EE-612:
Lecture 13
Threshold Voltage and MOSFET Capacitances

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outline

1) Threshold Voltage
2) Body Effect
3) MOSFET Capacitances

threshold voltage

$$V_T = V_{FB} + 2\psi_B - Q_D(2\psi_B)/C_{OX}$$

$$V_T = V_{FB} + 2\psi_B + \sqrt{2q\varepsilon_{Si}N_A(2\psi_B)}/C_{OX}$$

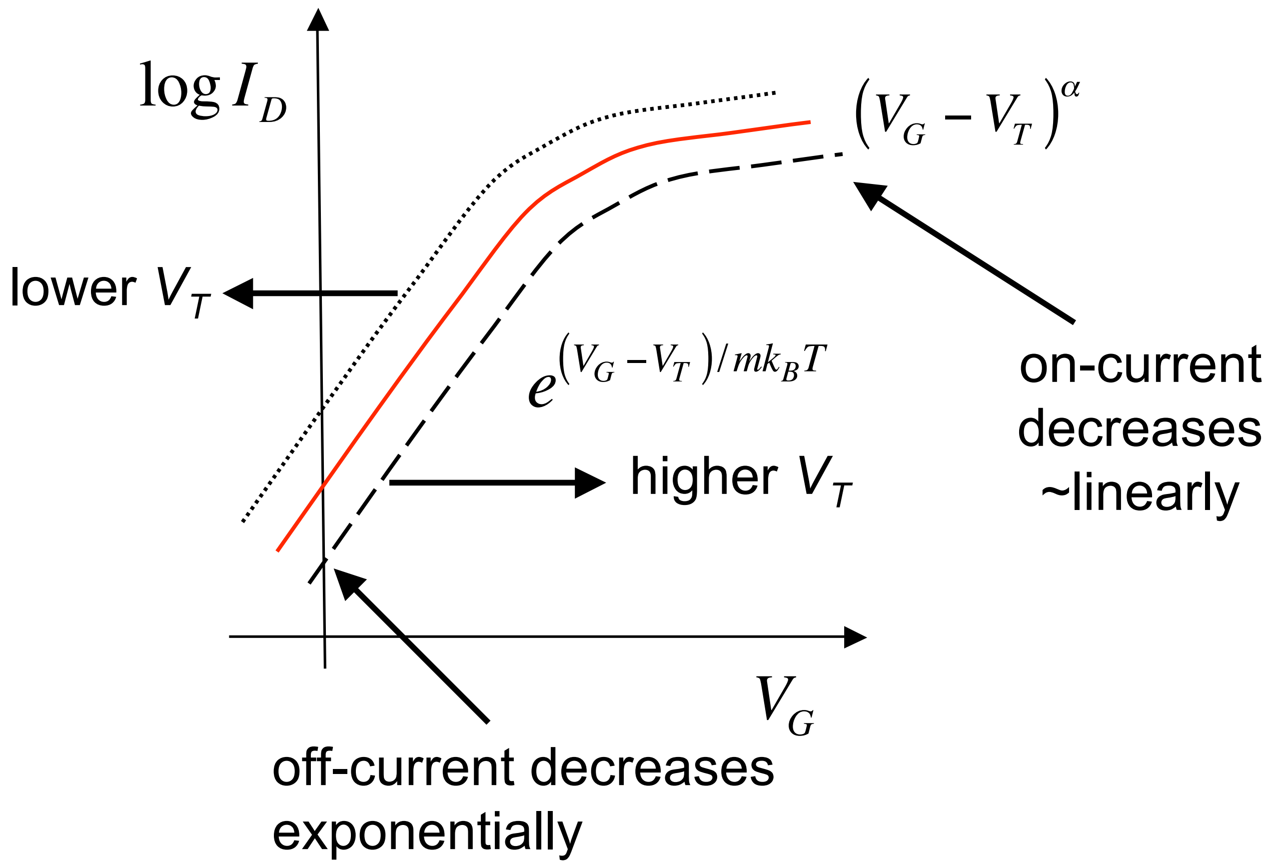
70 nm technology node (ITRS 2005 Ed.)

$N_A \approx 4.6\times10^{18}\ \mathrm{cm^{-3}}$ $\qquad EOT \approx 1.1\ \mathrm{nm}$

$\psi_B \approx 0.52\ \mathrm{V}$ $\qquad V_{FB} = \phi_{ms} \approx -0.55 - \psi_B \approx -1.07\ \mathrm{V}$

