} \simeq X_{i}
$$

For $q_{B}=0$, i.e., for $R_{i}=0, R=0$, and $X_{L S}=0$.
The conclusions are as follows.

1) $R \leq X_{i} / 2=4 X_{L 3} / \pi^{2}$ at any value of $R_{i}$, i.e., at any value of $R_{L}$.
2) For $q_{B} \geq 1, R$ decreases as $R_{i}$ increases, i.e., as $R_{L}$ increases. Thus the circuit acts like an impedance inverter.
3) For $q_{B}^{2} \gg 1, R$ is inversely proportional to $R_{i}$ and $X_{L S} \simeq X_{i}=8 X_{L 3} / \pi^{2}$ is approximately independent of $R_{i}$, i.e., $R_{L}$.
4) If $R_{\text {opt }}=R_{\max }=X_{i} / 2$, the range of $R$ is $0 \leq R \leq$ $R_{\text {opt }}$ for $0 \leq R_{i} \leq \infty$, i.e., for $0 \leq R_{L} \leq \infty$. Thus, the Class E inverter acts under lossless (zero-voltage-switching) conditions at any $R_{i}$, i.e., at any $R_{L}$, including a short circuit ( $R_{L}=0$ ) and an open circuit ( $R_{L}=\infty$ ).
5) If $R_{\text {opt }}<R_{\text {max }}=X_{i} / 2$, the lossless converter operation can be obtained for $0 \leq R_{i} \leq R_{i \text { max }}$ and $R_{i \text { min }} \leq R_{i}$ $\leq \infty$. Thus, there is a gap in $R_{i}$ (and therefore also in $R_{L}$ ) in which the lossless operation cannot be achieved. However, $R$ is less sensitive to $R_{i}$ for $R_{i \text { min }} \leq R_{i} \leq \infty$ and thereby a narrower bandwidth of the switching frequency $f$ is required to control $V_{o}$.

Note that for $R_{L}=\infty, R_{i}=\infty$ and therefore the load network consists of $C_{1}, L_{2}$, and $L_{3}$. For $R_{L}=0, R_{i}=0$ and thereby the load network consists of $C_{1}, L_{2}$ and $C_{2}$. In both cases, the series-resonant circuit is closed. In the basic topology of the Class E converter [31], the seriesresonant circuit is open at $R_{L}=\infty$.

## IV. DC Voltage Transfer Function

First, the ac-to-dc voltage transfer function will be determined for the Class E inverter of Fig. 1(d). The dc
input power of the inverter is $P_{i}=I_{i} V_{i}=V_{i}^{2} / R_{\mathrm{dc}}$, where $R_{\mathrm{dc}}=V_{i} / I_{i}$ is the dc input resistance of the inverter. The ac output power of the inverter is $P_{o A}=V_{R}^{2} / R$, where $V_{R}$ is the rms value of the inverter output voltage $v_{R}$ which may be either sinusoidal or nonsinusoidal depending on $Q_{1}$ and $D$ [17]. Hence, $V_{R}=\sqrt{P_{o A} R}$. Assuming that the inverter efficiency is $\eta_{A}=100$ percent, $P_{o A}=P_{i}$. Thus, the ac-to-dc voltage transfer function of the Class E inverter is

$$
\begin{equation*}
M_{A}=\frac{V_{R}}{V_{i}}=\sqrt{\frac{P_{o A} R}{V_{i}^{2}}}=\sqrt{\frac{P_{o A} R}{P_{i} R_{\mathrm{dc}}}}=\sqrt{\frac{R}{R_{\mathrm{dc}}}} \tag{33}
\end{equation*}
$$

The values of $M_{A}^{2}=P_{o A} R / V_{i}^{2}$ are given in [17] for optijmum operation at any values of $Q_{1}$ and $D$. Using these results, the values of $M_{A}$ were calculated and are given in Table I. As seen, $M_{A}$ increases with $D$ and $Q_{1}$. A close examination reveals that the values of $M_{A}$ are in the range of $0<M_{A}<\sqrt{2}$ for optimum operation. For $Q_{1} \geq 5$, the voltage $v_{R}$ is nearly a sine wave [17].

