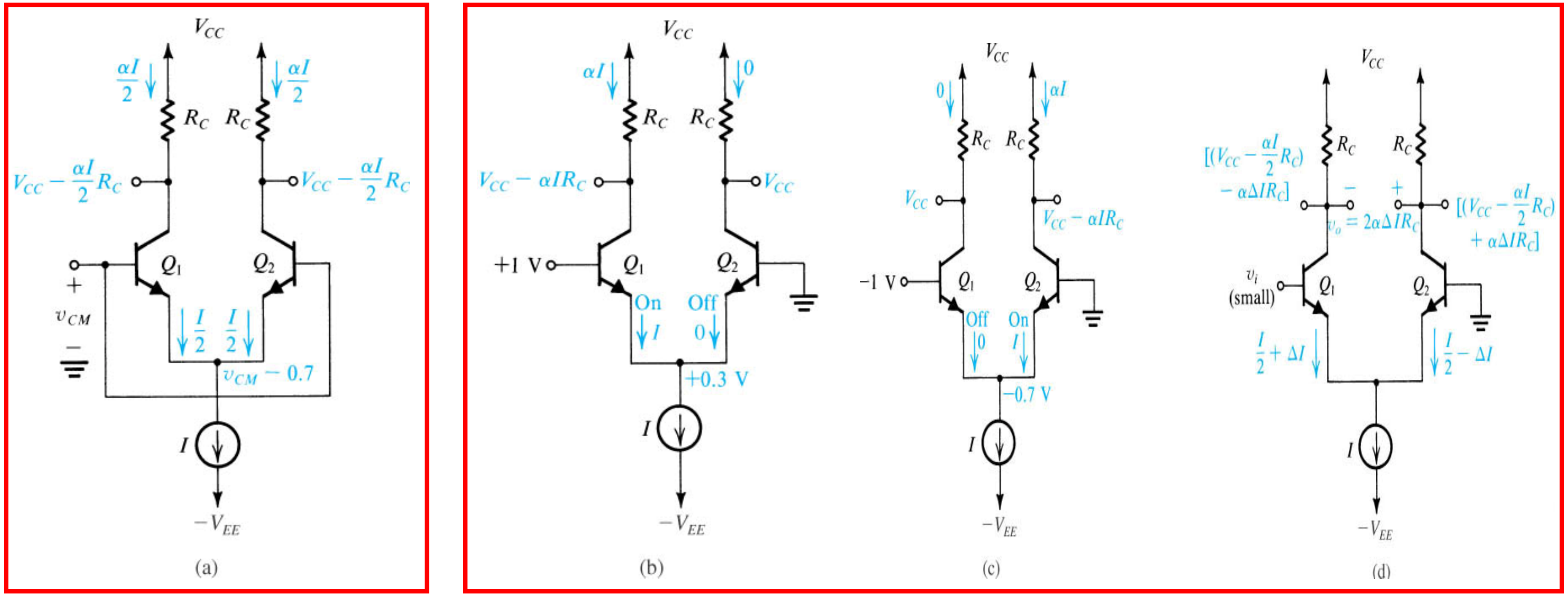
Bipolar Differential Amplifiers: Qualitative Analysis



Common Mode

Differential Mode

Different modes of operation of the BJT differential pair: **(a)** The differential pair with a common-mode input signal v_{CM}. **(b)** The differential pair with a "large" differential input signal. **(c)** The differential pair with a large differential input signal of polarity opposite to that in (b). **(d)** The differential pair with a small differential input signal v_i. Note that we have assumed the bias current source I to be ideal (i.e., it has an infinite output resistance) and thus I remains constant with the change in v_{CM}.

Bipolar Differential Amplifiers: Large Signal Analysis

The exponential relationship applied to each of the two transistors may be written as:

$$i_{E1} = \frac{I_S}{\alpha} e^{(v_{B1}-v_E)/V_T} \quad \text{and} \quad i_{E2} = \frac{I_S}{\alpha} e^{(v_{B2}-v_E)/V_T}$$

These two equations can be combined to obtain

$$\frac{i_{E1}}{i_{E2}} = e^{(v_{B1}-v_{B2})/V_T}$$

which can be manipulated to yield

$$\frac{i_{E1}}{i_{E1}+i_{E2}} = \frac{1}{1+e^{(v_{B2}-v_{B1})/V_T}} \quad \text{and} \quad \frac{i_{E2}}{i_{E1}+i_{E2}} = \frac{1}{1+e^{(v_{B1}-v_{B2})/V_T}}$$