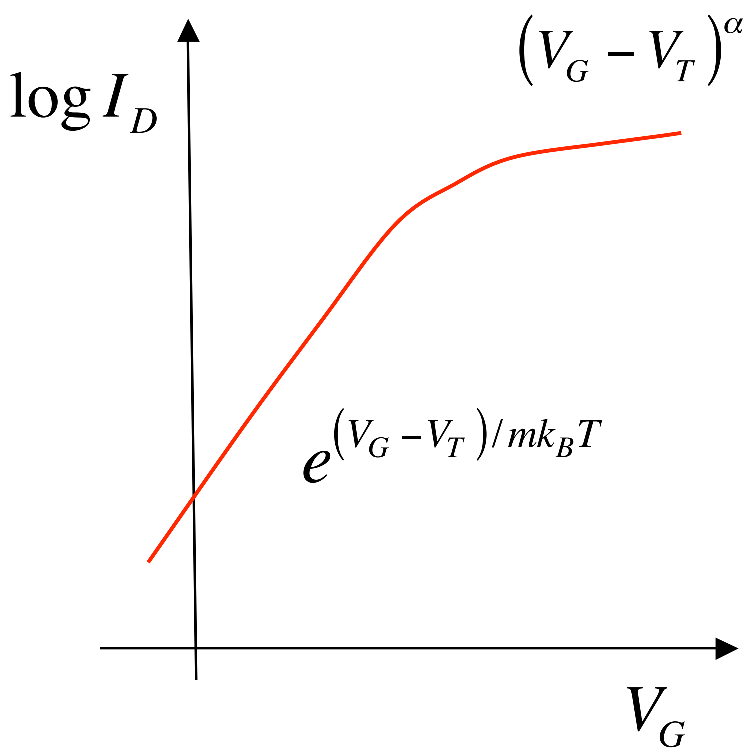
$V_T \approx 0.38\ \mathrm{V}$

required V_T



$\log I_D$
$(V_G - V_T)^{\alpha}$
lower V_T
$e^{(V_G - V_T)/mk_BT}$
on-current
decreases
~linearly
higher V_T
V_G
off-current decreases
exponentially

required V_T



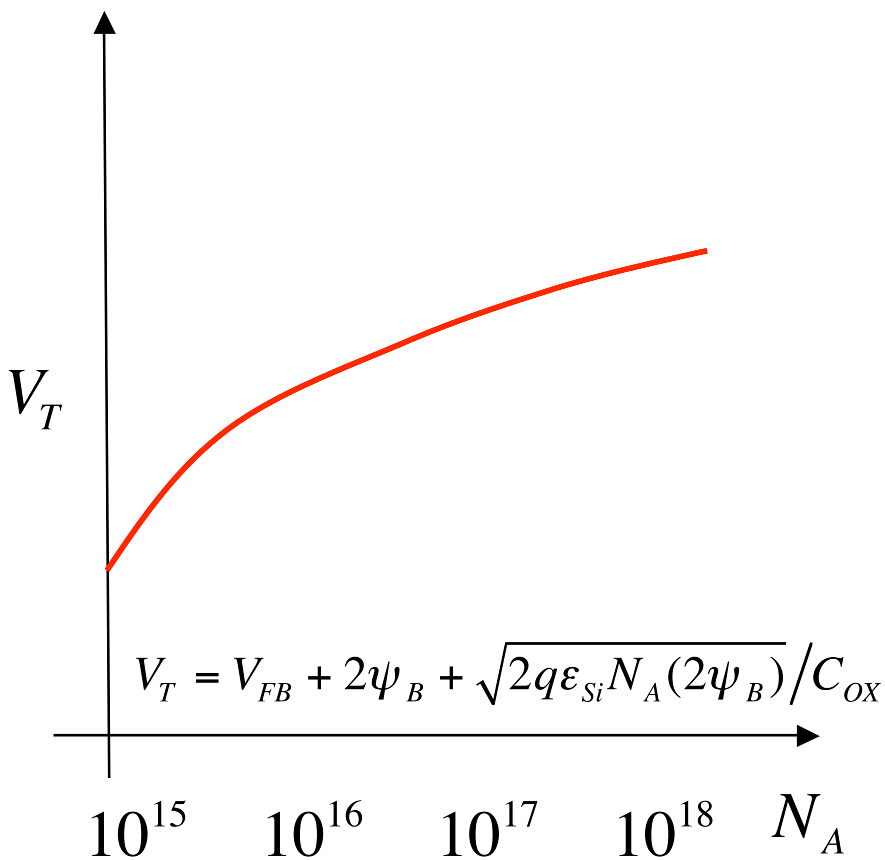
V_T selection is a trade-off between high on-current (low V_T) and low off-current (high V_T).