The efficiency of the actual inverter $\eta_{A}=P_{o A} / P_{i}$ is less than 100 percent. Hence, from (33),

$$
\begin{equation*}
M_{A}=\sqrt{\frac{\eta_{A} R}{R_{\mathrm{dc}}}} \tag{34}
\end{equation*}
$$

For example, at $\eta_{A}=95$ percent (which is a typical value of $\eta_{A}$ for $f \leq 3 \mathrm{MHz}$ and $V_{i} \geq 12 \mathrm{~V}$ ), $\sqrt{\eta_{A}}=0.975$. Thus, $M_{A}$ of the actual inverter will be lower than that of the ideal inverter by 2.5 percent.

The conduction power loss in a MOSFET is $P_{C D S}=$ $2.37 r_{D S} I_{i}^{2}$ at $D=0.5$ and $Q \geq 5$ for optimum operation, where $r_{D S}$ is the transistor on-resistance. The efficiency of the resonant circuit is $\eta_{o}=1-Q_{1} / Q_{o}$, where $Q_{o}$ is the unloaded quality factor of the resonant circuit associated with a series equivalent ac resistance $r_{L}$ of $L$ and a series ac resistance $r_{C}$ of $C_{2}$. For instance, at $Q_{1}=5$ and $Q_{o}=$ $200, \eta_{o}=97.5$ percent. To obtain high-efficiency $\eta_{o}, Q_{o}$ should be as high as possible and $Q_{1}$ should be low. In contrast, the bandwidth of FM regulation of the dc output voltage $V_{o}$ is narrower if $Q_{1}$ is higher. Therefore, the choice of $Q_{1}$ involves a reasonable trade-off.

Now the transfer function of the impedance inverter will be derived. As stated before, the power at the input of the impedance inverter $L_{3}$ for $Q_{1} \geq 5$ contains the fundamental component only and is expressible as $P_{R i}=V_{1}^{2} / R_{i}$, where $V_{1}$ is the rms value of the fundamental component $v_{1}$ given by (5). Neglecting power loss in $L_{3}, P_{R i}=P_{o A}$. Thus, from (29) and (30), the ac-to-ac voltage transfer function of the impedance inverter is

$$
\begin{equation*}
M_{I}=\frac{V_{i}}{V_{R}}=\sqrt{\frac{R_{i}}{R}}=\sqrt{1+q_{B}^{2}}=\sqrt{1+\tan ^{2} \phi} \tag{35}
\end{equation*}
$$

It is apparent that $M_{I} \geq \sqrt{2}$ for $q_{B} \geq 1$.
Using (6), (33), and (35), one obtains the dc-to-dc volt-

TABLE I
$M_{A}$ as a Function of $Q_{1}$ and $D$ for Optimum Operation

age transfer function of the Class E converter of Fig. 1(a):

$$
\begin{equation*}
M=\frac{V_{o}}{V_{i}}=\sqrt{\eta_{A}} M_{A} M_{I} M_{R}=\frac{\pi M_{A} \sqrt{\eta} \sqrt{1+q_{B}^{2}}}{2 \sqrt{2} n} \tag{36}
\end{equation*}
$$

where $\eta$ is the total converter efficiency and the values of $M_{A}$ are given in Table I. For instance, for optimum operation at $D=0.5$ and $Q_{1}=5, M=0.8047$ - $\sqrt{\eta} \sqrt{1+q_{B}^{2}} / n$. Hence, $M=1.138 \sqrt{\eta} / n$ at $q_{B}=1$.

## V. Design Equations

The following converter specifications are usually supplied for the designer: $V_{i}, V_{o}$, and $P_{o \text { max }}$. The converter can be designed so that the maximum output power $P_{o \text { max }}$ occurs for optimum operation of the Class E inverter. In general, the operating point for $P_{o \text { max }}$ should be located on the graph of Fig. 5(a) to the left of the maximum value (i.e., $q_{B} \geq 1$ ) to obtain lossless operation for $R_{L \text { min }} \leq$ $R_{L} \leq \infty$. The minimum value of the load resistance is

$$
\begin{equation*}
R_{L \min }=\frac{V_{o}^{2}}{P_{o \max }} \tag{37}
\end{equation*}
$$

Assuming $n$ and substituting (37) into (20), we have

$$
\begin{equation*}
R_{i \min }=\frac{8 n^{2}}{\pi^{2}} R_{L \min } . \tag{38}
\end{equation*}
$$

