EXPERIMENT 9: LISSAJOUS FIGURES

PREPARATION

Read the attached *Notes on Lissajous Figures.*
EP-9: Find the amplitude ratio and phase shift for each of the three Lissajous figures in Fig. E9-4.

EXPERIMENT

Part A -- Phase-Shift Network

The *RC* phase-shift network in Fig. E9-1 is prewired in a box, with $R = 1\ \text{k}\Omega$ and $C = 110$ nF. Your task is to measure the amplitude and phase of v_B while simultaneously monitoring the amplitude and phase of v_A when terminal *X* is grounded with a jumper wire. Use the function generator to supply a sinusoidal input v_A. Do *not* connect the SYNC output to the scope. Make sure the "Probe attenuation factor" is set at 1 for both channels of the scope.

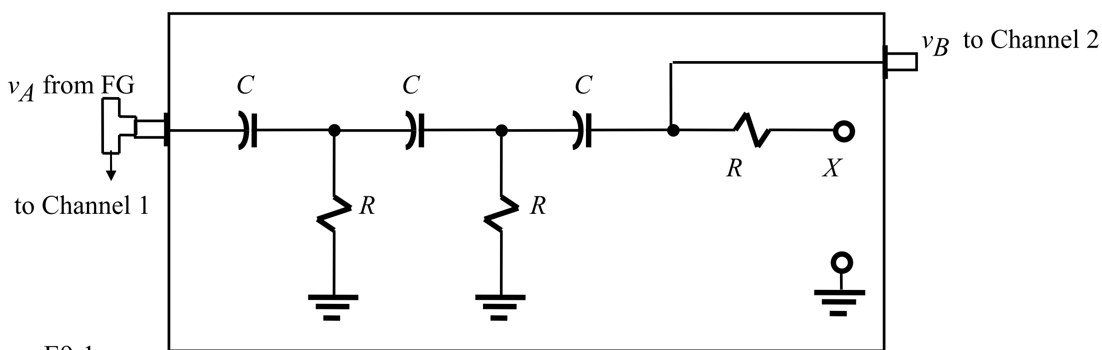

Figure E9-1

1. Keeping terminal *X* at ground with a jumper wire, set the FG frequency at $f = 2$ kHz. Apply v_A to Channel 1 of the scope and v_B to Channel 2. Set the scope for the XY mode (so v_A is the horizontal or X deflection). Adjust v_A for an amplitude of about 8 Vpp. Calculate the amplitude ratio and phase shift from cursor measurements of the resulting Lissajous figure.

2. Press **Autoscale** to switch the scope to dual-trace mode, displaying v_A and v_B. Use cursors to measure the time shift t_d of v_B by comparing like zero-crossings. Calculate the phase shift via

$$\theta(°) = t_d \times f \times 360°$$

3. Return to the XY mode, and decrease *f* until the Lissajous figure becomes a straight line corresponding to $\theta = \pm 180°$. (This occurs when v_B has a much smaller amplitude than v_A, so you'll need to reduce the volts/div of Channel 2.) Record *f* and determine the amplitude ratio for this case.

Part B -- Phase-Shift Oscillator

The circuit in Fig. E9-2 will spontaneously oscillate at a particular frequency f_{osc} if $\underline{V}_B/\underline{V}_A \times \underline{V}_{out}/\underline{V}_B = 1$ so $\underline{V}_{out} = \underline{V}_A$. Since the inverting op-amp circuit has $\underline{V}_{out}/\underline{V}_B = -R_F/R = (R_F/R)\ \underline{/\pm 180^\circ}$, the phase-shift network must have $\underline{V}_B/\underline{V}_A = a\ \underline{/\pm 180^\circ}$ at f_{osc} and oscillation requires $R_F/R = a^{-1}$ or $R_F = R/a$.

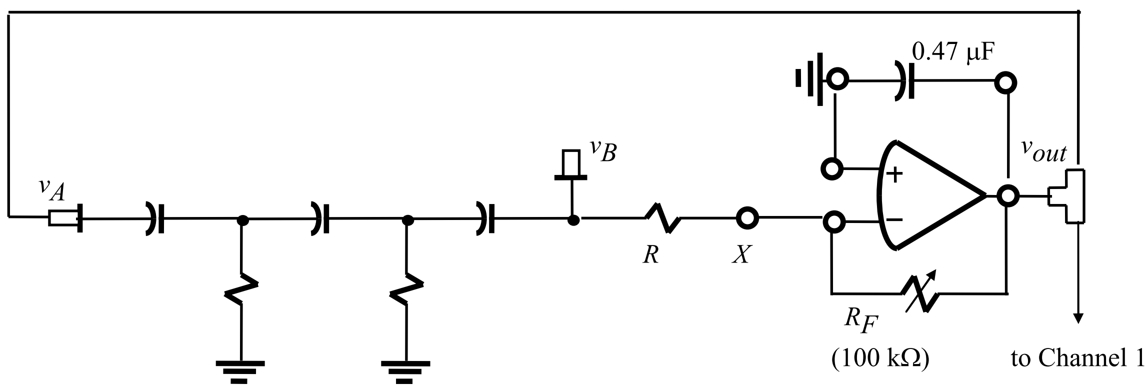

Figure E9-2

Build the circuit in Fig. E9-2 with an op-amp and the *RC* phase-shift network (from part A). Use a jumper wire (*not* a BNC cable) to connect *X* on the *RC* phase-shift network box directly to the op-amp's inverting input. Use a 100-kΩ digital pot for R_F, and apply v_{out} to Channel 1. Connect the ±5 V supplies to the op-amp and turn the outputs off via the **Output On/Off** button. Also connect a 0.47-µF capacitor between the op-amp's output and ground to prevent unwanted high-frequency oscillation.

