

Task One

1) Explain the time constant of a circuit that contains a capacitor and resistor in series.

A capacitor is made up out of two metal plates separated by either air or some other non-conductive substance. When connected to a battery, the battery charges one of the plates in a positive manner and the other plate in a negative manner. The resistive-capacitive, or CR, time constant is a measure of how long it takes to either charge or discharge a capacitor by 63.2 percent. This length of time depends on both the capacitance of the capacitor and the resistance of the circuit. The CR time constant is equal to the resistance multiplied by the capacitance.

2) A 100 μF capacitor is connected in series with an 8000 Ω resistor. Determine the time constant of the circuit.

If the circuit is suddenly connected to a 100v dc supply find

- I. The initial rise of P.D across the capacitor, (rate of voltage rise per volt to the first time constant)**

$$T = CR = 100\mu\text{F} * 10^{-6} * 8000\Omega = 0.8\text{s}$$

$$\frac{63.2\text{v}}{0.8\text{S}} = 79\text{v rise}$$

- II. The initial charging current, (at zero time)**

$$I = \frac{V}{R} = \frac{100\text{v}}{8000\Omega} = 0.0125\text{a}$$

- III. The ultimate charge in the capacitor**

$$Q = C V = 100 * 10^{-6} * 100\text{v} = 0.01 \text{ coulomb}$$

- IV. The ultimate energy stored in the capacitor**

$$W = \frac{1}{2} C V^2 = \frac{100 * 10^{-6} * 100\text{v}^2}{2} = 0.5 \text{ joules}$$

3) Explain the term time constant in connection with an inductive circuit.

A coil, also known as an inductor, is a spiral of wire. It is usually wound around a length of metal such as iron. When a current passes through the coil, it creates a magnetic field around the coil. If the current changes, this magnetic field also changes. The resistive-inductive, or LR, time constant is a measure of how rapidly current in the coil builds up or dies away by 63.2 percent. This length of time depends not only on the inductance of the coil but also on the resistance of the circuit. The LR time constant is equal to the inductance divided by the resistance.

- 4) A coil of 10H is connected to a 100Ω resistor across a 300v d.c supply instantaneously.
Calculate:

- i. The time constant of the circuit

$$\frac{L}{R} = \frac{10}{100} = 0.1s = 100ms$$

- ii. The current after 2ms

$$I = \frac{V}{R} = \frac{300v}{100\Omega} = 3a$$

$$i = I (1 - e^{-\frac{t}{T}})$$

$$i = 3a (1 - e^{-\frac{2ms}{100ms}}) = 0.02$$

$$i = 3a (1 - e^{-0.02}) = 0.059a$$

- iii. The voltage drop across the resistor after 5ms

$$VR = V (1 - e^{-\frac{t}{T}})$$

$$VR = 300v (1 - e^{-\frac{5ms}{100ms}}) = 0.05$$

$$VR = 300v (1 - e^{-0.05}) = 14.63v$$

- iv. The time it takes for the current to settle down

$$5 * 0.1 = 0.5s$$

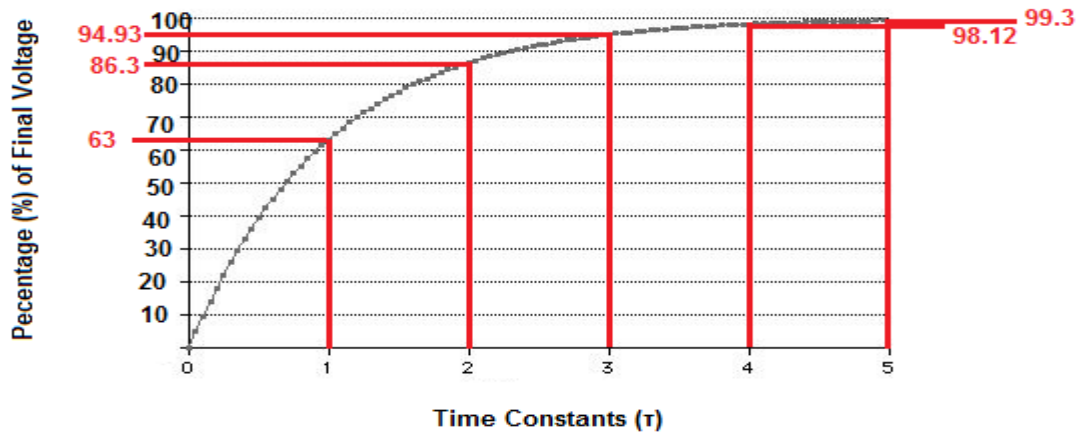
- v. The final current in the circuit

$$I = \frac{V}{R} = I = \frac{300}{100} = 3a$$

$$0.632 * 3a = 1.89a$$

- 5) Give Rough sketch graph, with explanatory notes, to show how current and voltage vary with time in series CR and LR circuits both on connecting a supply and shorting out the circuits.