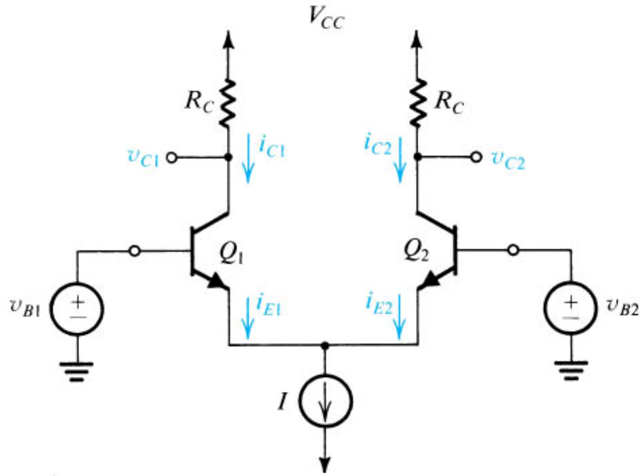
From the circuit we have

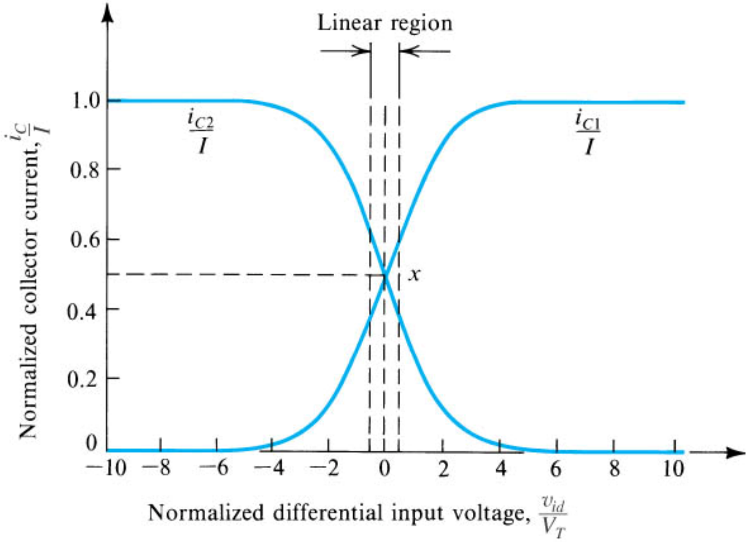
$$i_{E1} + i_{E2} = I$$

Which may be used to obtain the following expressions for i_{E1} and i_{E2}

$$i_{E1} = \frac{I}{1+e^{(v_{B2}-v_{B1})/V_T}} \quad \text{and} \quad i_{E2} = \frac{I}{1+e^{(v_{B1}-v_{B2})/V_T}}$$