Let us define the following parameters: $b=V_{i}^{2} R / P_{o}$, $c=\omega C_{1} R, d=\omega C R=\omega C_{2} R$, and $e=\omega L / R$, where $C_{2}$ $=C$ and $L=L_{2}+L_{s}$. The values of all of these parameters are given in [17] at any $Q_{1}$ and $D$ for optimum operation of the Class E inverter. Assuming $Q_{1}, D$, and $f$, one can determine

$$
\begin{equation*}
R_{\max }=\frac{b V_{i}^{2}}{P_{o \text { max }}} \tag{39}
\end{equation*}
$$

$$
\begin{equation*}
C_{1}=\frac{c}{\omega R_{\max }}, \tag{40}
\end{equation*}
$$

$$
\begin{align*}
C_{2} & =\frac{d}{\omega R_{\max }}  \tag{41}\\
L & =\frac{e R_{\max }}{\omega} \tag{42}
\end{align*}
$$

From (30) and (39),

$$
\begin{equation*}
q_{B \min }=\sqrt{\frac{R_{i \min }}{R_{\max }}-1} \tag{43}
\end{equation*}
$$

Using (31),

$$
\begin{equation*}
X_{i}=\frac{R_{i \min }}{q_{B \min }} \tag{44}
\end{equation*}
$$

which leads to

$$
\begin{equation*}
L_{i}=\frac{X_{i}}{\omega} . \tag{45}
\end{equation*}
$$

From (31),

$$
\begin{equation*}
X_{L S \min }=\frac{X_{i}}{1+\frac{1}{q_{B \min }^{2}}} \tag{46}
\end{equation*}
$$

which leads to

$$
\begin{equation*}
L_{S \min }=\frac{X_{L S \min }}{\omega} \tag{47}
\end{equation*}
$$

Hence, from (32),

$$
\begin{equation*}
L_{2}=L-L_{S \min } \tag{48}
\end{equation*}
$$

From (22) and (44),

$$
\begin{equation*}
L_{3}=\frac{\pi^{2}}{8} L_{i} \tag{49}
\end{equation*}
$$

Let us define $k=R_{L_{\text {max }}} / R_{L_{\text {min }}}=R_{i \text { max }} / R_{i \text { min }}=$ $q_{B \text { max }} / q_{B \text { min }}$. Hence, from (30), one obtains $R_{\text {max }} / R_{\text {min }}=$ $\left(1+k^{2} q_{B \text { min }}^{2}\right)\left[k\left(1+q_{B \text { min }}^{2}\right)\right]$.
To obtain lossless operation at any value of $R_{L}$, one can assume $q_{B}=1$ at $R_{i \text { min }}$. In this case, the operating point for $P_{o \text { max }}$ is located at the maximum value of the graph shown in Fig. 5(a). Equations (37)-(42) and (47)-(49) remain the same. From (43), (44), and (46),

$$
\begin{align*}
X_{i} & =R_{i \min }=2 R_{\max }  \tag{50}\\
X_{L S \min } & =\frac{X_{i}}{2}=R_{\max } \tag{51}
\end{align*}
$$

From (38) and (50),

$$
\begin{equation*}
n=\pi \sqrt{\frac{R_{i \min }}{8 R_{L \min }}}=\frac{\pi}{2} \sqrt{\frac{R_{\max }}{2 R_{L \min }}} \tag{52}
\end{equation*}
$$

Using (30) and (33), one obtains $R_{\text {max }} / R_{\text {min }}=(1+$ $\left.k^{2}\right) /(2 k)$. For example, at $k=3, R_{\max } / R_{\min }=1.67$; at $k=5, R_{\max } / R_{\min }=2.6$; and at $k=10, R_{\max } / R_{\text {min }}=5$. Thus variations in $R$ are much smaller than those in $R_{L}$.