1. Set $R_F = R/a$, where $R = 1$ kΩ and a is the amplitude ratio found in part A.3. Turn on the power supply outputs (providing ±5V supplies for ±Vcc) and the circuit should oscillate. (If not, then increase R_F until oscillation occurs.) Measure f_{osc} on the scope and compare it with the value found in part A.3.
2. Increase R_F and note any changes in the shape or frequency of v_{out}. Then decrease R_F until oscillation stops. What is the minimum value of R_F required to sustain oscillation? How/Why does it differ from the value required to start oscillation?

POST-LAB:

1. Try to find the transfer functions V_B/V_A (you may want to use a math modeling tool – e.g. Maple, MathCAD) and V_{out}/V_B for the circuit shown in Figure E9-2.
2. How could you use these transfer functions to predict what frequency the circuit will oscillate at?
3. How would you design a circuit to produce a square wave oscillator (with an output frequency of 440 Hz) – using (at most) the above circuits, another op-amp and a few more passive components?

NOTES ON LISSAJOUS FIGURES

Consider any two branch variables in a circuit under ac steady-state conditions, which can be written in general as

$$x(t) = X_m \cos(\omega t + \phi_x) \qquad y(t) = Y_m \cos(\omega t + \phi_y)$$

The corresponding phasor ratio is $\underline{Y}/\underline{X} = (Y_m \underline{/\phi_y})/(X_m \underline{/\phi_x}) = a\,\underline{/\theta}$, where

$$a = Y_m/X_m \qquad \theta = \phi_y - \phi_x$$

which are the amplitude ratio and phase shift, respectively.

One way of measuring these quantities involves the elliptical Lissajous figure produced by applying $x(t)$ to the horizontal input of a scope and $y(t)$ to the vertical input. As illustrated in Fig. E9-3, y_{max} appears in the first quadrant when $-90° < \theta < 90°$ and in the second quadrant when $90° < \theta < 270°$.

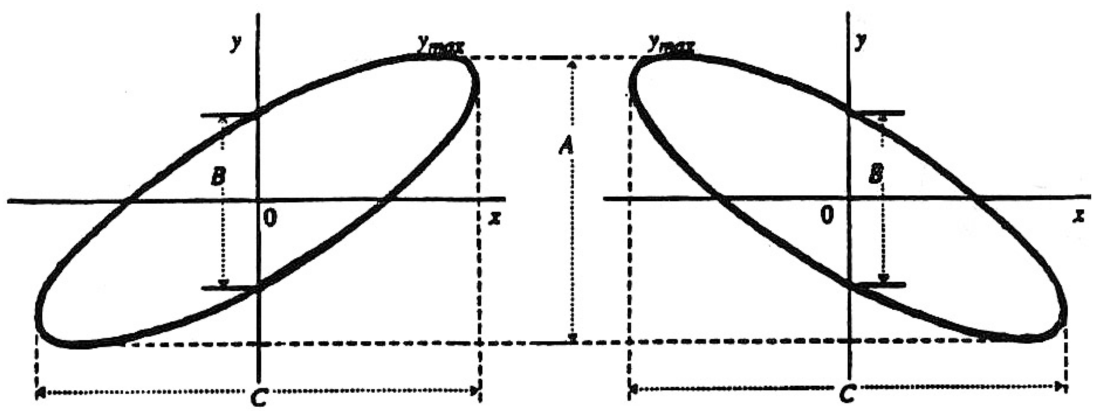

Figure E9-3 (a) $-90° < \theta < 90°$ (b) $90° < \theta < 270°$

The peak-to-peak vertical excursion is $A = 2Y_m$, and the peak-to-peak horizontal excursion is $C = 2X_m$. Hence, we can calculate the amplitude ratio via

$$a = Y_m/X_m = A/C$$

To find the phase shift, we note that $x(t) = 0$ when $\omega t + \phi_x = \pm 90°$, so $\omega t = -\phi_x \pm 90°$. Correspondingly, we have $y(t) = Y_m \cos(\phi_y - \phi_x \pm 90°) = Y_m \cos(\theta \pm 90°) = \pm Y_m \sin\theta$, so $B = 2Y_m|\sin\theta| = A|\sin\theta|$. Hence, we can calculate the phase shift via

$$\begin{aligned} \theta &= \pm \sin^{-1}(B/A) & -90° < \theta < 90° \\ &= 180° \pm \sin^{-1}(B/A) & 90° < \theta < 270° \end{aligned}$$

The quadrant of θ must be determined by a dual-trace display or other means.

Figure E9-4 shows Lissajous figures for various values of amplitude ratio and phase shift.

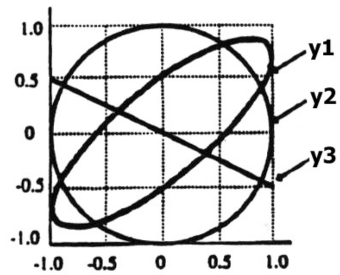

Figure E9-4