Method For Calculating Output Voltage Tolerances in Adjustable Regulators

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Introduction

When working with voltage regulator circuits, the designer is often confronted with the need to calculate the tolerance of the regulated output voltage. For fixed voltage regulators this problem is easily managed because the required information is directly supplied on the semiconductor manufacturer's datasheet.

The tolerancing of adjustable regulators can be more complicated because of the introduction of an external feedback resistor network, effects of adjust pin current, and the difficulties associated with combining these terms to obtain an overall estimate of output voltage tolerance.

Traditional "worst case" analysis methods, although valid, result in unrealistic and excessively conservative estimates of the total tolerances leading to unnecessary added circuit costs.

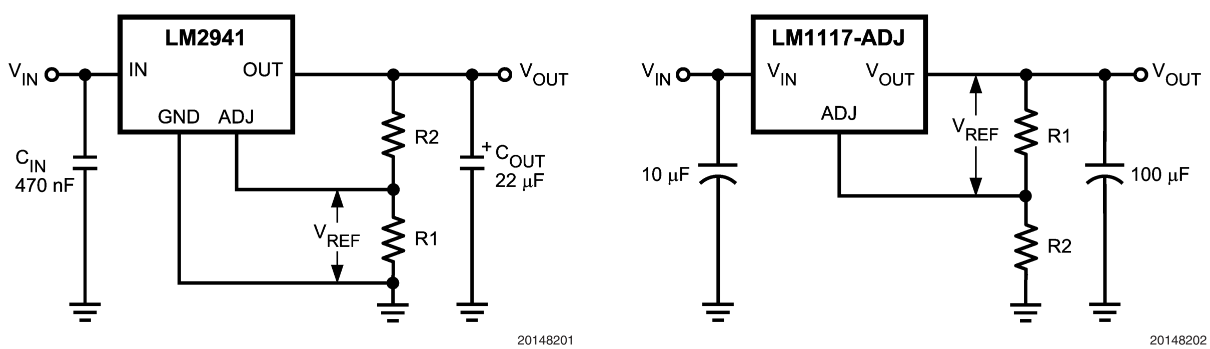
Reduced voltages for modern microprocessors are further increasing the demands on available voltage tolerances. As such, a more detailed understanding of the tolerancing problem is needed.

The Commonly Encountered Circuits

The most commonly encountered adjustable regulator circuits are shown in *Figure 1*. We will examine these circuits and the statistical effects on V_{out} that are caused by variations in R1, R2, and V_{adj}. Be aware that regulators without ground pins (the LM317 and LM1117 for example) have the same mathematical relationship between V_{out} and V_{adj}, however, V_{ref} is referenced to ground in one case and to the output pin the other. Reversing the positions of R1 and R2 makes the mathematical relationship for both circuits identical (equation 1). Be aware that many datasheets are not consistent in the identification of R1 and R2 which further compounds confusion regarding the correct relationship.

For the circuits in *Figure 1*, V_{out} is related to V_{ref}:

$$V_{out} = V_{ref} \times \frac{R_1 + R_2}{R_1} \quad (1)$$



Ground Referenced **Not Ground Referenced**

FIGURE 1. Adjustable Regulators: Common Topologies

The Worst Case Approach

The overly conservative approach to this problem is to take the known relationship between reference voltage tolerances and output voltage and calculate the worst case output voltage that can occur in the unlikely case that all tolerances are simultaneously at their worst case extremes. Remember that exact values for the desired R1 and R2 may not be available so an additional, but static, voltage error may have to be tolerated.

Equations 2 and 3 are commonly applied using plus and minus 1% for values of R_{min} and R_{max}. The drawback of this approach is that it results in excessively conservative tolerance limits. We will show later that this is especially true since V_{ref}, R1, and R2 are uncorrelated random variables.

