Using residue theorem to find the integral of $g(z) = \dfrac{\sinh(z)}{z^9}$

Given contour C: | z + j| = 2, traversed positively (CCW)

Your work using L'Hospital's rule is as follows: $\lim_{z\to 0} \dfrac{\sinh(z)}{z^9} = \lim_{z\to 0} \dfrac{\sinh(z)}{9\cdot z^8} = 0$

No, this is not correct. It should be as follows: $\lim_{z\to 0} \dfrac{\sinh(z)}{z^9} = \lim_{z\to 0} \dfrac{\cosh(z)}{9\cdot z^8} = \infty$

Your work for pole testing is as follows: $\lim_{z\to 0} [z-0]^8 \cdot g(z) = \lim_{z\to 0} (z-0)^8 \cdot \sinh(z)/9z^8$

$= \sinh(0)/9 \quad = 0$

There is no pole at order 8 at z = 0

No, this is not correct. It should be as follows: $\lim_{z\to 0} \dfrac{\sinh(z)}{z} = \lim_{z\to 0} \dfrac{\cosh(z)}{1} = 1$

There is a degenerate pole at z=0 of order 8,
so there is one pole/zero cancellation

Your work for Laurent series expansion is as follows

$\sinh(z)/z^9 = 1/z^9 = \{ z - z^3/3! + z^5/5! - z^7/7! + z^9/9! + \ldots\}$

As you know, this is not correct

It should be $\dfrac{\sinh(z)}{z^9} = \dfrac{1}{z^8} + \dfrac{1}{3!\cdot z^6} + \ldots = \sum_{i=1}^{\infty} \dfrac{z^{i-9}}{i!}$

Eg: Q(b)

Using residue theorem to show that the value of the real trigonometric integrate from 0 to 2Π { $3/(5 + 2 \cos \theta)$} $d\theta$ is $6 \Pi/\sqrt{21}$

My working is as followed:
Integrate from 0 to 2Π { $3/(5 + 2 \cos \theta)$} $d\theta$ is $6 \Pi/\sqrt{21} = 1.31 \Pi$
Let unit circle $|z| = 1 \to z = e^{j\theta}$
$d\theta = dz/jz$
$2\cos\theta = e^{j\theta} - e^{-j\theta}$ **(Note there is a sign error here $2\cos\theta = e^{j\theta} + e^{-j\theta}$)**

Substitue $d\theta = dz/jz$
$2\cos\theta = e^{j\theta} - e^{-j\theta}$ into { $3/(5 + 2 \cos \theta)$} $d\theta$ is $6 \Pi/\sqrt{21}$

$f(z) = 3jz/z^2 + 5Z + \mathbf{1}$ **(no, this step is wrong $f(z) = -3j/z^2 + 5z + 1$)**
$= 3jz / (z - za)(z-zb)$ **(no)**
where $za = - 0.21$ **(don't use approximations, use square root notation)**
$zb = -4.79$ **(don't use approximations, use square root notation)**

only za is inside c: $|z| = 1$

then residue of $(f,za) = \lim_{z \to (-0.21)} (z + 0.21) \times (3jz/(z + 0.21)(z + 4.79)$
$= - (0.63j/4.58)$

$I = 2 \Pi j \times 0.63j/4.58$
$= 2 \Pi (-1)(0.63) / 4.58$
= answer does not get 1.31Π

Eg: Q(c)

Evaluate contour integrate f(z)dz for f(z) = [z.exp(z)/sin(z)] with C: | z + 2| = 12 traversed CCW.

My working is as followed:

For sin z = 0, z = 0, z = +/- Π , z = +/- 3 Π, z = +/- 4 Π **(you mean 0, ±*π*, ±2*π*, ±3*π*)**

7 poles (z = 0, z = +/- Π , z = +/- 3 Π, z = - 4 Π) is inside the circle
contour integrate f(z)dz for f(z) = [z.exp(z)/sin(z)]
= 2 Πj [Res (f(z),-4 Π) + Res (f(z),-3 Π) + Res (f(z),-2 Π) + Res (f(z),- Π) + Res (f(z),+ Π) + Res (f(z),+2 Π) + Res (f(z),+3 Π)]

For z = -4 Π
Res(f(z),-4 Π) = lim (z + 4 Π) (z.exp(z)/sinz)
z - > -4 Π

(I got one question here: how do I do the limit for exp(z) function)??

Please advise! Thanks so much!

Note: exp(z) goes to 1 in the limit as z goes to zero, that the only place you need to take the limit.