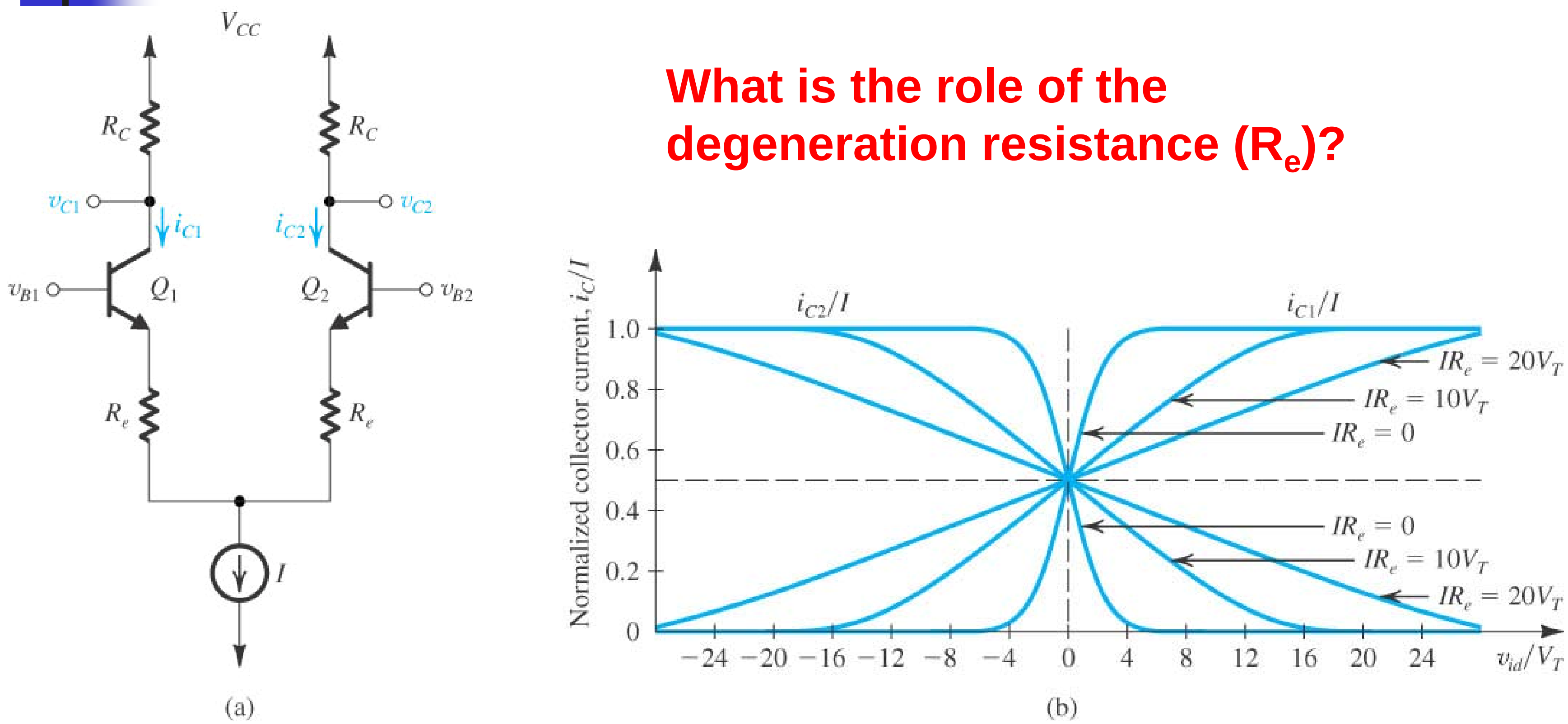
i_{C1} and i_{C2} may be obtained by multiplying i_{E1} and i_{E2} by α which is almost unity and plotted as shown in the figure





Bipolar Differential Amplifiers: Linearization

For Your Information



What is the role of the degeneration resistance (R_e)?

The transfer characteristics of the BJT differential pair **(a)** can be linearized **(b)** (i.e., the linear range of operation can be extended) by including resistances in the emitters.

Bipolar Differential Amplifiers: DC Analysis (Example 1)

- **Problem:** Find the Q-points of transistors in the shown differential amplifier.
- **Given data:** $V_{CC} = V_{EE} = 15$ V, $R_{EE} = R_C = 75\text{k}\Omega$, $\beta = 100$
- **Analysis:**

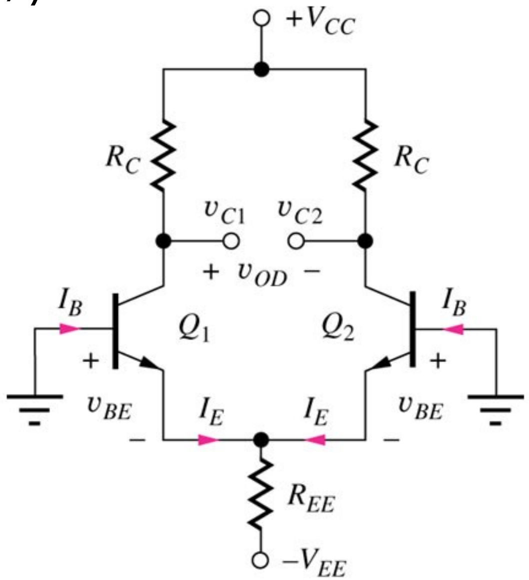
$$I_E = \frac{V_{EE} - V_{BE}}{2R_{EE}} = \frac{(15 - 0.7)\text{V}}{2(75\times10^3)\Omega} = 95.3\mu\text{A}$$