Results for the worst case approach are tabulated in *Table 1*. The minimum and maximum deviations are not exactly symmetric so only the worst case is tabulated. The worst case always occurs on the maximum side. Centering of the nominal value at the actual mean can take advantage of this fact and gain a small improvement in worst case tolerancing.

$$V_{out\_max} = V_{ref\_max} \times \frac{R_{1\_min} + R_{2\_max}}{R_{1\_min}} \quad (2)$$

$$V_{out\_min} = V_{ref\_min} \times \frac{R_{1\_max} + R_{2\_min}}{R_{1\_max}} \quad (3)$$

TABLE 1. Values of Worst Case Error [%] for Common V_{out}'s (V_{ref} = 1.275 V)

V_{out} = 1.8 V				V_{out} = 2.5 V			
	Resistor Tolerance				Resistor Tolerance		
	0.5%	1%	5%		0.5%	1%	5%
ΔV_{ref} [%]	ΔV_{out} [%]	ΔV_{out} [%]	ΔV_{out} [%]	ΔV_{ref} [%]	ΔV_{out} [%]	ΔV_{out} [%]	ΔV_{out} [%]
± 0.5	0.82	1.14	3.85	**± 0.5**	1.01	1.53	5.87
± 1.0	1.32	1.65	4.37	**± 1.0**	1.52	2.04	6.40
± 2.0	2.32	2.65	5.40	**± 2.0**	2.52	3.05	7.45
± 5.0	5.33	5.67	8.50	**± 5.0**	5.54	6.08	10.61

V_{out} = 3.3 V				V_{out} = 5.0 V			
	Resistor Tolerance				Resistor Tolerance		
	0.5%	1%	5%		0.5%	1%	5%
ΔV_{ref} [%]	ΔV_{out} [%]	ΔV_{out} [%]	ΔV_{out} [%]	ΔV_{ref} [%]	ΔV_{out} [%]	ΔV_{out} [%]	ΔV_{out} [%]
± 0.5	1.13	1.77	7.14	**± 0.5**	1.26	2.03	8.48
± 1.0	1.64	2.28	7.67	**± 1.0**	1.77	2.54	9.02
± 2.0	2.64	3.29	8.73	**± 2.0**	2.77	3.55	10.10
± 5.0	2.64	6.33	11.93	**± 5.0**	5.80	6.60	13.33

V_{out} = 12 V				V_{out} = 15 V			
	Resistor Tolerance				Resistor Tolerance		
	0.5%	1%	5%		0.5%	1%	5%
ΔV_{ref} [%]	ΔV_{out} [%]	ΔV_{out} [%]	ΔV_{out} [%]	ΔV_{ref} [%]	ΔV_{out} [%]	ΔV_{out} [%]	ΔV_{out} [%]
± 0.5	1.41	2.32	9.99	**± 0.5**	1.43	2.36	10.21
± 1.0	1.91	2.83	10.54	**± 1.0**	1.93	2.87	10.76
± 2.0	2.92	3.85	11.64	**± 2.0**	2.94	3.89	11.86
± 5.0	5.95	6.90	14.92	**± 5.0**	5.97	6.95	15.15

Sensitivity Analysis

So how do small changes in R1, R2, and V_{out} translate to the output voltage? Sensitivity analysis reveals the underlying nature of the circuit.

Taking the partial derivatives of equation 1 with respect to each of its variables lets us calculate the sensitivity of V_{out} to small changes in each variable. This is done by dividing the partial derviatives by V_{out}, then substituting equation 1 back into the equation, and finally solving for the fractional change of V_{out} with respect to the fractional change in each of the three variables:

$$\frac{\delta V_{out}}{V_{out}} = 1\left(\frac{\delta V_{ref}}{V_{ref}}\right) \quad \text{therefore: } S^{V_{out}}_{V_{ref}} = 1 \tag{4}$$

$$\frac{\delta V_{out}}{V_{out}} = \frac{-R_2}{R_1 + R_2}\left(\frac{\delta R_1}{R_1}\right) \quad \text{therefore: } S^{V_{out}}_{R1} = \frac{-R_2}{R_1 + R_2} \tag{5}$$

$$\frac{\delta V_{out}}{V_{out}} = \frac{R_2}{R_1 + R_2}\left(\frac{\delta R_2}{R_2}\right) \quad \text{therefore: } S^{V_{out}}_{R2} = \frac{R_2}{R_1 + R_2} \tag{6}$$

The result from equation 4 is obvious. That is, variation in V_{ref} translate directly to variations in V_{out}. Equations 5 and 6 are a bit more interesting. These show that variations in the voltage setting resistors R1 and R2 will translate to the output with a sensitivity ranging from zero to one. The highest sensitivity to resistor variation occurs when output voltages are high and lowest when the output voltage equals the reference voltage (*Figure 2*).