High-performance (high I_{ON}):
V_{DD} = 1.1V
V_T = 0.17V   (15% of V_{DD})

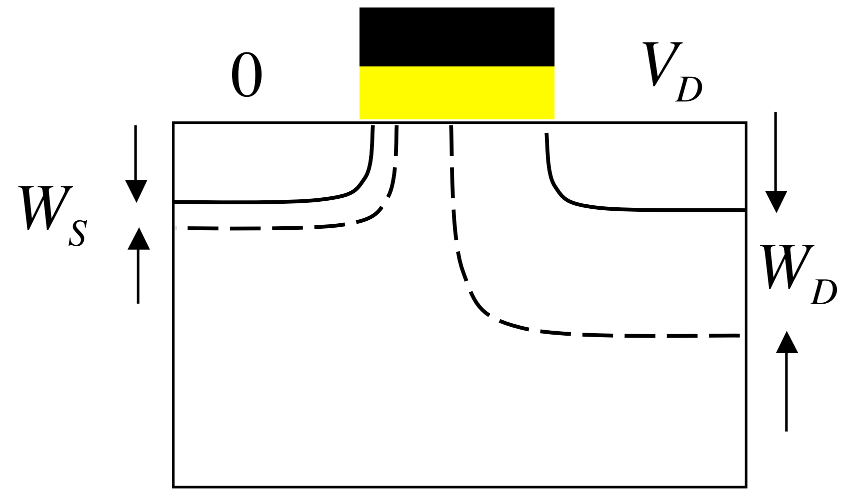
Low-power (low I_{OFF}):
V_{DD} = 1.2V
V_T =0.52V   (43% of V_{DD})

60 nm node from ITRS 2005 Ed.

V_T engineering



$$V_T = V_{FB} + 2\psi_B + \sqrt{2q\varepsilon_{Si}N_A(2\psi_B)}\Big/C_{OX}$$



$N_A\left(\min\right)$: punch through

$$W_S + W_D < L$$

$N_A\left(\max\right)$: tunneling

(also C_J and body effect)

V_T temperature dependence

$$V_T = V_{FB} + 2\psi_B + \sqrt{2q\varepsilon_{Si}N_A(2\psi_B)}\Big/C_{OX}$$

$$V_T = -\frac{E_G}{2q} + \psi_B + \sqrt{4q\varepsilon_{Si}N_A\psi_B}\Big/C_{OX}$$

$$\frac{dV_T}{dT} = -\frac{1}{2q}\frac{dE_G}{dT} + \left(1 + \sqrt{2\varepsilon_{Si}qN_A/\psi_B}\Big/C_{OX}\right)\frac{d\psi_B}{dT}$$

$$\frac{dV_T}{dT} = -\frac{1}{2q}\frac{dE_G}{dT} + \left(1 + 2C_{DM}/C_{OX}\right)\frac{d\psi_B}{dT}$$

$$m = 1 + C_{DM}/C_{OX}$$

V_T temperature dependence (ii)

$$\frac{dV_T}{dT} = -\frac{1}{2q}\frac{dE_G}{dT} + (2m-1)\frac{d\psi_B}{dT}$$

$$\psi_B = \frac{k_B T}{q}\ln\left(N_A/n_i\right) \qquad n_i = \sqrt{N_C N_V}\, e^{-E_G/2k_B T}$$

$$\frac{dV_T}{dT} = -(2m-1)\frac{k_B}{q}\left[\ln\left(\frac{\sqrt{N_C N_V}}{N_A}\right) + \frac{3}{2}\right] + \frac{(m-1)}{q}\frac{dE_G}{dT}$$

eqn. (3.45) of Taur and Ning

V_T temperature dependence (iii)

$$\left.\begin{array}{l} N_A \approx 10^{16}\ \text{cm}^{-3} \\ m \approx 1.1 \end{array}\right\} \quad \frac{dV_T}{dT} \approx -1\ \text{mV/K}$$

$$\left.\begin{array}{l} N_A \approx 10^{18}\ \text{cm}^{-3} \\ m \approx 1.3 \end{array}\right\} \quad \frac{dV_T}{dT} \approx -0.7\ \text{mV/K}$$

$$V_T\left(100^\circ\text{C}\right) \approx V_T\left(25^\circ\text{C}\right) - \left(55 - 75\ \text{mV}\right)$$

$$I_{OFF}\left(100^\circ\text{C}\right) \approx 30 - 50 \times I_{OFF}\left(25^\circ\text{C}\right)$$