$$I_C = \alpha I_E = \frac{100}{101} I_E = 94.4\mu\text{A}$$

$$I_B = \frac{I_C}{\beta} = \frac{94.4\mu\text{A}}{100} = 0.944\mu\text{A}$$

$$V_C = 15 - I_C R_C = 7.92\text{V}$$

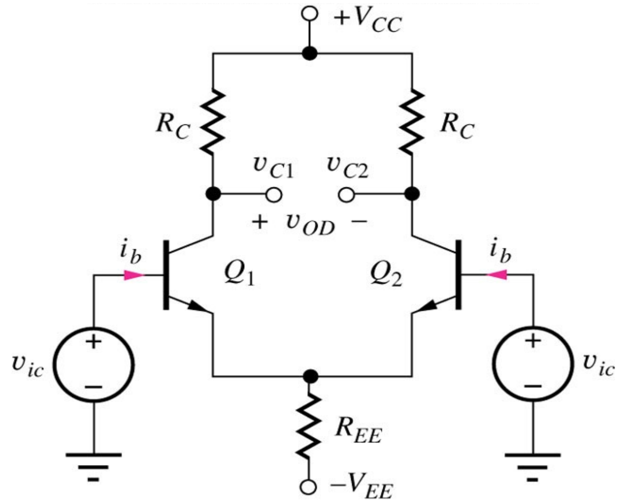
$$V_{CE} = V_C - V_E = 7.92\text{V} - (-0.7\text{V}) = 8.62\text{V}$$



Due to symmetry, both transistors are biased at Q-point (94.4 μA, 8.62V)

Bipolar Common-mode Input Voltage Range

- **Problem:** Find the max. V_{IC} before saturation in the shown differential amplifier.
- **Given data:** $V_{CC}=V_{EE}=15$ V, $R_{EE}=R_C=75\text{k}\Omega$, $\beta=100$
- **Analysis:**



We want to find max. V_{IC} while the C-B junction is reverse biased.

$$V_{CB}=V_{CC}-I_C R_C-V_{IC}\geq 0$$

$$I_C=\alpha\frac{V_{IC}-V_{BE}+V_{EE}}{2R_{EE}}$$

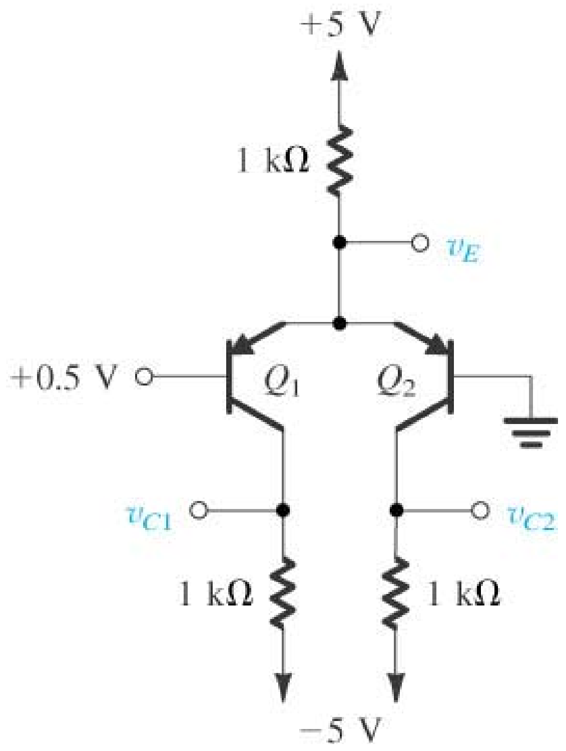
$$\therefore V_{IC}\leq V_{CC}\frac{1-\alpha\dfrac{R_C}{2R_{EE}}\dfrac{\left(V_{EE}-V_{BE}\right)}{V_{CC}}}{1+\alpha\dfrac{R_C}{2R_{EE}}}$$

For symmetrical power supplies ($V_{EE}=V_{CC}$) , $V_{EE}>>V_{BE}$, and $R_C=R_{EE}$,

$$V_{IC}\leq\frac{V_{CC}}{3}=5V$$

Bipolar Differential Amplifiers: DC Analysis (Example 2)

- **Problem:** Find v_E, v_{c1}, and v_{c2} in the shown differential amplifier.
- **Given data:** V_{CC}= V_{EE}= 5 V, R_{EE}=R_C=1kΩ, $\alpha \approx$, $|V_{BE}|$=0.7 V
- **Analysis:**