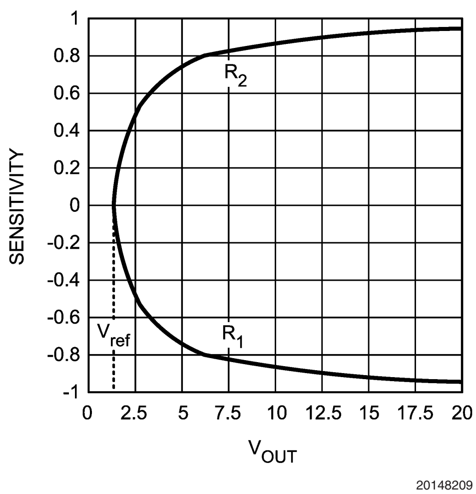


FIGURE 2. – Sensitivity to Resistor Variations vs.V_{out}

Review of Random Variable Mathematics

A few glances at *Table 1* and it becomes clear that, for adjustable regulators, the worst case deviations of V_{out} can be quite large. For example, look at the case where V_{out} = 3.3 V, ΔV_{ref} = ±1%, and ΔR = ±1%. For this case, the total output worst case error could be as high as ΔV_{out} = ±2.28%! We can show that this number, although conservative, is a gross exaggeration of the true variation in V_{out}. This is because V_{ref}, R1, and R2 are all independent random variables. Some argue that R1 and R2 may not be independent random variables. This is especially true if they are fed from the same reel or supply bin. There is some truth to this, however, R1 and R2 are rarely the same value and even if they are, their sensitivities (equations 5 and 6) have equal magnitudes and opposite polarities so any correlation would tend to cancel rather than add!

Statistical Variation of Resistors, Semiconductors, and Systems

To calculate actual variation, we will need to make some assumptions about the statistics of V_{ref} and the resistors we buy. This information may be available from the vendor, however, in many cases, the vendor may be reluctant to release this data.

For fundamental electronic components like resistors it is reasonable to assume that these are produced under a "six sigma" paradigm and have Gaussian variation. Variations in V_{ref} are also approximately Gaussian (*Figure 3*). Distribution data for a typical linear regulator is shown in *Figure 3* and has variation against room temperature specifications on the order of ±6σ. Variation against the full temperature range specification is even more impressive and can be as high as ±10σ (to accommodate variations with temperature).

Because components like regulators and chip resistors are made in very high volumes, tight process control is no less than mandatory.

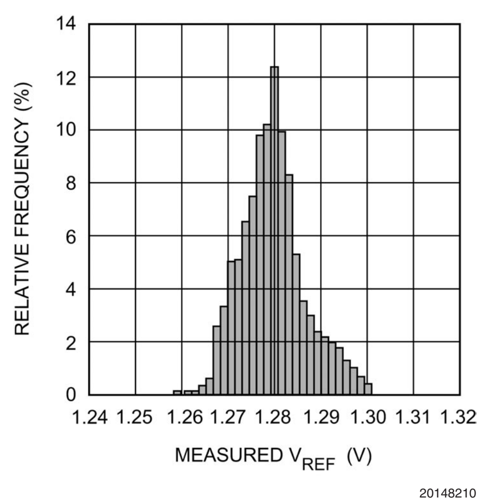


FIGURE 3. Typical V_{ref} Variation

Statistical Variation of Resistors, Semiconductors, and Systems

(Continued)

A similar histogram supporting our assumption for 0805 general purpose surface mount resistors is shown in *Figure 4*. The nominal value for a 1% resistor is shown here controlled to greater than ±6σ and is also approximately Gaussian.