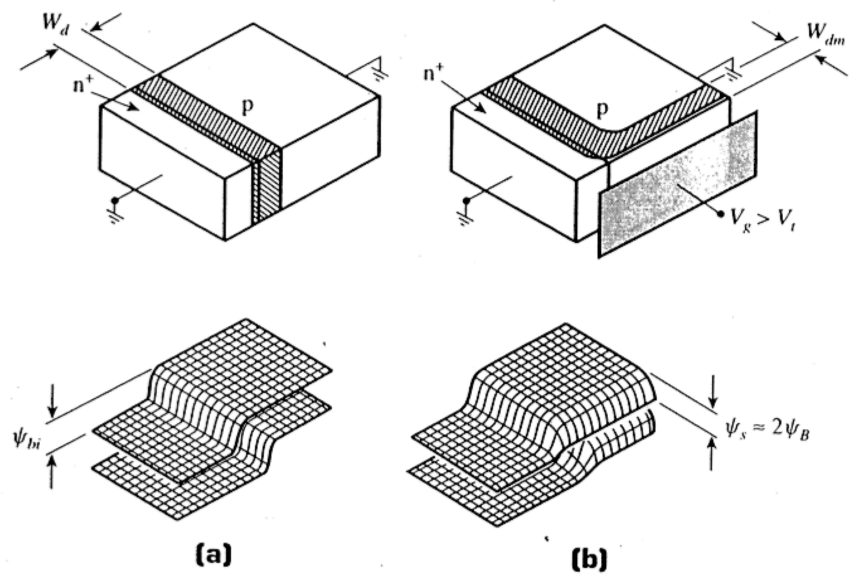
outline

1) Threshold Voltage

2) Body Effect

3) MOSFET Capacitances

recall



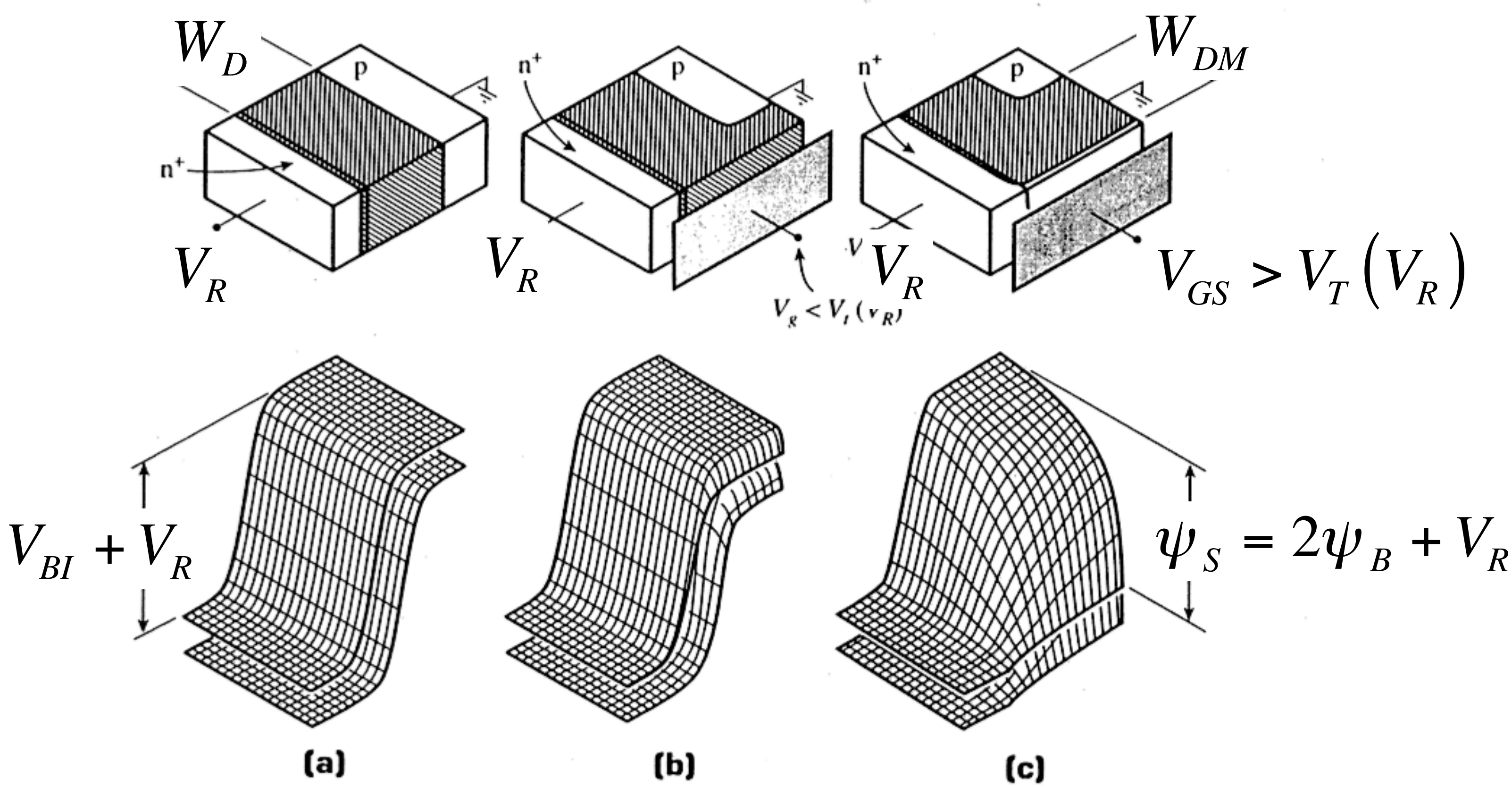
Gated doped or p-MOS with adjacent n^+ region

