Faculty of Computing,
Engineering & Technology

Continuous-Time Signals & Systems

Signal Processing
SP (CE00039-2)

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Processes, Symbols & Functions - Contents

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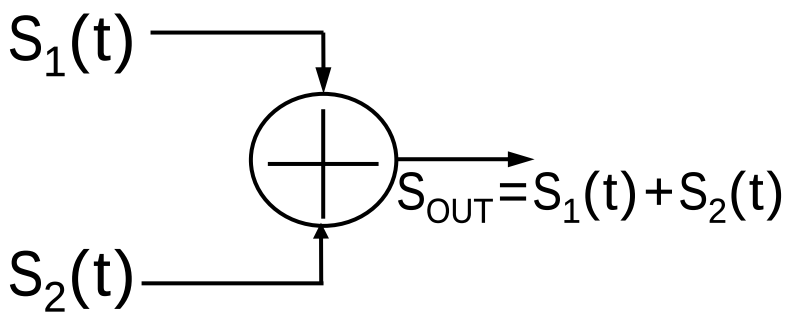
Introduction

- Present some relevant continuous time processes, symbols & functions, as buildings blocks on which later sections of the course may be based.
- Many of these processes & functions are based on mathematics
  - i.e. an equation or mathematical process that describes the function.

- The format is as follows:
  - Process title
  - Symbol and mathematical equation where appropriate
  - ‘Rules of thumb’

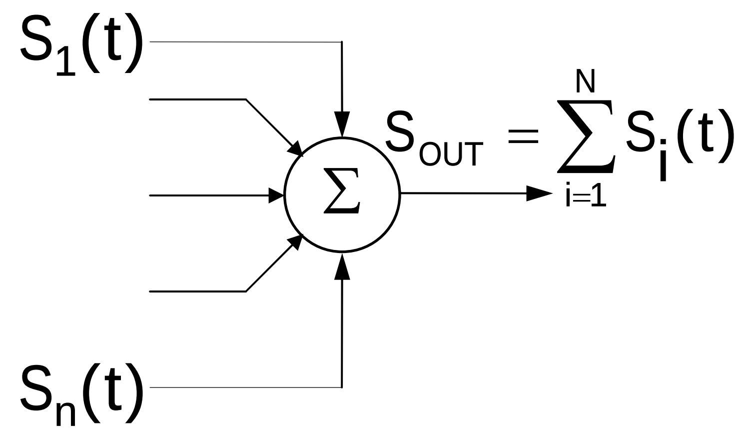
Summation

- Summer (Addition)



- Both may be implemented with a summing operational amplifier (see handout notes)

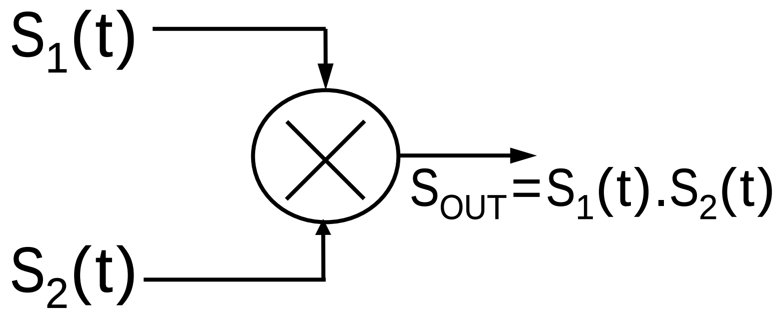
- Summation
  - Summing N inputs



- **Rules Of Thumb:** The output = sum of the inputs.
  If the inputs are analogue signals, the summer could be an operational amplifier.
  If the inputs are digital, i.e. binary numbers, the summation could be carried out by a digital computer or a DSP (digital signal processor).