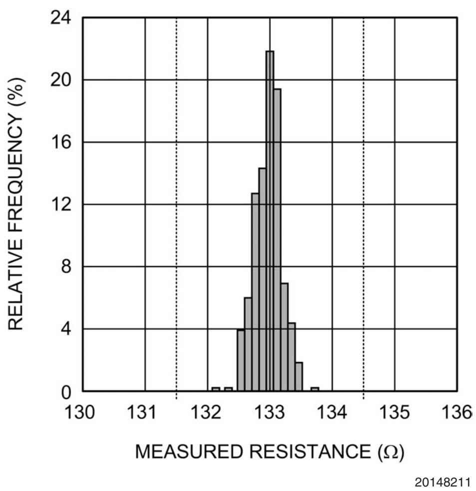


FIGURE 4. Typical Resistor Variation: 1% General Purpose 0805 SMD

Note: *Figure 3* and *Figure 4* are presented to support the assumption that V_{ref} and the voltage setting resistors are Gaussian and have variations on the order of six sigma or better.

The RSS Method

The RSS (root sum squares) method is only valid for the case when independent Gaussian random variables are combined as sums. Since equation 1 contains products, quotients and sums of random variables, this method is not valid for equation 1.

Random Variable Theory vs. Worst Case Over PVT

In the worst case method, we saw that total output voltage tolerances could be substantially larger than expected (*Table 1*). Is it realistic to use these limits? Random variable theory can be used to show that these excessive limits are not necessary. In particular, the concept of "worst case" is not exactly appropriate when dealing with random processes where there is always a (very small) probability that a sample could fall outside of the worst case limits.

Rather than look at worst case limits, it is more appropriate to look at the equivalent ±6σ points for the regulator's output voltage. For this purpose, let's look again at the output voltage equation.

$$V_{out} = V_{ref} \times \frac{R_1 + R_2}{R_1}$$

Be reminded that V_{ref}, R1, and R2 are all independent random variables. As such, V_{out} is a function of three random variables. To complicate matters, summation, multiplication, and division are all involved. Although summation of two Gaussian random variables produces a Gaussian result, this is not the case for multiplication or division. As such, the true distribution of V_{out} could be quite complicated. Fortunately, approximations exist for calculating the mean value and deviation for sums, products, and quotients. These approximations are especially accurate for the case where V(x) << E(x) which is the case for linear regulators and resistors. In particular, consider these relationships:

$$E(x) = \text{mean}(x)$$

$$V(x) = \text{variance}(x) = \sigma^2$$

For uncorrelated Gaussian random variables, the following relationships apply:

Gaussian Random Variable Operations

Operation	Mean and Variance	Resulting Distribution
SUM	$E(x + y) = E(x) + E(y)$	Gaussian
	$V(x + y) = V(x) + V(y)$	
PRODUCT	$E(x \bullet y) = E(x) \bullet E(y)$	Gaussian & Modified Bessel
	$V(x \bullet y) = E(x)^2 \times V(y) + E(y)^2 \times V(x) + V(x) \times V(y)$	
QUOTIENT	$E\left[\frac{x}{y}\right] \approx \frac{E(x)}{E(y)} + \frac{E(x) \times V(y)}{E(y)^3}$	Cauchy for zero mean x and y
	$V\left[\frac{x}{y}\right] \approx \left[\frac{E(x)}{E(y)}\right]^2 \times \left[\frac{V(x)}{E(x)^2} + \frac{V(y)}{E(y)^2}\right]$	

Notice that the resulting distribution after these operations is not always Gaussian. It is possible to calculate the distribution function for the resulting random variable, however, this is quite complicated and unnecessary since we are only interested in the mean and variance of the result. Since V(x) << E(x), the resulting distribution will somewhat resemble a Gaussian distribution so our Gaussian based SPC (Statistical Process Control) concepts will still be valid.