Example: Design a dc/dc converter with the following specifications: $V_{i}=28 \mathrm{~V}, V_{o}=5 \mathrm{~V}, P_{o \text { max }}=10 \mathrm{~W}$, and $k=R_{L \text { max }} / R_{L \text { min }}=3$.
Let us design the Class E dc/dc converter of Fig. 1(a) operating at the switching frequency $f=2 \mathrm{MHz}$ for optimum operation. From (37), $R_{L \text { min }}=V_{o}^{2} / P_{o \text { max }}=5^{2} / 10$ $=2.5 \Omega . D 2$ and $D 3$ are assumed to be Schottky diodes with forward voltage drops $V_{F}=0.5 \mathrm{~V}$. Hence, the rectifier efficiency $\eta_{R}=1 /\left(1+V_{F} / V_{o}\right)=1 /(1+0.5 / 5)$ $=91$ percent. Assuming that the transformer efficiency is $\eta_{t}=95$ percent and the Class E inverter efficiency is $\eta_{A}$ $=95$ percent, the total converter efficiency is $\eta=\eta_{A} \eta_{t} \eta_{R}$ $=0.95 \times 0.95 \times 0.91=0.82=82$ percent. Hence, the dc input power is $R_{i \text { max }}=P_{o \text { max }} / \eta=10 / 0.82=12.2$ W and the dc input current is $I_{i \text { max }}=P_{i \text { max }} / V_{i}=12.2 / 28$ $=0.44 \mathrm{~A}$. The Class E inverter will be designed for optimum operation at $P_{o \text { max }}$ with $D=0.5$ and $Q_{1}=5$. From [17], $b=V_{i}^{2} R / P_{o}=0.5249, c=\omega C_{1} R=0.2067, d=$ $\omega C R=\omega C_{2} R=0.2269$, and $e=\omega L / R=5.673$. Hence, from (39)-(42), $R_{\max }=b V_{i}^{2} / P_{i \max }=0.5249 \times 28^{2} / 12.2$ $=33.73 \Omega, C_{1}=c /\left(\omega R_{\max }\right)=0.2067 /(2 \times \pi \times 2 \times$ $\left.10^{6} \times 33.73\right)=448 \mathrm{pF}, C_{2}=d /\left(\omega R_{\max }\right)=0.2269 /(2$ $\left.\times \pi \times 2 \times 10^{6} \times 33.73\right)=535 \mathrm{pF}$, and $L=e R_{\text {max }} / \omega$ $=5.673 \times 33.73 /\left(2 \times \pi \times 2 \times 10^{6}\right)=15.23 \mu \mathrm{H}$. Assume that $q_{B}=1$ at $R_{L \text { min }}$. From (50) and (51), $X_{i}=$ $R_{i \text { min }}=2 R_{\text {max }}=2 \times 33.73=67.46 \Omega$ and $X_{L S \text { min }}=R_{\text {max }}$ $=33.73 \Omega$. Thus, $L_{i}=X_{i} / \omega=67.46 /(2 \times \pi \times 2 \times$ $10^{6}$ ) $=5.37 \mu \mathrm{H}$ and $L_{S \text { min }}=L_{i} / 2=2.68 \mu \mathrm{H}$. From (48), $L_{2}=L-L_{S \text { min }}=15.23-2.68=12.55 \mu \mathrm{H}$. From (49), $L_{3}=\pi^{2} L_{i} / 8=\pi^{2} \times 5.37 / 8=6.62 \mu \mathrm{H}$. From (50), $n=\pi \sqrt{R_{i \text { min }} /\left(8 R_{L \text { min }}\right)}=\pi \sqrt{64.46 /(8 \times 2.5)}=5.8$. From [17], $V_{S M}=3.61 V_{i}=3.61 \times 28=101 \mathrm{~V}$ and $I_{S M}$ $=2.783 I_{i \max }=2.783 \times 0.44=1.225 \mathrm{~A}$. The maximum load current is $I_{o \text { max }}=P_{o \text { max }} / V_{o}=10 / 5=2 \mathrm{~A}$. Hence, from (26) at $\phi=45^{\circ}$, the peak currents of the rectifier diodes are $I_{D 2 M}=I_{D 3 M}=1.665 I_{o \text { max }}=1.665 \times 2=$ 3.33 A. From (28), $V_{D 2 M R}=V_{D 3 M R}=-2 \times 5=-10$ V . The values of $V_{S M}, I_{S M}, I_{D 2 M}$, and $I_{D 3 M}$ decrease with $R_{L}$ as will be shown in Section VII. Therefore, their maximum values occur at $R_{L \text { min }}$. The maximum value of the peak voltage across $C_{1}$ is equal to $V_{S M}=101 \mathrm{~V}$. The peak voltage across $L_{3}$ is $n V_{o}=5.8 \times 5=29 \mathrm{~V}$. Thus the peak-to-peak input voltage of the transformer is 58 V . Since $q_{B}=1$ at $R_{L \text { min }}$, (29) gives $\phi=45^{\circ}$. Hence, from (10), $I_{m}=\pi I_{o \text { max }} /(2 n \cos \phi)=\pi \times 2 /(2 \times 5.8 \times \cos$ $\left.45^{\circ}\right)=0.766 \mathrm{~A}$. The reactances of $C_{2}$ and $L_{2}$ are $X_{C 2}=$ $1 / \omega C_{2}=149 \Omega$ and $X_{L 2}=\omega L_{2}=158 \Omega$ and therefore the peak value of the voltage across $C_{2}$ is $X_{C 2} I_{m}=149$ $\times 0.766=114 \mathrm{~V}$ and the peak value of the voltage across $L_{2}$ is $X_{L 2} I_{m}=158 \times 0.766=121 \mathrm{~V}$. These two voltages can be reduced by choosing a smaller value of $Q_{1}$ at $R_{L \text { min }}$. As $R_{L}$ increases, the two voltages decrease because the value of $I_{m}$ decreases with $R_{L}$. The maximum load resistance is $R_{L \text { max }}=3 R_{L \text { min }}=3 \times 2.5=7.5 \Omega$ and therefore $R_{i \text { max }}=3 R_{i \text { min }}=3 \times 67.46=202.38 \Omega$. From (29), $q_{B \text { max }}=R_{i \text { max }} / X_{i}=3 R_{i \text { min }} / X_{i}=3$, from which $\phi=$ $71.56^{\circ}$. Since $k=3, R_{\text {max }} / R_{\text {min }}=\left(1+k^{2}\right) /(2 k)=(1$ $\left.+3^{2}\right) /(2 \times 3)=1.67$, which gives $R_{\text {min }}=R_{\max } / 1.67$ $=33.73 / 1.67=20.2 \Omega$.

