Mini Tutorial
MT-223

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State Variable Filters

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IN THIS MINI TUTORIAL

Three sample state variable filters are designed in this mini tutorial, one in a series of mini tutorials describing discrete circuits for precision op amps.

The state variable realization as described by Tow (see the References section) is shown in Figure 1 along with the design equations.

This configuration offers the most precise implementation of the filter function, at the expense of many more circuit elements. All three major parameters (gain, Q, and ω_0) can be adjusted independently, and low-pass, high-pass, and band-pass outputs are available simultaneously. Note that the low-pass and high-pass outputs are inverted in phase while the band-pass output maintains the phase.

The gain of each of the outputs of the state variable filter is also independently variable. With an added amplifier section summing the low-pass and high-pass sections, the notch function can also be synthesized. By changing the ratio of the summed sections, low-pass notch, standard notch, and high-pass notch functions can be realized. A standard notch may also be realized by subtracting the band-pass output from the input with the added op amp section. An allpass filter may also be built with the four amplifier configuration by subtracting the band-pass output from the input. In this instance, the band-pass gain must equal 2.

Since all parameters of the state variable filter can be adjusted independently, component spread can be minimized. In addition, variations due to temperature and component tolerances are minimized.

Tuning the resonant frequency of a state variable filter is accomplished by varying R4 and R5. While you do not have to tune both, it is generally preferable if you are varying over a wide range. Holding R1 constant, tuning R2 sets the low-pass gain and tuning R3 sets the high-pass gain. Band-pass gain and Q are set by the ratio of R6 and R7.

Since the parameters of a state variable filter are independent and tunable, it is easy to add electronic control of frequency, Q and ω_0. This adjustment is accomplished by using an analog multiplier, multiplying DACs (MDACs) or digital pots (see MT-208).

In selecting an amplifier for building the state variable filter, the rule of thumb is to try to have a minimum of 20 dB of loop gain at the center frequency, looking at the band-pass output. In the state variable filter, the amplifiers are used as an integrator (in the band-pass output). which sets the bandwidth requirement.

A close relative of the state variable filter is the biquad filter (see MT-205).

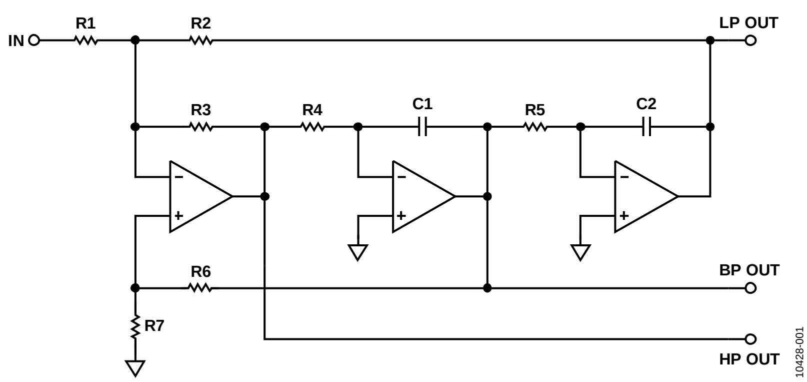

Figure 1. State Variable Filter

STATE VARIABLE DESIGN EQUATIONS

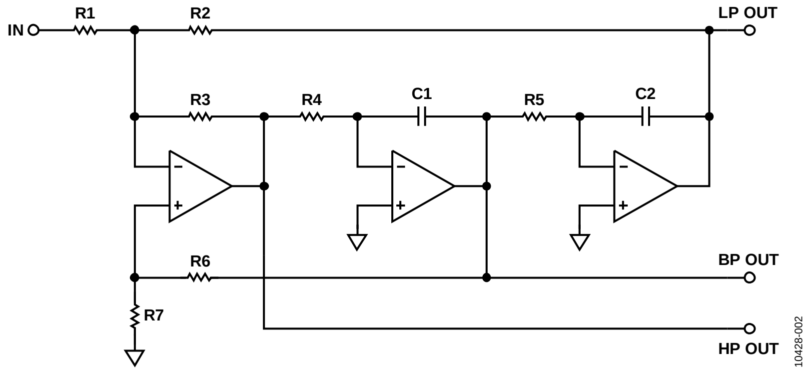

Figure 2.

$$A_{LP(s=0)} = -\frac{R2}{R1}$$

$$A_{HP(s=\infty)} = -\frac{R3}{R1}$$

To design the filter, choose $R1$.

$$R2 = A_{LP}\, R1$$

$$R3 = A_{HP}\, R1$$

$$\omega_0 = \sqrt{\frac{R3}{R2\ R4\ R5\ C1\ C2}}$$

Let $R4 = R5 = R$ and let $C1 = C2 = C$

Now, choose C.

$$R = \frac{2\pi F_0}{C}\sqrt{\frac{A_{HP}}{A_{LP}}} \qquad A_{BP(s=\omega_0)} = \frac{\dfrac{R6+R7}{R7}}{R1\left(\dfrac{1}{R1}+\dfrac{1}{R2}+\dfrac{1}{R3}\right)}$$

Now, choose R7.

$R6 =$

$$R7\sqrt{R2\ R3}\ Q\left(\frac{1}{\dfrac{1}{R1}+\dfrac{1}{R2}+\dfrac{1}{R3}}\right)$$

STATE VARIABLE DESIGN EQUATIONS FOR NOTCH FILTERS

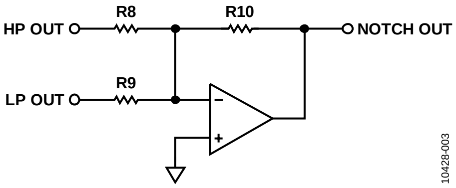

Figure 3.

To design the filter, choose $R10$.

Then, choose A_{HP}, A_{LP}, $A_{NOTCH} = 1$.

For $\omega_Z = \omega_0$: $R8 = R9 = R10$

For $\omega_Z < \omega_0$: $R9 = R10$

$$R8 = \frac{\omega_0^{\ 2}}{\omega_Z^{\ 2}} R10$$

For $\omega_Z > \omega_0$: $R8 = R10$

$$R9 = \frac{\omega_Z^{\ 2}}{\omega_0^{\ 2}} R10$$

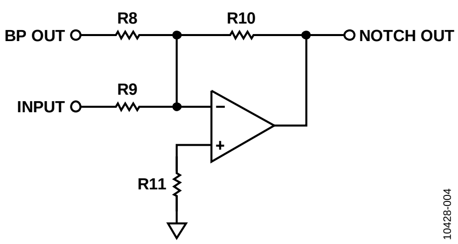

Figure 4.

To design the filter, choose $A_{NOTCH} = 1$.

Then, choose *R10:*

$R8 = R9 = R11 = R10$.

REVISION HISTORY

4/12—Revision 0: Initial Version

STATE VARIABLE DESIGN EQUATIONS FOR ALLPASS FILTERS

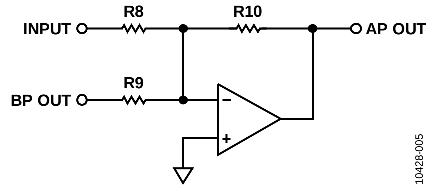

Figure 5.

$H = 1$

$R8 = R10$

$R9 = R8/2$

REFERENCES

Tow, J. "Active RC Filters–A State-Space Realization", Proc. IEEE, 1968, Vol.56, pp. 1137-1139.

Zumbahlen, Hank, editor, 2008. *Linear Circuit Design Handbook*, Newnes, ISBN 978-0-7506-8703-4.

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