Since equation 1 involves a sum, product, and quotient, we cannot use the relationships above and, instead, must calculate a specific approximating equation for $E(V_{out})$ and $V(V_{out})$ using these relationships.

$$E(V_{out}) \approx V_{out} + \frac{1}{2} \times V[R_1] \times \frac{\delta^2 V_{out}}{\delta R_1^2} + \frac{1}{2} \times V[R_2] \times \frac{\delta^2 V_{out}}{\delta R_2^2} + \frac{1}{2} \times V[V_{ref}] \times \frac{\delta^2 V_{out}}{\delta V_{ref}^2} \tag{7}$$

$$V(V_{out}) \approx V[R_1] \times \left[\frac{\delta V_{out}}{\delta R_1}\right]^2 + V[R_2] \times \left[\frac{\delta V_{out}}{\delta R_2}\right]^2 + V[V_{ref}] \times \left[\frac{\delta V_{out}}{\delta V_{ref}}\right]^2 \tag{8}$$

Equations 7 and 8 originate from Taylor series expansion (see Mood, Graybill, and Boes).

Notice that the expected value for V_{out} will be slightly different than the simple value calculated in equation 1. The third and fourth terms in equation 7 are zero. However, there is a very small positive error caused by the second term which is not exactly zero. Since we are working with six sigma processes, it is easy to show that this second term is virtually zero and the expected value of V_{out} is essentially as calculated with equation 1.

To evaluate equations 7 and 8, we will need the following partial derivatives of equation 1.

Random Variable Theory vs. Worst Case Over PVT (Continued)

Partial Derivatives of Equation 1

$\frac{\delta V_{out}}{\delta R_1} = -\frac{R_2}{R_1^2} \times V_{ref}$	$\frac{\delta V_{out}}{\delta R_2} = \frac{1}{R_1} \times V_{ref}$	$\frac{\delta V_{out}}{\delta V_{ref}} = \frac{R_1 + R_2}{R_1}$
$\frac{\delta^2 V_{out}}{\delta R_1^2} = \frac{2 \times R_2}{R_1^3} \times V_{ref}$	$\frac{\delta^2 V_{out}}{\delta R_2^2} = 0$	$\frac{\delta^2 V_{out}}{\delta V_{ref}^2} = 0$

Substituting into equations 7 and 8:

$$E(V_{out}) \cong V_{out} + V[R_1] \times R_2 / R_1^3 \times V_{ref} \qquad (9)$$

$$V(V_{out}) \approx V[R_1] \times \left[\frac{R_2}{R_1^2} \times Vref\right]^2 + V[R_2] \times \left[\frac{V_{ref}}{R_1}\right]^2 + V[V_{ref}] \times \left[\frac{R_1 + R_2}{R_1}\right]^2 \qquad (10)$$

Using equation 10, the six-sigma based error for common V_{out}'s is tabulated in *Table 2*. Notice that the results using this method are far more practical than those obtained with the worst case method **and a six sigma paradigm is still assured for the resulting output voltage.**

Values for $E(V_{out})$ are not tabulated because the difference from the calculated value of V_{out} is very small. The worst case for this error occurs at high output voltages. For example, with V_{out} = 15 V, R1 = 10 kΩ, R2 = 107 kΩ, and V_{ref} = 1.275 V the expected value of the output voltage will only be 40 µV higher than predicted with equation 1. This is an error of only 0.000254%. As such, this error predicted by equation 9 is ignored.

TABLE 2. Equivalent Six Sigma Output Tolerance for V_{out} [%] for Common V_{out}'s (V_{ref} = 1.275 V)

V_{out} = 1.8 V				V_{out} = 2.5 V			
	Resistor Tolerance				Resistor Tolerance		
	0.5%	1%	5%		0.5%	1%	5%
ΔV_{ref} [%]	ΔV_{out} [%]	ΔV_{out} [%]	ΔV_{out} [%]	ΔV_{ref} [%]	ΔV_{out} [%]	ΔV_{out} [%]	ΔV_{out} [%]
± 0.5	0.54	0.65	2.12	**± 0.5**	0.61	0.85	3.50
± 1.0	1.02	1.08	2.29	**± 1.0**	1.06	1.22	3.61
± 2.0	2.01	2.04	2.87	**± 2.0**	2.03	2.12	4.00
± 5.0	5.00	5.02	5.41	**± 5.0**	5.01	5.05	6.08