## VI. Family of Class E DC/DC Converters with an Inductive Impedance Inverter

Fig. 6 shows a family of the Class $E$ resonant $\mathrm{dc} / \mathrm{dc}$ converters with an impedance inverter. This family can be derived from the basic circuits of Class $\mathrm{E} \mathrm{dc} / \mathrm{dc}$ converters by adding the inductance $L_{3}$. In the converters of Fig. 6(a) and (b), a coupling capacitor $C_{f}$ is also added between the inductor $L_{3}$ or the transformer secondary winding and the half-wave rectifiers for the following reason. Consider the converter of Fig. 6(a). For the continuous mode of operation, diodes $D 2$ and $D 3$ conduct alternately with a conduction duty cycle of 50 percent. Therefore, the input voltage of the rectifier is a square wave whose low level is zero and high level is $V_{o}$. Hence, the average value of the rectifier input voltage is $V_{o} / 2$. On the other hand, the average value of the voltage across $L_{3}$ or the transformer secondary winding is zero. The coupling capacitor $C_{f}$ removes this contradiction. The voltage across $C_{f}$ is $V_{o} / 2$. Consequently, the voltage $v$ is a square wave with peak values $n V_{o} / 2$ and $-n V_{o} / 2$. Thus, the rectifier acts as a voltage doubler. For the converters of Fig. 6(a) and (b),

$$
\begin{align*}
\tan \phi & =\frac{n^{2} R_{L}}{4 X_{L L}}  \tag{53}\\
R_{i} & =\frac{2 n^{2}}{\pi^{2}} R_{L} \approx \frac{n^{2} R_{L}}{5}  \tag{54}\\
M_{R} & =\sqrt{\frac{R_{L}}{R_{i}}}=\frac{\pi}{n \sqrt{2}} \approx \frac{2.22}{n} . \tag{55}
\end{align*}
$$

For optimum operation at $D=0.5$ and $Q_{1}=5, M=$ $1.609 \sqrt{\eta} \sqrt{1+q_{B}^{2}} / n$; at $q_{B}=1, M=2.276 \sqrt{\eta} / n$. The continuous mode of operation occurs for $R_{L}<2 \pi X_{L 3} / n^{2}$.