Multiplier (1)

- Multiplier



- Often, $S_1(t)$ & $S_2(t)$ are signals that vary with time.
- Typical application:
  - Analogue amplitude modulation
  - A mixer in radio transmitters and receivers.
- Mathematically we could define:

$$S_1(t) = V_1\cos(\omega_1 t) \qquad S_2(t) = V_2\cos(\omega_2 t)$$

  - V_1 & V_2 are the amplitudes,
  - ω_1 & ω_2 are the angular frequencies in radians per second.
- Note that the angular frequency $\omega=2\pi f$, where f is the frequency in Hertz (Hz)

Multiplier (2)

- Thus $S_{OUT} = S_1(t).S_2(t)$
  $= V_1\cos(\omega_1 t).V_2\cos(\omega_2 t)$
- A trigonometric identity is:
  $\cos A \,.\, \cos B = ½\cos(A+B) + ½\cos(A - B)$
- So: $$S_{OUT} = \frac{V_1V_2}{2}\cos(\omega_1 + \omega_2)t + \frac{V_1V_2}{2}\cos(\omega_1 - \omega_2)t$$

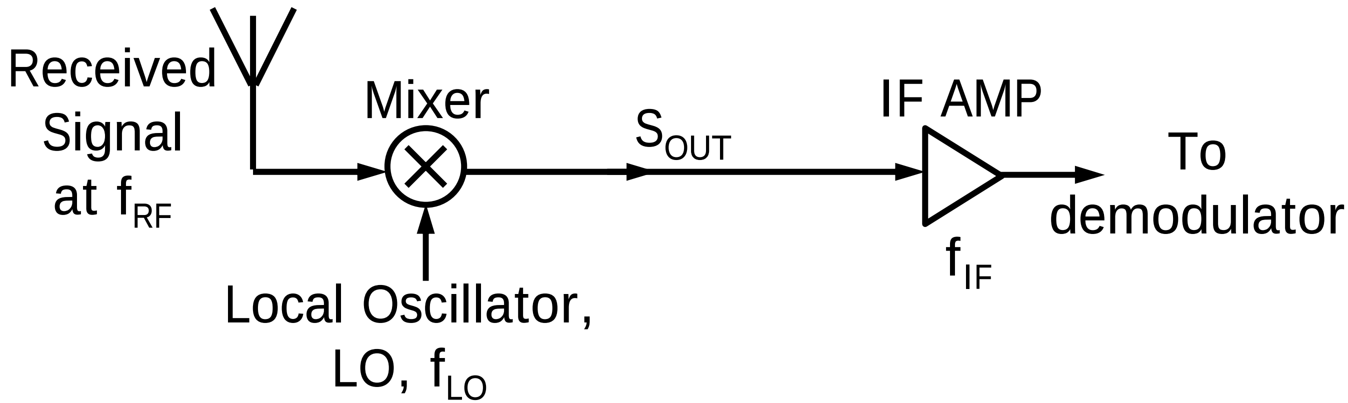
Sum Frequency=(f_1+f_2)Hz

Difference Frequency=(f_1-f_2)Hz

- **Rules Of Thumb**
  - The analysis shows that the product of 2 signals at frequencies f_1 & f_2Hz gives an output containing components
    - (f_1+f_2)Hz, (the sum frequency) &
    - $(f_1–f_2)$Hz (the difference frequency).

Multiplier (3)

- The ‘idea’ of sum and difference frequencies is very important in many applications of signal processing.
- E.g. a portion of a radio receiver:



- The signals at S_{OUT} contain:
  - SUM: $f_{RF}+f_{LO}$
  - DIFFERENCE: $f_{RF}-f_{LO}$
- The Intermediate Frequency Amplifier, IF Amp, selects the difference frequency:
  - $f_{IF}=f_{RF}-f_{LO}$

Example

- RF Frequency = 88 MHz = f_{RF}
- LO Frequency = 98 MHz = f_{LO}
- S_{OUT} contains 186 MHz = $f_{RF} + f_{LO}$
- & 10 MHz = $f_{RF} – f_{LO}$
- The IF Amp would be tuned to select 10 MHz
  - This 10 MHz signal contains the information, down converted from the RF signal.