V_{out} = 3.3 V				V_{out} = 5.0 V			
	Resistor Tolerance				Resistor Tolerance		
	0.5%	1%	5%		0.5%	1%	5%
ΔV_{ref} [%]	ΔV_{out} [%]	ΔV_{out} [%]	ΔV_{out} [%]	ΔV_{ref} [%]	ΔV_{out} [%]	ΔV_{out} [%]	ΔV_{out} [%]
± 0.5	0.66	1.00	4.37	**± 0.5**	0.73	1.17	5.29
± 1.0	1.09	1.32	4.45	**± 1.0**	1.13	1.45	5.36
± 2.0	2.05	2.18	4.78	**± 2.0**	2.07	2.26	5.63
± 5.0	5.02	5.07	6.62	**± 5.0**	5.03	5.11	7.26

V_{out} = 12 V				V_{out} = 15 V			
	Resistor Tolerance				Resistor Tolerance		
	0.5%	1%	5%		0.5%	1%	5%
ΔV_{ref} [%]	ΔV_{out} [%]	ΔV_{out} [%]	ΔV_{out} [%]	ΔV_{ref} [%]	ΔV_{out} [%]	ΔV_{out} [%]	ΔV_{out} [%]
± 0.5	0.81	1.36	6.34	**± 0.5**	0.82	1.39	6.49
± 1.0	1.18	1.61	6.40	**± 1.0**	1.19	1.64	6.55
± 2.0	2.10	2.37	6.63	**± 2.0**	2.10	2.38	6.77
± 5.0	5.04	5.16	8.06	**± 5.0**	5.04	5.16	8.18

Errors Caused by Adjust Pin Current

For all adjustable regulators, there is always a small amount of current that flows at the adjust pin. Ideally, this current would be zero. For many parts, this current is very low and is not specified. For a bipolar part like the LM1117-ADJ, the adjust pin current is typically 60 µA. For the LM2941 it is about 5 µA. For CMOS regulators, the adjust pin current is much less of a concern and is usually 100 nA or less.

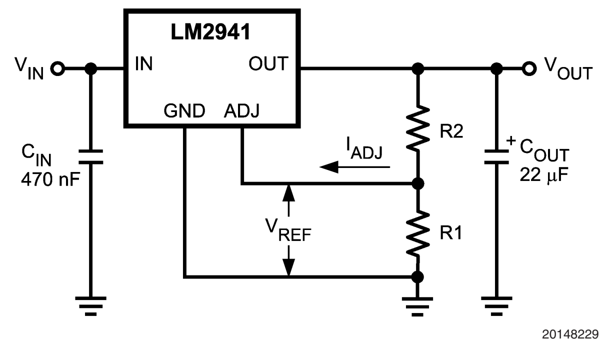
Equation 1 can be modified to include the effects of the unwanted adjust pin current.

$$V_{out} = V_{ref} \times \frac{R_1 + R_2}{R_1} + I_{adj} \times R_2 \tag{11}$$

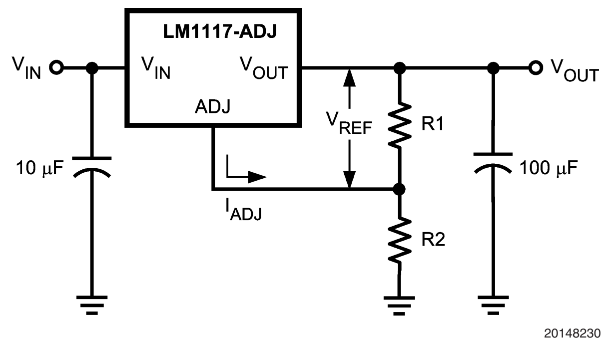
This equation applies to both circuits. For both circuits, the polarity of I_{adj} is positive, however, the direction of flow is as defined in *Figure 5*. Again be reminded that the location of R1, R2 and the V_{ref} is different for the two circuits, however, the resulting equation 11 is the same. For both circuits, the adjust pin current will cause positive errors in V_{out}. If the polarity of the adjust pin current is in question, some data sheets contain a transistor level "equivalent circuit diagram." The actual bias current polarity can usually be determined from the equivalent circuit diagram by examining the polarity of the transistor junction at the adjust pin. Depending upon how much adjust pin current error is tolerable, it is possible to calculate the largest sensible value for R2 (equation 12 and *Table 3*).