For the converter of Fig. 6(c),

$$
\begin{align*}
\tan \phi & =\frac{n^{2} R_{L}}{2 X_{L 3}}  \tag{56}\\
R_{i} & =\frac{4 n^{2}}{\pi^{2}} R_{L} \approx \frac{n^{2} R_{L}}{2.5}  \tag{57}\\
M_{R} & =\sqrt{\frac{R_{L}}{R_{i}}}=\frac{\pi}{2 n} \approx \frac{1.57}{n} . \tag{58}
\end{align*}
$$

For optimum operation at $D=0.5$ and $Q_{1}=5, M=$ $1.138 \sqrt{\eta} \sqrt{1+q_{B}^{2}} / n$; at $q_{B}=1, M=1.609 \sqrt{\eta} / n$. The continuous mode of operation occurs for $R_{L}<\pi X_{L 3} / n^{2}$. In addition, $I_{D M} / I_{o}$ of the converters in Fig. 6(a)-(c) is twice that of the converter in Fig. 1(a). For the converter of Fig. 6(e), $R_{i}$ is given by (17). All design equations given in Section $V$ are the same for all converters of Fig. 6. The only difference is that (38) should be replaced by (53) for the converters of Fig. 6(a) and (b), and by (54) for the converter of Fig. 6(c).


Fig. 6. Family of Class E resonant dc/dc converters with an inductive impedance inverter.

## VII. Experimental Results

To demonstrate the feasibility of the Class E dc/dc converter with an inductive impedance inverter, the transformerless converter of Fig. 6(a) was designed and built, using an IRF620 MOSFET, IR31DQ06 diodes, $L_{1}=100$ $\mu \mathrm{H}, C_{1}=1.35 \mathrm{nF}, C_{2}=1.8 \mathrm{nF}, L_{2}=15 \mu \mathrm{H}, L_{3}=7.85$ $\mu \mathrm{H}, C_{f}=66 \mathrm{nF}, V_{i}=28 \mathrm{~V}$, and for optimum operation $D=0.5, Q_{1}=5, f=1 \mathrm{MHz}$, and $R_{L \text { min }}=200 \Omega$. The measured converter parameters for optimum operation were: $V_{o}=55 \mathrm{~V}, P_{o}=15.1 \mathrm{~W}, \eta=89.3$ percent, and $M=55 / 28=1.964$. Using the measured efficiency $\eta$, the theoretical value of the dc-to-dc voltage transfer function was $M=2.276 \sqrt{0.893}=2.15$. Thus, the error was 9.47 percent. Next, the load resistance $R_{L}$ was gradually increased from $R_{L}=200 \Omega$ to infinity and the switching frequency $f$ was also increased so that the dc output voltage $V_{o}$ was held constant at 55 V . The switching frequency $f$ at an open circuit at the output was 1.12 MHz . The relative bandwidth required to maintain $V_{o}$ at 55 V over a wide range of $R_{L}$ (from $200 \Omega$ to infinity) was only 12 percent. Thus, the circuit is a narrow-band FM converter. Fig. 7 shows the current and voltage waveforms at $R_{L \text { min }}=200 \Omega$ (optimum operation), $R_{L}=300 \Omega$ (the continuous mode of operation), and $R_{L}=3 \mathrm{k} \Omega$ (the discontinuous mode of operation). As seen, the experimental waveforms were close to those predicted theoretically. The peak values of all the currents and voltages decreased with $R_{L}$; the only exception was the peak-to-peak value of


Fig. 7. Experimental current and voltage waveforms in converter of Fig. 6(a). (a) For optimum operation ( $R_{l}=200 \Omega$ ). (b) For continuous mode of operation ( $R_{L}=300 \Omega$ ). (c) For discontinuous mode of operation ( $R_{L}=3 \mathrm{k} \Omega$ ). Vertical: $1 \mathrm{~A} / \mathrm{div}$. for all currents, $20 \mathrm{~V} / \mathrm{div}$. for $v_{s}$ and $v$, and $1 \mathrm{~V} / \mathrm{div}$. for ac component of $V_{o}$; horizontal: $200 \mathrm{~ns} / \mathrm{div}$.



Fig. 9. Peak values of switch voltage $V_{S M}$ as a function of $R_{l}$ at constant values of $V_{o}$.


Fig. 10. Peak values of switch current $I_{S M}$ as a function of $R_{L}$ at constant values of $\boldsymbol{V}_{o}$


Fig. 11. Waveforms of drain current $i_{S}$ and drain-to-source voltage $v_{S}$ at various values of $R_{L}$. (a) For $V_{o}=37 \mathrm{~V}$. (b) For $V_{o}=55 \mathrm{~V}$. (c) For $V_{o}$ $=80 \mathrm{~V}$. Vertical: 1 A and $20 \mathrm{~V} /$ div.; horizontal: $200 \mathrm{~ns} /$ div.