a) gate biased at flat-band

b) gate biased in inversion

A. Grove, *Physics of Semiconductor Devices*, 1967.

recall



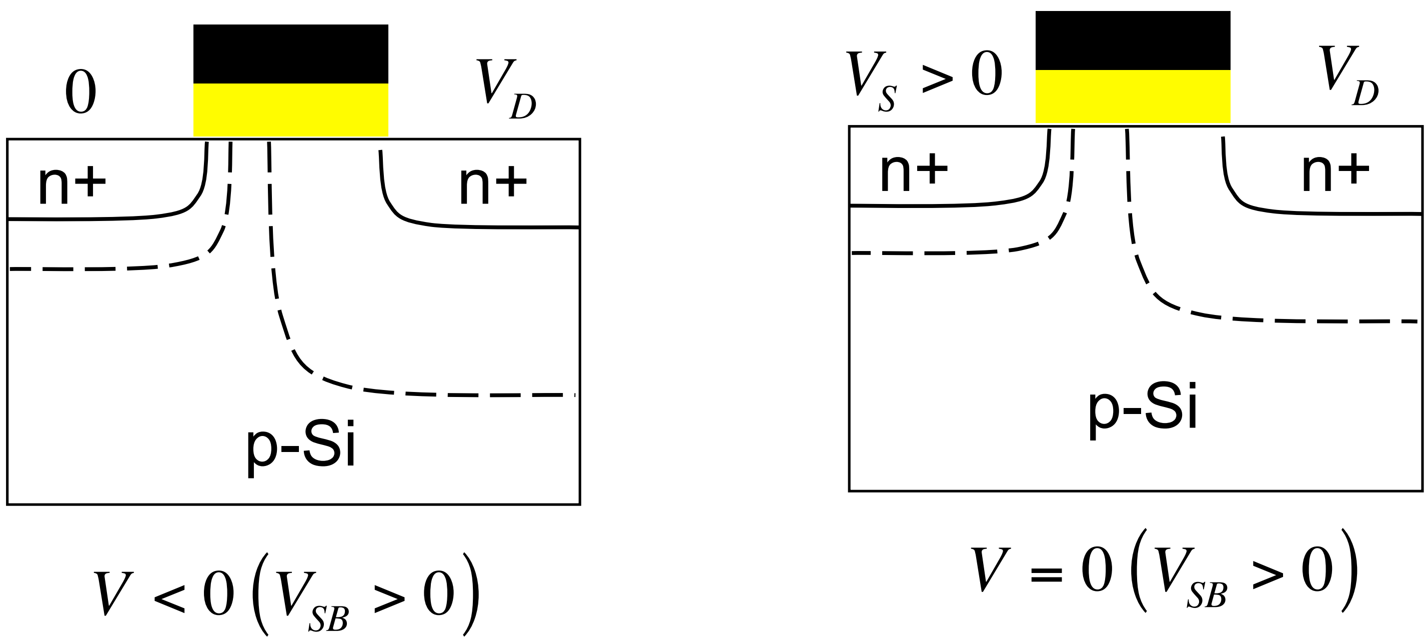
Gated doped or p-MOS with adjacent, reverse-biased n^+ region

a) gate biased at flat-band
b) gate biased in depletion
b) gate biased in inversion

A. Grove, *Physics of Semiconductor Devices*, 1967.

backgate voltage

$$V_T = V_{FB} + 2\psi_B + \sqrt{2q\varepsilon_{Si}N_A(2\psi_B + V_{SB})}\Big/C_{OX}$$



body effect

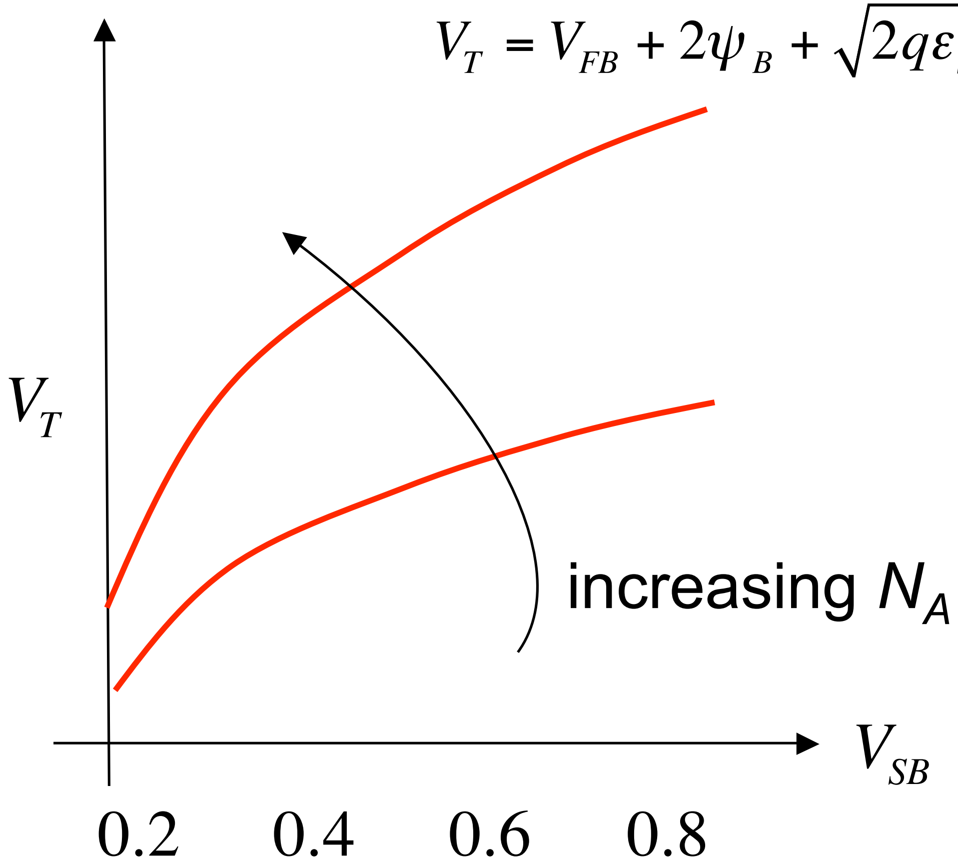
$$V_T = V_{FB} + 2\psi_B + \sqrt{2q\varepsilon_{Si}N_A(2\psi_B + V_{SB})}\Big/C_{OX}$$