$$R_{2\_MAX} = \frac{\%\ \text{Error}}{100} \times \frac{V_{out}}{I_{adj}} \tag{12}$$

For most circuits, a small value for R2 is not a problem. For battery powered circuits, the wasted current flowing through feedback resistors R1 and R2 may become an issue. If this is the case then it will be desirable to select the largest reasonable value for R2.



Ground Referenced



Not Ground Referenced

FIGURE 5. Adjust Pin Currents (Arrow Indicates Actual Direction of Current Flow)

Errors Caused by Adjust Pin Current (Continued)

TABLE 3. Largest Sensible Values of R2 for Various I_{adj} and V_{out}

For No More Than **0.1%** Additional Error Select R_2 Less Than:						
	V_{out}					
I_{adj} (µA)	**1.8 V**	**2.5 V**	**3.3 V**	**5.0 V**	**12.0 V**	**15.0 V**
0.01	180 kΩ	250 kΩ	330 kΩ	500 kΩ	1.2 MΩ*	1.5 MΩ*
0.1	18 kΩ	25 kΩ	33 kΩ	50 kΩ	120 kΩ	150 kΩ
1	1.8 kΩ	2.5 kΩ	3.3 kΩ	5 kΩ	12 kΩ	15 kΩ
10	180 Ω	250 Ω	330 Ω	500 Ω	1.2 kΩ	1.5 kΩ

For No More Than **0.5%** Additional Error Select R_2 Less Than:						
	V_{out}					
I_{adj} (µA)	**1.8 V**	**2.5 V**	**3.3 V**	**5.0 V**	**12.0 V**	**15.0 V**
0.01	900 kΩ	1.25 MΩ*	1.65 MΩ*	2.5 MΩ	6.0 MΩ*	7.5 MΩ*
0.1	90 kΩ	125 kΩ	165 kΩ	250 kΩ	600 kΩ	750 kΩ
1	9 kΩ	12.5 kΩ	16.5 kΩ	25 kΩ	60 kΩ	75 kΩ
10	900 Ω	1.25 kΩ	1.65 kΩ	2.5 kΩ	6 kΩ	7.5 kΩ

For No More Than **1%** Additional Error Select R_2 Less Than:						
	V_{out}					
I_{adj} (µA)	**1.8 V**	**2.5 V**	**3.3 V**	**5.0 V**	**12.0 V**	**15.0 V**
0.01	1.8 MΩ*	2.5 MΩ*	3.3 MΩ*	5.0 MΩ*	12.0 MΩ*	15.0 MΩ*
0.1	180 kΩ	250 kΩ	330 kΩ	500 kΩ	1.2 MΩ*	1.5 MΩ*
1	18 kΩ	25 kΩ	33 kΩ	50 kΩ	120 kΩ	150 kΩ
10	1.8 kΩ	2.5 kΩ	3.3 kΩ	5 kΩ	12 kΩ	15 kΩ

*Values of R2 greater than 1 MΩ may be inappropriate because of the difficultly associated with maintaining high impedances with surface mount resistors. In particular, ionic PC board contaminants may limit the highest attainable on-board resistance figures that can be reliably maintained.

Conclusion

The presented method for calculating voltage tolerances in adjustable regulators results in substantial improvements in the available output voltage tolerance while maintaining tight process control and a six sigma paradigm.

Complete understanding of the commonly used methods for combining tolerances and sources of error is the only way to get the most from any design.

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