Integrate

- Integration

$$V_{IN}(t) \longrightarrow \boxed{\int V_{IN}.dt} \longrightarrow V_{OUT}(t) = \frac{1}{\tau}\int V_{IN}(t).dt$$

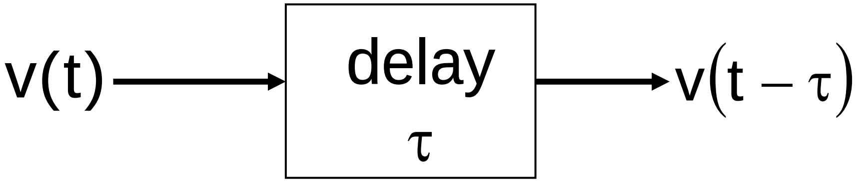
- Integrate and Dump

$$V_{IN}(t) \longrightarrow \boxed{\frac{1}{T}\int V_x.dt} \longrightarrow V_{OUT}(t)$$

Both may be implemented with
an operational amplifier
(See handout notes).

Delay

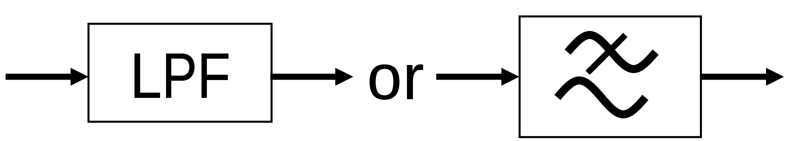
- Delay

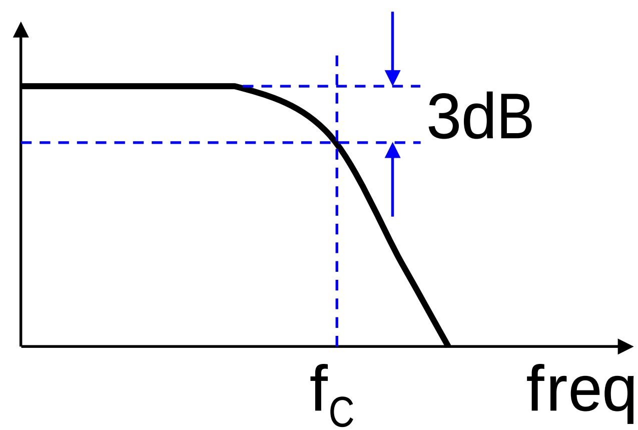


  - The output is the same as the input but delayed by a time τ
  - Ideally all frequencies present in the input are delayed by the same time
  - Methods of implementation include all pass filters (See handout notes).

Filters (1)

- Processors that have a frequency response, which in some desired way will affect the frequency content of a signal.
- There are several types of filter
  - The input signal is assumed to occupy a wide frequency spectrum & the effect of the filter is shown at the output, **assuming ideal** filters
- Low Pass Filter



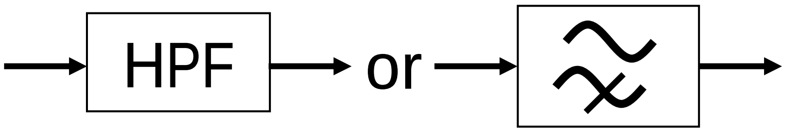


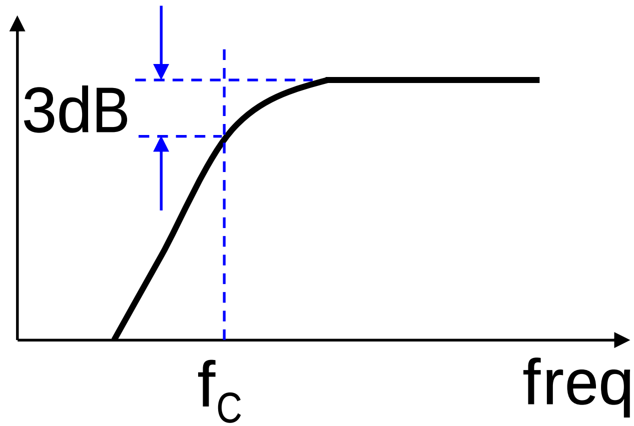
f_C=cut off, where gain drops 3dB from max