$$V_T = V_{FB} + 2\psi_B + \gamma\sqrt{2\psi_B + V_{SB}}\Big/C_{OX}$$

$$\gamma = \sqrt{2q\varepsilon_{Si}N_A}$$ body effect coefficient

V_T vs. backgate voltage

$$V_T = V_{FB} + 2\psi_B + \sqrt{2q\varepsilon_{Si}N_A(2\psi_B + V_{SB})}\Big/C_{OX}$$



substrate sensitivity

$$V_T = V_{FB} + 2\psi_B + \sqrt{2q\varepsilon_{Si}N_A(2\psi_B + V_{SB})} \big/ C_{OX}$$

$$\frac{dV_T}{dV_{SB}} = \frac{\sqrt{q\varepsilon_{Si}N_A / 2(2\psi_B + V_{SB})}}{C_{OX}}$$

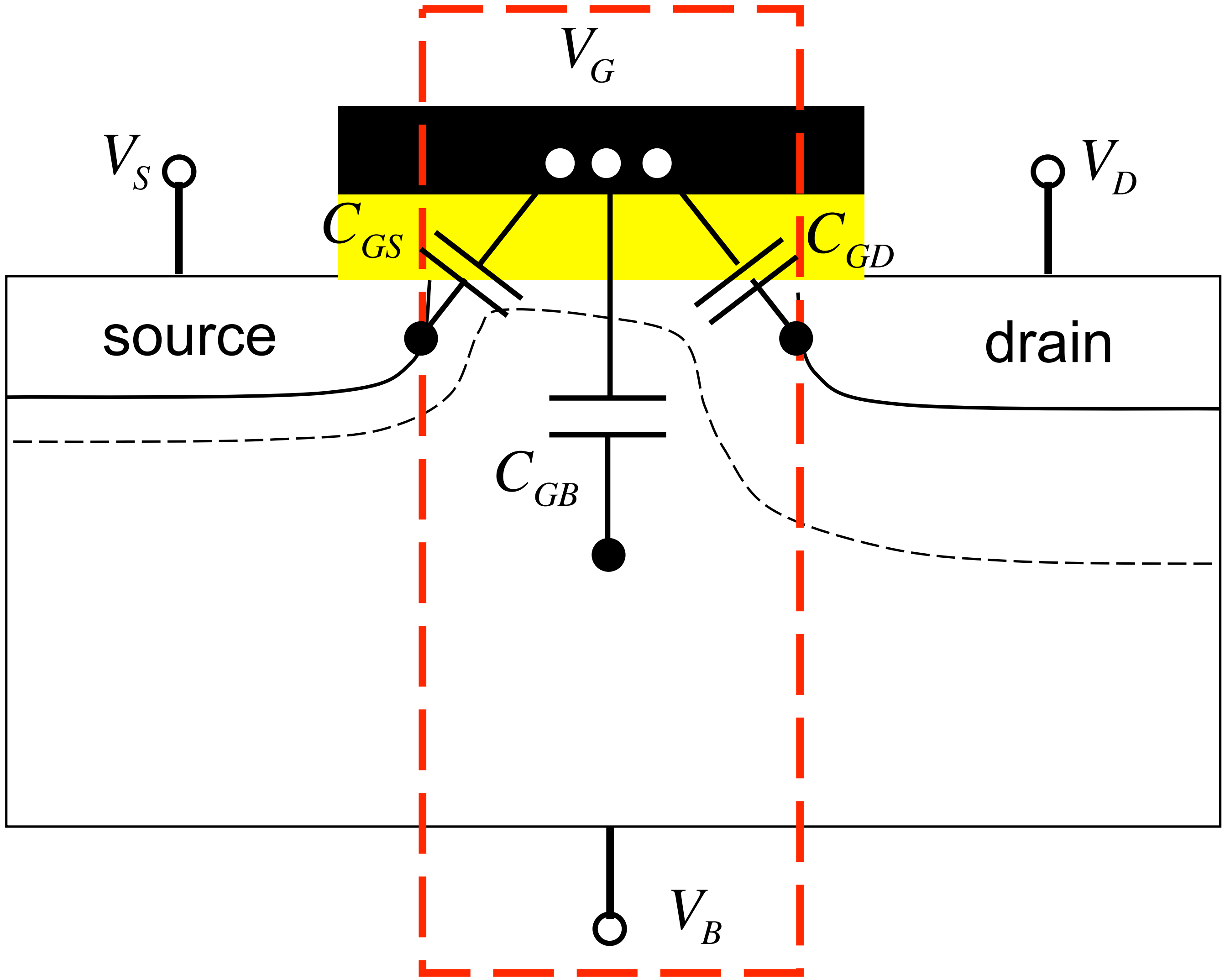
$$\left.\frac{dV_T}{dV_{SB}}\right|_{V_{SB=0}} = \frac{C_{DM}}{C_{OX}} = m - 1$$