We can assume Q1 to be off and Q2 on

$$v_E = +0.7 \text{ V}$$

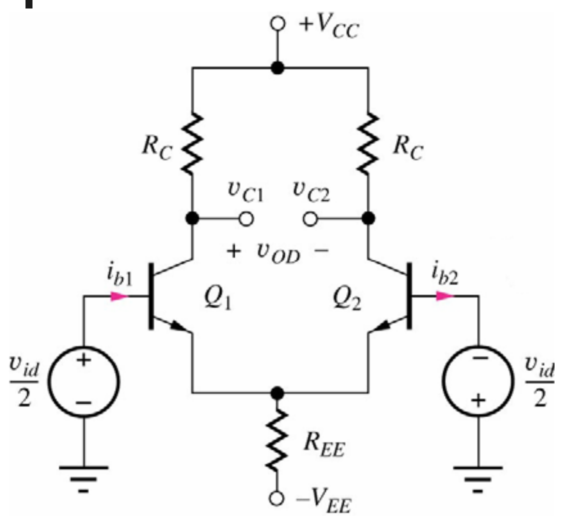
$$v_{C1} = -5 + R_C * I_{C1} = -5 + 0 = -5 \text{ V}$$

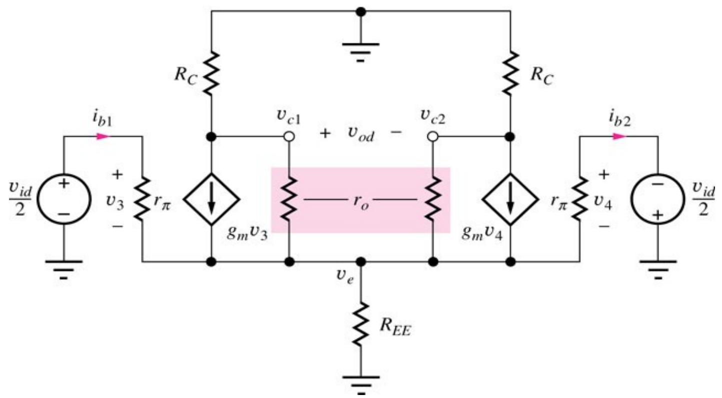
$$I_{E2} = I_E = (5 - 0.7)/1 = 4.3 \text{ mA}$$

$$I_{C2} = \alpha I_{E2} = 4.3 \text{ mA}$$

$$v_{C2} = -5 + R_C * I_{C2} = -5 + 4.3 = -0.7 \text{ V}$$

Differential-mode Gain and Input Resistance





$$v_3 = \frac{v_{id}}{2} - v_e \qquad v_4 = -\frac{v_{id}}{2} - v_e$$

$$(g_m + 1/r_\pi)(v_3 + v_4) = 1/R_{EE} v_e$$

$$\therefore v_e(1/R_{EE} + 2/r_\pi + 2g_m) = 0 \rightarrow v_e = 0$$

Emitter node in differential amplifier represents virtual ground for differential-mode input signals.

$$\therefore v_3 = \frac{v_{id}}{2} \qquad v_4 = -\frac{v_{id}}{2}$$

Output signal voltages are:

$$v_{c1} = -g_m R_C \frac{v_{id}}{2} \qquad v_{c2} = +g_m R_C \frac{v_{id}}{2}$$

$$\therefore v_{od} = -g_m R_C v_{id}$$

Differential-mode Gain and Input Resistance (contd.)

Differential-mode gain for balanced output, $v_{od} = v_{c1} - v_{c2}$ is:

$$A_{dd} = \left.\frac{v_{od}}{v_{id}}\right|_{v_{ic}=0} = -g_m R_C$$

If either v_{c1} or v_{c2} is used alone as output, output is said to be single-ended.

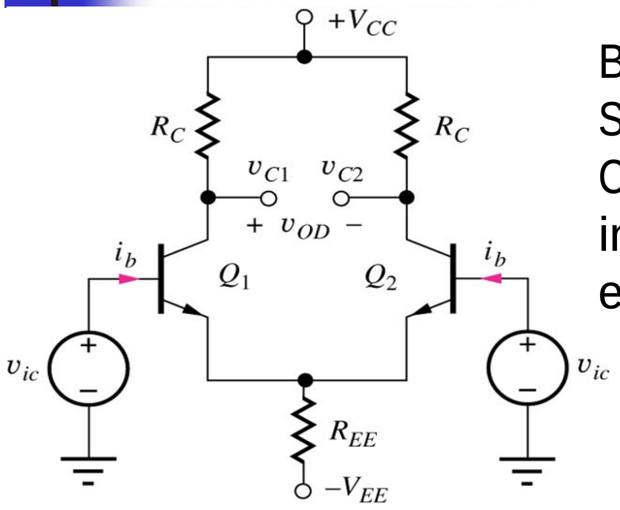
$$A_{dd1} = \left.\frac{v_{c1}}{v_{id}}\right|_{v_{ic}=0} = -\frac{g_m R_C}{2} = \frac{A_{dd}}{2} \qquad A_{dd2} = \left.\frac{v_{c2}}{v_{id}}\right|_{v_{ic}=0} = \frac{g_m R_C}{2} = -\frac{A_{dd}}{2}$$