LPF Response

Filters (2)

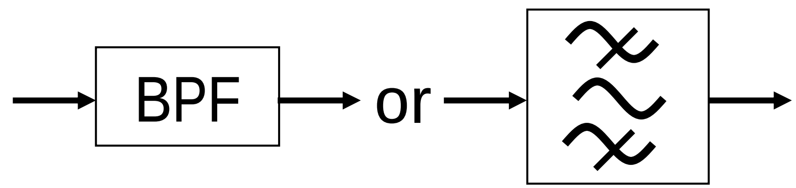
- High Pass Filter

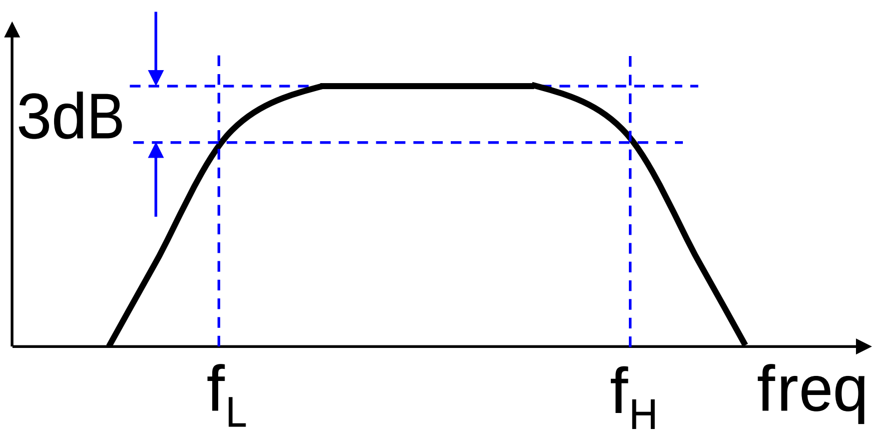




HPF Response

- Band Pass Filter



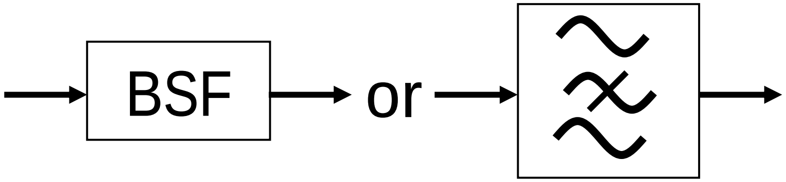


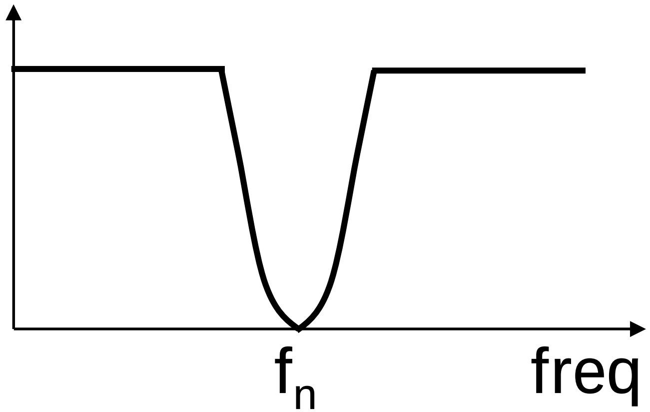
BPF Response

Bandwidth between 3dB points = f_H-f_L

Filters (3)

