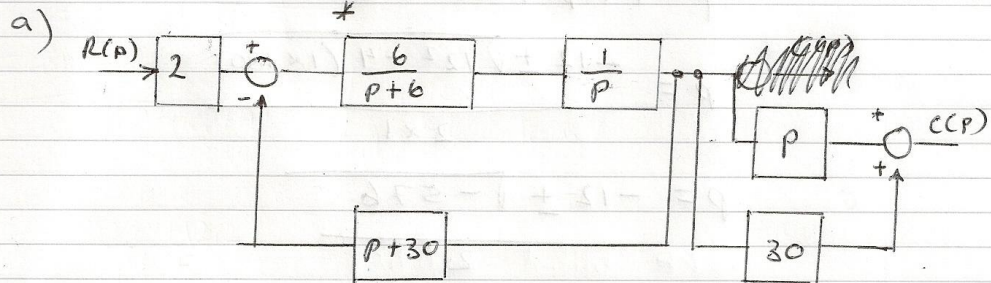


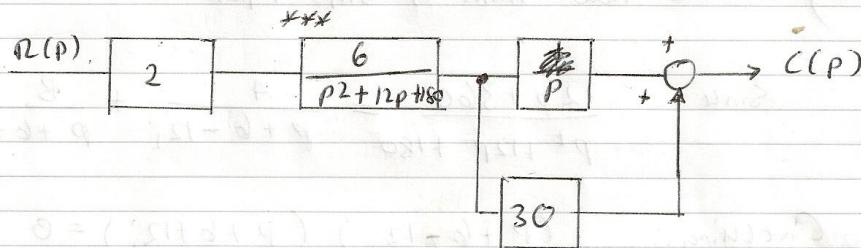
1)



$$* \quad \frac{6/p}{1 + \frac{6}{p} \cdot 1} = \frac{6}{p+6}$$

$$** \quad \frac{6}{p+6} \cdot \frac{1}{p} = \frac{6}{p^2+6p}$$

$$*** \quad \frac{\left( \frac{6}{p^2+6p} \right)}{1 + \frac{6}{p^2+6p} \cdot p+30} = \frac{6}{p^2+6p+6p+180} = \frac{6}{p^2+12p+180}$$



$$C(p) = \frac{12p}{p^2+12p+180} R(p) + \frac{360}{p^2+12p+180} R(p)$$

$$C(p) = \frac{12p + 360}{p^2+12p+180} R(p)$$

$$T(p) = \frac{C(p)}{R(p)} = \frac{12p + 360}{p^2+12p+180}$$

b) Characteristic equation:

$$p^2 + 12p + 180 = 0$$

$$p = \frac{-12 \pm \sqrt{12^2 - 4(1 \times 180)}}{2 \times 1}$$

$$p = \frac{-12 \pm \sqrt{-576}}{2}$$

ii)  $p = -6 \pm j12$

The general char eqn for a second order system is:

$$p^2 + 2\zeta\omega_0 p + \omega_0^2 = 0$$

i)  $\omega_0 = \sqrt{180} = 13.42 \text{ rad/s}$  ?

c) We now have a step input:

Since:  $\frac{12p + 360}{p^2 + 12p + 180} = \frac{A}{p + 6 - 12j} + \frac{B}{p + 6 + 12j}$

Checking...  $(p + 6 - 12j)(p + 6 + 12j) = 0$

$$p^2 + 6p + 12jp + 6p + 36 + 72j$$

$$+ -12jp - 72j + 144 = 0$$

$$\underline{p^2 + 12p + 180 = 0}$$

$$12p + 360 = A(p + 6 + 12j) + B(p + 6 - 12j)$$

when  $p = -6 + 12j$

$$-72 + 144j + 360 = A(24j) + B(0) \quad \begin{matrix} 6 - 12j \\ A = 12/j \end{matrix}$$

→

B7 c) Continued...