Mark on the graph the appropriate time scales.



The initial state of a CR circuit there will be no charge on the capacitor so there will be no potential voltage developed across its terminals.

The current through the capacitor is given by $I(t) = dv/dt$. So with $t = 0$ the capacitor looks like a short circuit to the supply.

The current will, however, be limited by the series resistor to V_{supply}/R initially, but as the charge on the plates starts to build up, a voltage, $V = Q/C$, across the capacitor will accumulate.

But the supply voltage has not changed so the voltage across the series resistor will immediately start to fall; $V_{\text{resistor}} = V_{\text{supply}} - V_{\text{capacitor}}$.

Hence the current available to charge the capacitor will also fall in response ($I = V_{\text{res}}/R = (V_{\text{supply}} - V_{\text{capacitor}})/R$) and the rate of charge slows as does the rate of rise of the voltage across the capacitor.

The voltage across the cap' follows an inverse exponential increase with a time constant given by $t = CR$ reaching 63.2% of the supply voltage in that time.

The current into the capacitor thus follows an exponential fall from an initial value of $I = V_{\text{supply}}/R$ and decays to zero following the same $t = CR$ time constant. i.e. it will fall to 63.2% of the initial current in that time.

LR circuits behave like an open circuit when the magnetic field is changing with time; once it stabilizes, it behaves like a short circuit. When you just connect this LR circuit to the DC voltage supply, at $t=0$, the inductor is like an open circuit; as $t \rightarrow \infty$ (steady state) the inductor is just a short circuit. This happens regardless of the size of the inductor of course, the larger the inductor, the longer it takes to stabilize.

The reason for this behaviour is that, when the magnetic field is changing with time, there is an induced voltage on the terminals of the inductor which will oppose the current. This is called the Lenz Law. Any transients in this magnetic field will affect this voltage.

Task Three

1. A coil of inductance 4H and resistance of 80Ω is connected in parallel with a 200Ω of resistor of negligible inductance across a 200v dc supply. The switch connecting these to the supply is then opened; the coil and resistor remain connected together. State in each case one for immediately before and one for immediately after opening the switch;

a) The current Through the resistor:

Immediately before Switch opening

$$200\text{v} / 200\text{ohm} = 1\text{a}$$

Immediately after Switch opening

$$-200\text{v} / 80\text{ohms} = -2.5\text{a}$$

b) The current through the coil:

Immediately before Switch opening

$$200\text{v} / 80\text{ohm} = 2.5\text{a}$$

Immediately after Switch opening

$$200\text{v} / 80\text{ ohm} = 2.5\text{a}$$

c) The e.m.f induced in the coil:

Immediately before Switch opening

$$2.5\text{a} * 80\text{ohms} - (200\text{v}) = 0\text{v}$$

Immediately after Switch opening

$$-2.5\text{a} * (200\Omega + 80\Omega) = -700\text{v}$$

d) The voltage across the coil:

Immediately before Switch opening

$$2.5\text{a} * 80\text{ ohm} = 200\text{v}$$

Immediately after Switch opening

$$-2.5\text{a} * 200\text{ohms} = -500\text{v}$$

2. For task 1 question 2 determine the voltage across the capacitor when the charging current has fallen to 90% of maximum.

$$V_C = 100 * (1 - \frac{90}{100})$$

$$V_C = 100 * (1 - 0.9)$$

$$V_C = 100 * 0.1$$

$$V_C = 10\text{v}$$

3. For task 1 question 4 determine the current through the resistor after 3.3τ (tau).

$$\frac{V}{R} * (1 - e^{-\frac{t}{T}})$$

$$\frac{V}{R} (1 - e^{-\frac{3.3}{100\text{ms}}}) = 0.033$$

$$\frac{300\text{v}}{100\Omega} * (1 - e^{-0.033}) = 0.097\text{a}$$