Fig. 12. Waveforms of $i_{S}$ and $v_{S}$ illustrating power losses in MOSFET for optimum operation at $D=0.5, f=1 \mathrm{MHz}, R_{L \text { min }}=200 \Omega$, and $V_{n}=$ 55 V . (a) $i_{S}$ and $v_{s}\left(1 \mathrm{~A} .20 \mathrm{~V}\right.$, and $200 \mathrm{~ns} /$ div.) (b) $v_{S}$ and $i_{S}$ illustrating conduction loss ( 1 V .1 A , and $200 \mathrm{~ns} / \mathrm{div}$.). (c) $v_{S}$ and $i_{S}$ at turn-on ( $20 \mathrm{~V}, 0.5 \mathrm{~A}$, and $20 \mathrm{~ns} / \mathrm{div}$.). (d) $i_{S}$ and $v_{S}$ at turn-off ( 0.5 A , 20 V , and $20 \mathrm{~ns} / \mathrm{div}$.).
$v$ which was constant. For optimum operation and the continuous mode of operation, the waveforms were approximately free of parasitic oscillations. For the discontinuous mode of operation, voltage $v$ differs from a square wave and current $i_{L}$ differs from a triangle wave. Nevertheless, it was easy to maintain $V_{o}$ at 55 V . The maximum value of the peak-to-peak ripple voltage was $V_{r \text { max }}=0.2$ V ; hence $V_{r \text { max }} / V_{o}=0.36$ percent. The output voltage was very smooth, without spikes and noise.

Fig. 8 depicts the switching frequency $f$ as a function of the load resistance $R_{L}$ at constant values of $V_{0}$. The characteristic at $V_{o}=55 \mathrm{~V}$ corresponds to $D=0.5, f=$ 1 MHz , and $R_{L \text { min }}=200 \Omega$ for optimum operation, as previously described. The characteristic at $V_{o}=37 \mathrm{~V}$ corresponds to $D=0.44, f=1.15 \mathrm{MHz}$, and $R_{L \text { min }}=460$ $\Omega$ for optimum operation. The characteristic at $V_{o}=80$ V corresponds to $D=0.61, f=0.949 \mathrm{MHz}$, and $R_{L \text { min }}$ $=265 \Omega$ for optimum operation (in this case, $D 2$ and $D 3$ were composed of two diodes connected in series). All other components were the same as at $V_{o}=55 \mathrm{~V}$. It can be seen that different values $V_{o}$ can be obtained by varying


Fig. 13. Waveforms of rectifier diodes at $f=1 \mathrm{MHz}, R_{I \text { min }}=200 \Omega$, and $V_{0}=55 \mathrm{~V}$. (a) $i_{D 2}, v_{D 2}, i_{D 3}, v_{D 3}(1 \mathrm{~A}, 20 \mathrm{~V}$, and $200 \mathrm{~ns} /$ div. $)$. (b) Waveforms illustrating conduction losses ( $1 \mathrm{~V}, 1 \mathrm{~A}$, and $200 \mathrm{~ns} /$ div.). (c) Waveforms illustrating turn-on switching losses ( $10 \mathrm{~V}, 0.1 \mathrm{~A}$, and $20 \mathrm{~ns} /$ div.). (d) Waveforms illustrating turn-off losses ( $10 \mathrm{~V}, 0.1 \mathrm{~A}$, and $20 \mathrm{~ns} /$ div.).
the value of $D$ and $f$ for optimum operation. The values of the dc voltage transfer function $M=V_{o} / V_{i}$ were 1.32 , 1.96 , and 2.86 at $V_{o}=37,55$, and 80 V , respectively. Figs. 9 and 10 show the peak values of the switch voltage $V_{S M}$ and the switch current $I_{S M}$ versus $R_{L}$ at constant values of $V_{o}$. As shown, both $V_{S M}$ and $I_{S M}$ decreased with $R_{L}$. Fig. 11 shows the waveforms of $i_{S}$ and $v_{S}$ at various values of $R_{L}$ for $V_{o}=37,55$, and 80 V . As seen, the lossless operation (i.e., zero-voltage-switching) was obtained in all three cases for the entire range of $R_{L}$, from full-load resistance $R_{L \min }$ to infinity ( $R_{L}=\infty$ ). The switching frequency $f$ was varied from 1.15 to 1.21 MHz at $V_{o}=37$ V and from 0.949 to 1.03 MHz at $V_{o}=80 \mathrm{~V}$. Thus, the values of $\Delta f / f_{\text {opt }}$ were 5.2 and 8.5 percent, respectively. The peak values of the voltages across $L_{2}$ and $C_{2}$ decreased with $R_{L}$. For instance, at $V_{o}=55 \mathrm{~V}$, the peak value of the voltage across $L_{2}$ decreased from 129 to 71 V and the peak value of the voltage across $C_{2}$ decreased from 102 to 56 V as $R_{L}$ was increased from $200 \Omega$ to infinity. The highest peak values of all currents and voltages occur at $R_{L \text { min }}$.