body effect coefficient

outline

1) Threshold Voltage
2) Body Effect
3) **MOSFET Capacitances**

intrinsic MOSFET C's



below threshold

$$\bar{Q}_G = W\int_0^L Q_G(y)dy \quad \text{F}$$

$$\bar{Q}_G \approx -W\,L\,Q_D$$

$$C_{GS} = \frac{\partial \bar{Q}_G}{\partial V_{GS}} \approx 0 \qquad C_{GD} = \frac{\partial \bar{Q}_G}{\partial V_{GD}} \approx 0$$

$$C_{GB} = \frac{\partial \bar{Q}_G}{\partial V_{GB}} = WL\left(\frac{1}{C_{OX}} + \frac{1}{C_D}\right)^{-1}$$

above threshold

$$\bar{Q}_G \approx -W\int_0^L C_G\left[V_G - V_T - V(y)\right]dy \quad \text{F}$$

$$I_D = -WC_G\left(V_G - V_T - V\right)\mu_{eff}\frac{dV}{dy}$$

$$I_D = \frac{W}{L}\mu_{eff}C_G\left(V_{GS} - V_T - 0.5V_{DS}\right)V_{DS} \qquad V_{GD} = V_{GS} - V_{DS}$$

$$I_D = \frac{W}{2L}\mu_{eff}C_G\left[\left(V_{GS} - V_T\right)^2 - \left(V_{GD} - V_T\right)^2\right] \qquad (1)$$

above threshold

$$\bar{Q}_G \approx -W\int_0^L C_G\left[V_G - V_T - V(y)\right]dy \quad \text{F}$$

$$I_D = -WC_G\left(V_G - V_T - V\right)\mu_{eff}\frac{dV}{dy}$$

$$dy = -WC_G\left(V_G - V_T - V\right)\mu_{eff}dV/I_D$$

$$\bar{Q}_G \approx \frac{\mu_{eff}W^2C_G^2}{I_D}\int_0^{V_{DS}}\left[V_G - V_T - V\right]^2 dV \qquad (2)$$

above threshold

$$I_D = \frac{W}{2L}\mu_{eff}C_G\left[\left(V_{GS}-V_T\right)^2-\left(V_{GD}-V_T\right)^2\right] \qquad (1)$$

$$\bar{Q}_G \approx \frac{\mu_{eff}W^2C_G^2}{I_D}\int_0^{V_{DS}}\left[V_G-V_T-V\right]^2 dV \qquad (2)$$

$$\bar{Q}_G \approx \frac{2}{3}WLC_G\left[\frac{\left(V_{GD}-V_T\right)^3-\left(V_{GS}-V_T\right)^3}{\left(V_{GD}-V_T\right)^2-\left(V_{GS}-V_T\right)^2}\right]$$

above threshold (linear)

$$C_{GS} = \frac{\partial \bar{Q}_G}{\partial V_{GS}} \approx \frac{2}{3} WLC_G \left[1 - \frac{\left(V_{GD} - V_T\right)^2}{\left(V_{GD} + V_{GS} - 2V_T\right)^2} \right]$$

$$C_{GD} = \frac{\partial \bar{Q}_G}{\partial V_{GD}} \approx \frac{2}{3} WLC_G \left[1 - \frac{\left(V_{GS} - V_T\right)^2}{\left(V_{GD} + V_{GS} - 2V_T\right)^2} \right]$$

$$C_{GB} = \frac{\partial \bar{Q}_G}{\partial V_{GB}} \approx 0$$

$$V_D, V_S \approx 0$$

$$C_{GS} = C_{GD} \approx WLC_G / 2$$

above threshold (saturation)

$$V_{GD} = V_{GS} - V_{DS} \qquad V_{DSAT} = V_{GS} - V_T$$

$$C_{GS} = \frac{\partial \bar{Q}_G}{\partial V_{GS}} \approx \frac{2}{3} WLC_G$$