$$p = -6 - 12j$$

$$-72 - 144j + 360 = -24jB, B = \cancel{12+6j} (6+12j)$$

$$\frac{12p+360}{p^2+12p+180} = \frac{12+6j}{p+6-12j} + \frac{12+6j}{p+6+12j}$$

$$\frac{12p+360}{p^2+12p+180} = \frac{6-12j}{p+6-12j} + \frac{6+12j}{p+6+12j}$$

now we have the transfer function as first order components:

$$r(t) \rightarrow \left[ \frac{k}{1+Tp} \right] \rightarrow k(1-e^{-t/T})r(t)$$

$$\text{Thus: } (1) \frac{6-12j}{p+6-12j} = \frac{1}{1 + \frac{1}{6-12j}p}$$

$$(2) \quad \text{---} = \frac{1}{1 + \frac{1}{6+12j}p}$$

The system response  $c(t)$  is:

$$c(t) = \left[ (1 - e^{-(6+12j)t}) + (1 - e^{-(6-12j)t}) \right] r(t)$$

when  $t < 0$

$$c(t) = 0$$

when  $t \geq 0$

$$c(t) = 2(5) = 10, \quad t=0, \quad c(t) = 0?$$

→ →



d)

when  $t = \infty$

$$i(\infty) = 2(5) = 10$$

2)

Initial Conditions:

Capacitor fully charged,  $V_c = 10V$

inductor short circuit,  $\frac{L di}{dt} = 0V$

Current through  $R_1 = 0A$

Assuming the 'input' is the (one resistor) switch closed:

$$V_{in} = i R_1 + \frac{L di}{dt} + \frac{1}{C} \int i dt$$

$$V_{out} = i R_1$$

$$V_{in}(p) = i R_1 + \frac{L p i}{C p} = i \left( R_1 + L p + \frac{1}{C p} \right)$$

$$V_{out}(p) = i R_1$$

$$\frac{V_{out}(p)}{V_{in}(p)} = \frac{R_1}{R_1 + L p + \frac{1}{C p}} = \frac{1}{1 + \frac{L}{R_1} p + \frac{1}{C R_1 p}}$$

$$\text{char eqn: } 1 + \frac{L}{R_1} p + \frac{1}{C R_1 p} = 0$$

$$p + \frac{L}{R_1} p^2 + \frac{1}{C R_1} = 0$$

$$\therefore \frac{L}{R_1} p^2 + p + \frac{1}{C R_1} = 0$$

$$p = \frac{-1 \pm \sqrt{1^2 - 4 \left( \frac{L}{R_1} \cdot \frac{1}{C R_1} \right)}}{2 \times \frac{L}{R_1}}$$

~~as we~~

2) a) Continued.

$$p = \frac{-1 \pm \sqrt{1 - \frac{2}{9}}}{2 \times \frac{1}{6}} = -3 \pm \sqrt{7}$$

$$p_1 = -0.35$$

$$p_2 = -5.65$$

how do we know its current or not voltage?  
The response is then of the form:  $i(t) = A e^{-0.35t} + B e^{-5.65t}$

we know at  $t=0$ ,  $i(t)=0$

$$0 = A + B, A = -B$$

$$i(t) = A (e^{-0.35t} - e^{-5.65t})$$

at  $t=0$ , KVL says:

$$V_C = 10V$$

$$0 = i R_1 + L \frac{di}{dt} + \frac{1}{C} \int i dt$$

$\downarrow$   
 $\int i dt = 0 \rightarrow$  const charge instantly

$$0 = L \frac{di}{dt} + 10, -10 = L \frac{di}{dt}$$

$$\frac{di}{dt} = \frac{-10}{L} = -\frac{20}{100} \text{ A/s}$$

$$\text{Thus: } \frac{di}{dt} = -0.35 A e^{-0.35t} - 5.65 B e^{-5.65t}$$

$$-\frac{20}{100} = -0.35 A - 5.65 B$$

$$-\frac{20}{100} = 400 \text{ B A} (-0.35 + 5.65), \text{ A}$$

$$A = \frac{400 \times 20}{3.77} = 3.77$$

$$i(t) = - \frac{3.77}{11325} (e^{-0.35t} - e^{-5.65t})$$

$t=0$  ← switch closed. . .

$$i(t) = 0 ?$$

b) current will reach maximum value when

$$\frac{di}{dt} = 0.$$

$$0 = 1.3195 e^{-0.35t} - 21.31 e^{-5.65t}$$

$$21.31 e^{-5.65t} = 1.3195 e^{-0.35t}$$

$$\ln(21.31) - 5.65t = \ln(1.3195) - 0.35t$$

$$\ln(21.31) - \ln(1.3195) = 5.3t$$

$$t = 0.525s$$

$$i(t) = -2.44A$$