Fig. 12 shows the waveforms of $i_{S}$ and $v_{S}$ in the enlarged scales to illustrate power losses in the MOSFET. The waveforms in Fig. 12(b) illustrate the conduction loss. The peak value of $v_{S}$ was 1 V when the transistor was on. Hence, the transistor on-resistance was $r_{D S}=$ $1 \mathrm{~V} / 2 \mathrm{~A}=0.5 \Omega$. The dc input current was 0.604 A . Thus, the conduction loss was $P_{C D S}=2.37 r_{D S} I_{i}^{2}=0.432$ W . The waveforms of $v_{S}$ and $i_{S}$ at turn-on are shown in Fig. 12(c). The turn-on switching loss was zero because $i_{S}$ and $v_{S}$ were not overlapping at turn-on. Fig. 12(d) shows the waveforms of $i_{S}$ and $v_{S}$ at turn-off. When the voltage $v_{S}$ begins to increase, the current $i_{S}$ decreases from $2 I_{i}$ to zero. The transition time interval was 20 ns , which was 2 percent of the period $T=1 / f=1000 \mathrm{~ns}$. The turnoff switching loss was nonzero because $i_{S}$ and $v_{S}$ were overlapping during turn-off [11].
Fig. 13 shows the waveforms of the rectifier diodes in the enlarged scales to illustrate power losses in these diodes. Fig. 13(b) illustrates the conduction losses. The forward diode voltages were approximately $V_{F}=0.5 \mathrm{~V}$. The dc output current was $I_{o}=0.275 \mathrm{~A}$. Hence, the conduction loss in each diode was $P_{C D R}=V_{F} I_{o} / 2=69 \mathrm{~mW}$. The waveforms during turn-on are shown in Fig. 13(c). As $v_{D 2}$ and $v_{D 3}$ increase from -55 to $0.5 \mathrm{~V}, i_{D 2}$ increases from zero to 0.3 A and $i_{D 3}$ increases from zero to 0.2 A . The transition time intervals were 50 ns for both diodes. Fig. 13(d) shows the waveforms during turn-off. The diode voltages begin to decrease from 0.5 to -55 V as the diode currents reach zero. Next, the diode currents are negative during the reverse-recovery time.

## VIII. Conclusion

The analysis, design procedure, and experimental results have been presented for the Class E resonant $\mathrm{dc} / \mathrm{dc}$ converters with an inductive impedance inverter. It is shown that lossless converter operation can be obtained over a wide range of the load resistance, from zero to infinity. However, the practical range of $R_{L}$ is from $R_{L \text { min }}$ $=V_{o}^{2} / P_{o \text { max }}$ to infinity. The bandwidth of the switching frequency $f$ required to regulate the dc output voltage $V_{o}$ is very narrow. The measured values of $\Delta f / f_{\text {opt }}$ for $R_{L \text { min }}$ $\leq R_{L} \leq \infty$ were $5.2,12$, and 8.5 percent at $V_{o}=37,55$, and 80 V , respectively. Therefore, the Class E converters are narrow-band frequency-modulated (NBFM) converters. The same rules can also be applied in other resonant converters, e.g., in converters derived from Class D tuned power amplifiers. The Class E dc/dc converters may be improved by using resonant rectifiers proposed in [30], [33].

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[^0]:    Manuscript received February 4, 1988; revised July 20, 1988. This work was supported under the Ohio State Research Challenge Grant, No. 660763.
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    IEEE Log Number 8824131