Differential-mode input resistance is small-signal resistance presented to differential-mode input voltage between the two transistor bases.

$$i_{b1} = \frac{(v_{id}/2)}{r_\pi} \qquad \therefore R_{id} = v_{id}/i_{b1} = 2r_\pi$$

If v_{id} =0, $R_{od} = 2(R_C \| r_o) \cong 2R_C$. For single-ended outputs, $R_{od} \cong R_C$

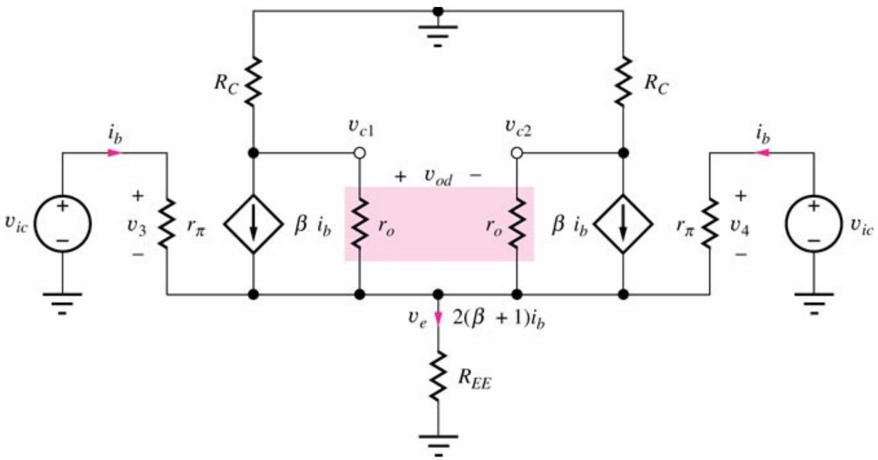
Common-mode Gain and Input Resistance



Both arms of differential amplifier are symmetrical. So terminal currents and collector voltages are equal. Characteristics of differential pair with common-mode input are similar to those of a C-E amplifier with large emitter resistor.

$$i_b = \frac{v_{ic}}{r_\pi + 2(\beta + 1)R_{EE}}$$

Output voltages are:



$$v_{c1} = v_{c2} = -\beta\, i_b R_C = \frac{-\beta R_C}{r_\pi + 2(\beta + 1)R_{EE}} v_{ic}$$

$$v_e = 2(\beta + 1) i_b R_{EE}$$

$$= \frac{2(\beta + 1)R_{EE}}{r_\pi + 2(\beta + 1)R_{EE}} v_{ic} \cong v_{ic}$$

Common-mode Gain and Input Resistance (contd.)

Common-mode gain is given by:

$$A_{cc} = \left.\frac{v_{oc}}{v_{ic}}\right|_{v_{id}=0} = -\frac{\beta R_C}{r_\pi + 2(\beta + 1)R_{EE}} \cong -\frac{R_C}{2R_{EE}}$$

For $R_C = R_{EE}$, common-mode gain =0.5. Thus, common-mode output voltage and A_{cc} is 0 if R_{EE} is infinite. This result is obtained since output resistances of transistors are neglected. A more accurate expression is:

$$A_{cc} \cong R_C\left(\frac{1}{\beta r_o} - \frac{1}{2R_{EE}}\right)$$

$v_{od} = v_{c1} - v_{c2} = 0$ Therefore, common-mode conversion gain is found to be 0.