- Band Stop or Notch Filter

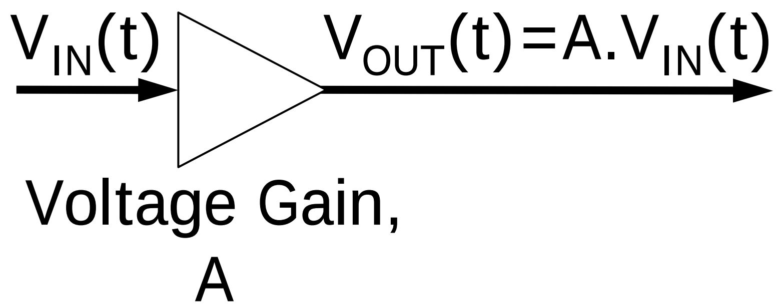




BSF Response

- A filter is often used to ‘band limit’ noise, i.e. reduce the noise as much as possible whilst allowing the signal to pass
- Aim is to increase the signal to noise ratio (S/N).
  - Process may be thought of as an ‘extrusion’ process where the BPF represents a tunnel, to pass the signal but remove as much noise and interference as possible.

Amplifier



- The output voltage is A times more than the input, where A is the amplifier voltage gain

Logarithms

- A logarithm is a way of writing or expressing a number.
- ‘Logs’ offer two main benefits:
  - It helps us to comprehend values more easily, for example power, that may cover an extremely wide range.
  - They are useful in communications because they allow powers, gains and losses to be added rather than multiplied/divided.
- Spectrum analysers usually have both a linear and a logarithmic (decibel) scale.

Logarithms

- Suppose we have the following values X, (they could represent units of power, ignore the units for now).

X	1	10	100	1000	10,000	100,000	1000,000
X	10^0	10^1	10^2	10^3	10^4	10^5	10^6

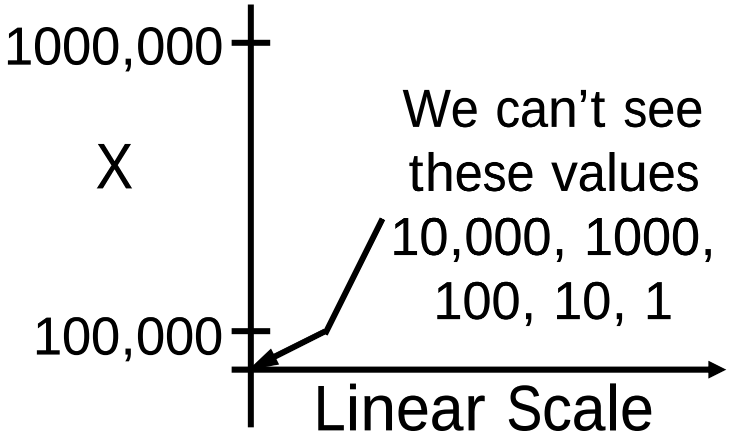
- To find the 'log' of X find $\log_{10}$ X:

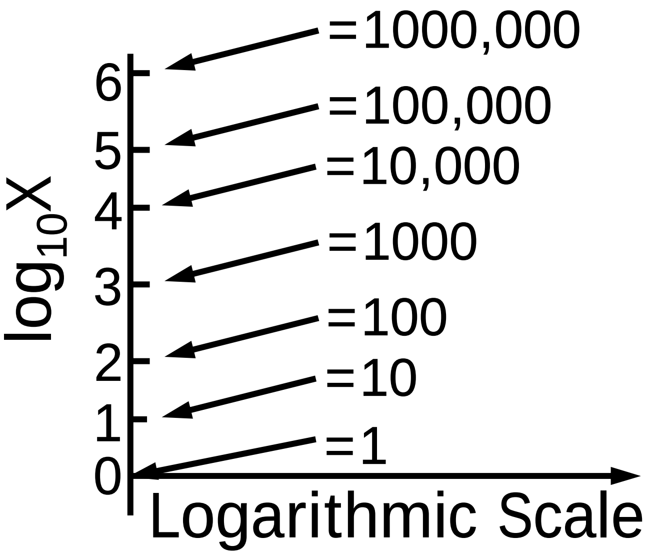
Log_{10} X	0	1	2	3	4	5	6

- Consider for example:
  - $10^3 = 10 \times 10 \times 10 = 1000$
  - $\mathrm{Log}_{10}\, 1000 = 3$
- Similarly
  - $\mathrm{Log}_{10}\, 100 = 2$