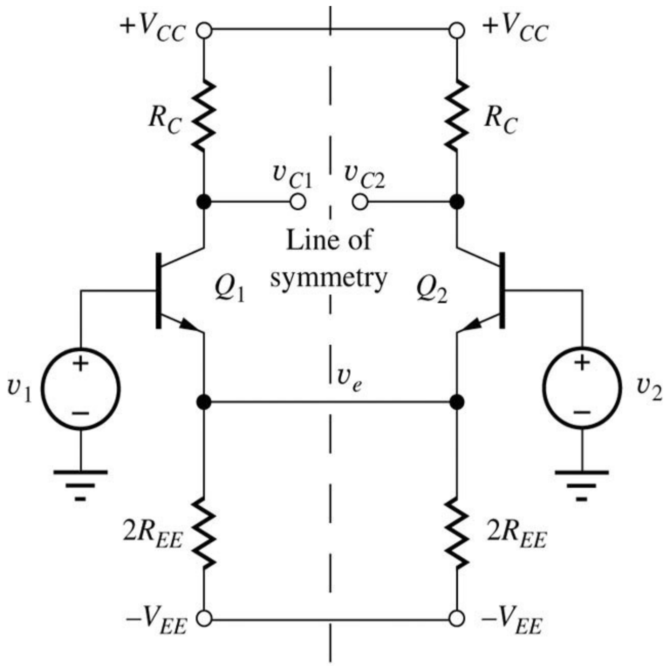
$$R_{ic} = \frac{v_{ic}}{2i_b} = \frac{r_\pi + 2(\beta + 1)R_{EE}}{2} = \frac{r_\pi}{2} + (\beta + 1)R_{EE}$$

Common-Mode Rejection ratio (CMRR)

- Represents ability of amplifier to amplify desired differential-mode input signal and reject undesired common-mode input signal.
- For differential output, common-mode gain of balanced amplifier is zero, CMRR is infinite. For single-ended output,

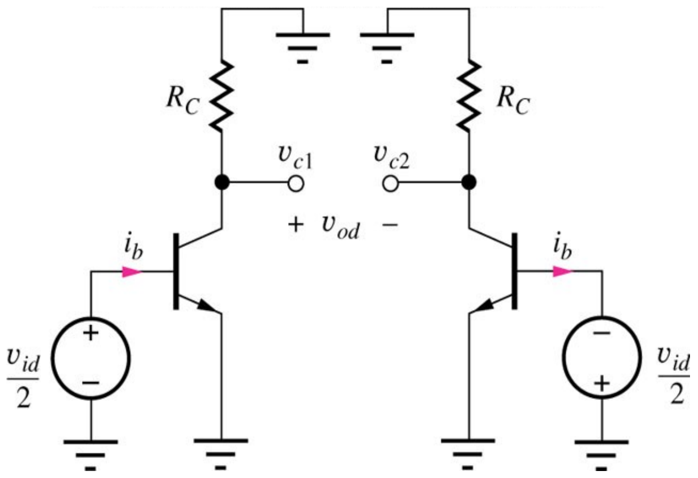
$$\mathrm{CMRR} = \left|\frac{A_{dm}}{A_{cm}}\right| = \left|\frac{A_{dd}/2}{A_{cc}}\right| = \frac{1}{2\left(\dfrac{1}{\beta r_o g_m} - \dfrac{1}{2 g_m R_{EE}}\right)} \cong g_m R_{EE}$$

Analysis of Differential Amplifiers Using Half-Circuits



- Half-circuits are constructed by first drawing the differential amplifier in a fully symmetrical form- power supplies are split into two equal halves in parallel, emitter resistor is separated into two equal resistors in parallel.
- None of the currents or voltages in the circuit are changed.
- For differential mode signals, points on the line of symmetry are virtual grounds connected to ground for ac analysis
- For common-mode signals, points on line of symmetry are replaced by open circuits.

Bipolar Differential-mode Half-circuits



Direct analysis of the half-circuits yield:

$$v_{c1} = -g_m R_C \frac{v_{id}}{2} \qquad v_{c2} = +g_m R_C \frac{v_{id}}{2}$$

$$v_{od} = v_{c1} - v_{c2} = -g_m R_C v_{id}$$

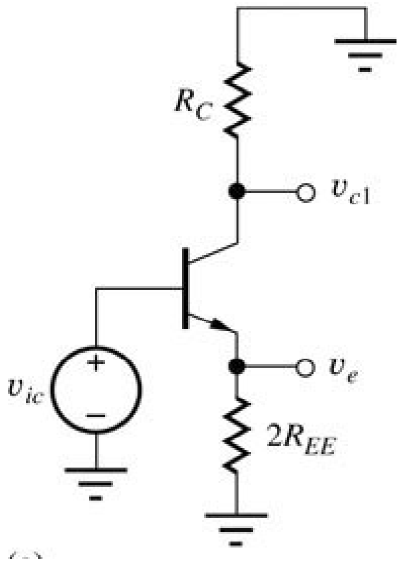
$$A_{dd} = \left.\frac{v_{od}}{v_{id}}\right|_{v_{ic}=0} = -g_m R_C$$

$$A_{dd1} = \left.\frac{v_{c1}}{v_{id}}\right|_{v_{ic}=0} = -\frac{g_m R_C}{2} = \frac{A_{dd}}{2}$$

Applying rules for drawing half-circuits, the two power supply lines and emitter become ac grounds. The half-circuit represents a C-E amplifier stage.

$$R_{id} = v_{id}/i_{b1} = 2r_\pi \qquad R_{od} = 2(R_C \| r_o)$$

Bipolar Common-mode Half-circuits



Direct analysis of the half-circuits yield:

$$v_{c1} = v_{c2} = -\beta\, i_b R_C = \frac{-\beta\, R_C}{r_\pi + 2(\beta+1)R_{EE}} v_{ic}$$

$$A_{cc} = \left.\frac{v_{oc}}{v_{ic}}\right|_{v_{id}=0} = -\frac{\beta\, R_C}{r_\pi + 2(\beta+1)R_{EE}} \cong -\frac{R_C}{2R_{EE}}$$

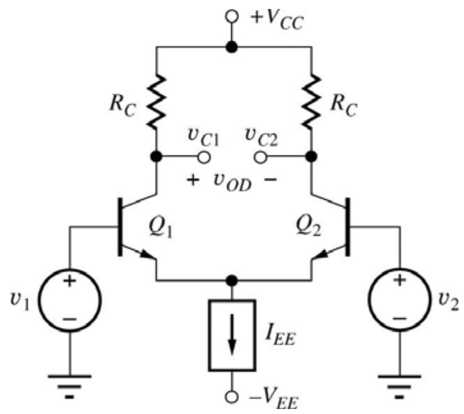
$$v_{od} = v_{c1} - v_{c2} = 0$$

$$R_{ic} = \frac{v_{ic}}{2i_b} = \frac{r_\pi + 2(\beta+1)R_{EE}}{2} = \frac{r_\pi}{2} + (\beta+1)R_{EE}$$

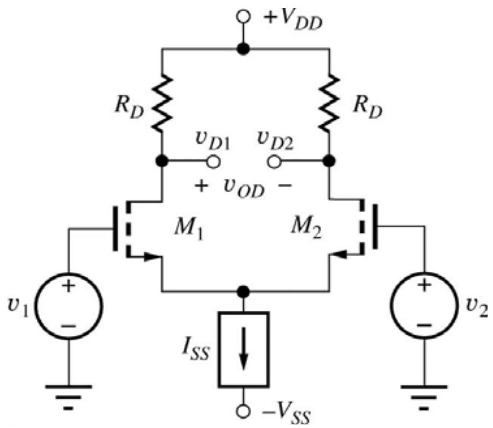
$$\mathrm{CMRR} = \left|\frac{A_{dm}}{A_{cm}}\right| = \left|\frac{A_{dd}/2}{A_{cc}}\right| \cong g_m R_{EE}$$

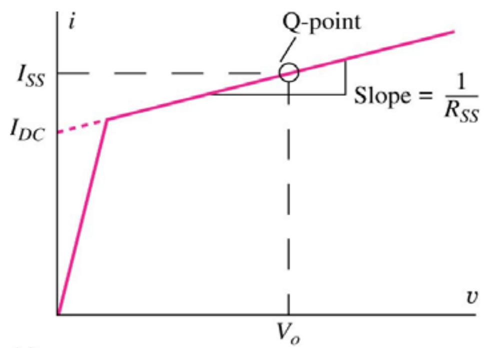
Applying rules for drawing half-circuits, the points at the line of symmetry are open circuited. The half-circuit represents a C-E amplifier stage with an emitter resistance.

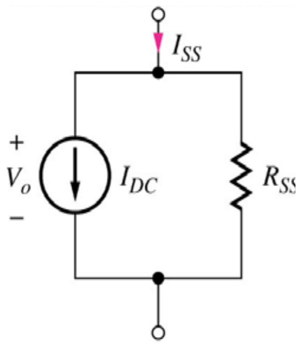
Biasing with Electronic Current Sources



- Differential amplifiers are biased using electronic current sources to stabilize the operating point and increase effective value of R_{EE} to improve CMRR
- Electronic current source has a Q-point current of I_{SS} and an output resistance of R_{SS} as shown.
- DC model of the electronic current source is a dc current source, I_{SS} while ac model is a resistance R_{SS}.







$$I_{DC} = I_{SS} - \frac{V_0}{R_{SS}}$$