Logarithmic Graphs

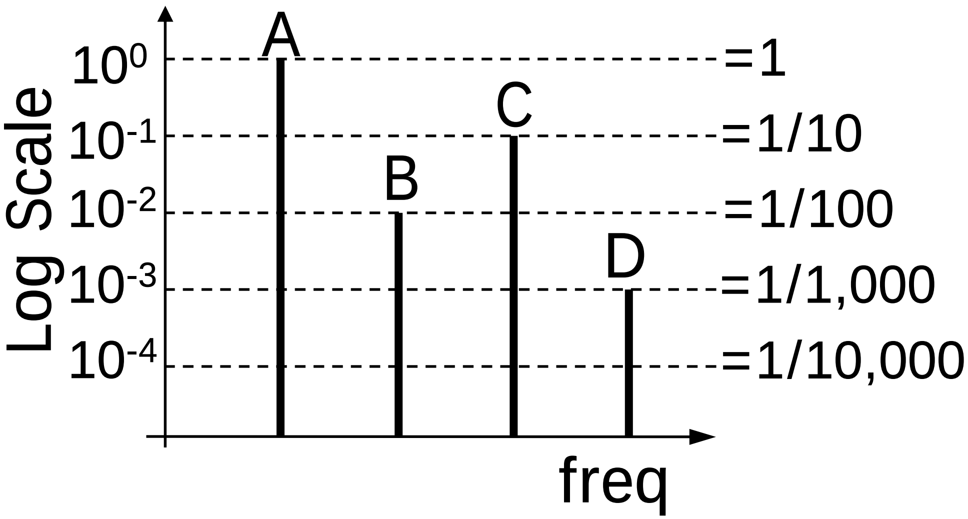
- Now suppose we plot both ‘linear’ and ‘log’ values as a scale.





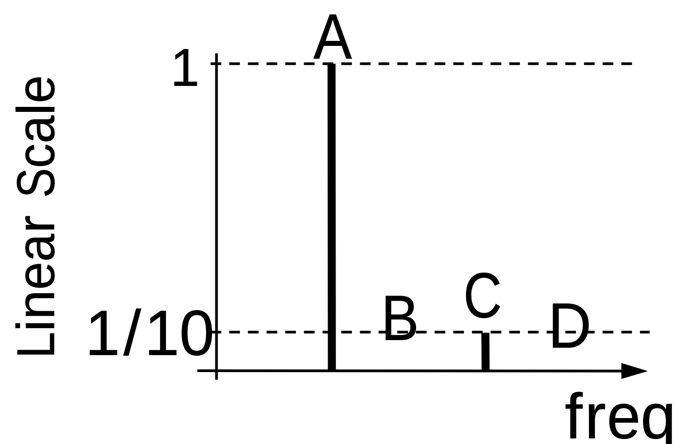
Logarithmic Spectra

- Suppose we saw the following component amplitudes (A, B, C & D) on a spectrum analyser, on a log scale.



Relative to A:
B = 1/100
C = 1/10
D = 1/1000

Can not 'see' B or D



Logarithmic Multiplication/Division rule

- Consider two numbers, X_1 & X_2
- Say:

  $X = X_1.X_2$ (product)
- Now:

  $\log_{10}X=\log_{10}(X_1.X_2) = \log_{10}X_1+\log_{10}X_2$

- Also if:

$$X = X_1 / X_2 \qquad \log_{10}\left(X_1 / X_2 \right) = \log_{10} X_1 - \log_{10} X_2$$

- This addition/subtraction property of logarithmic values rather than multiplying/dividing is useful when considering signals, gains & losses in communication systems.

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