$$C_{GD} = \frac{\partial \bar{Q}_G}{\partial V_{GS}} \approx 0$$

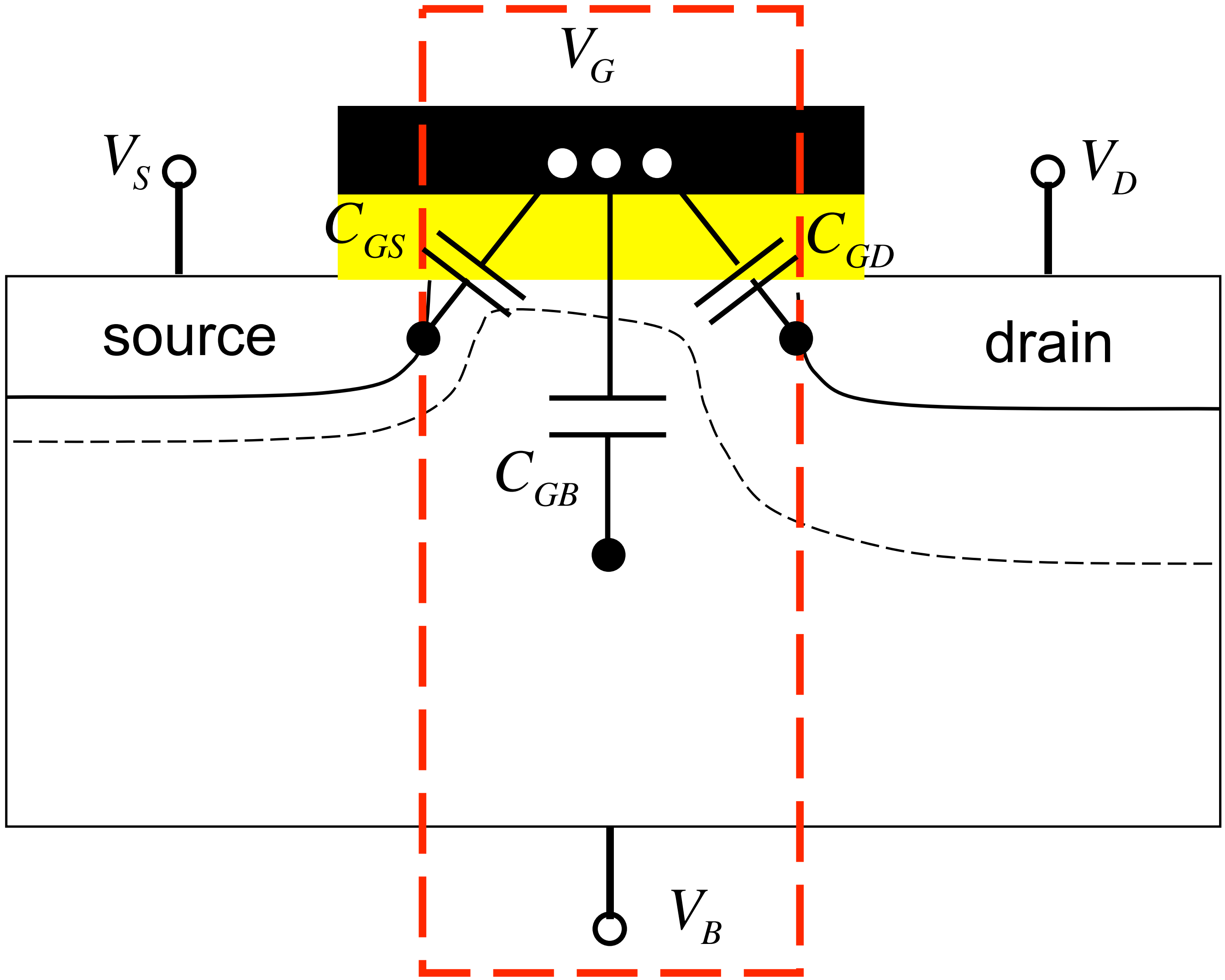
$$C_{GB} = \frac{\partial \bar{Q}_G}{\partial V_{GB}} \approx 0$$

‘Meyer model’

see:

MOSFET Models for VLSI Circuit Simulation, N. Arora, Springer-Verlag, 1993

intrinsic MOSFET C's



about MOSFET capacitances

1) charge conservation problems

$$Q = \int_{t_1}^{t_2} i\, dt = \int_{V_1}^{V_2} C(V)\, dV = \bar{C}\left[V\left(t_2\right) - V\left(t_1\right)\right]$$

2) ‘charge based models’

3) capacitance matrix

$$i_j = \frac{dQ_j}{dt} = \frac{\partial Q_j}{\partial V_G}\frac{\partial V_G}{\partial t} + \frac{\partial Q_j}{\partial V_S}\frac{\partial V_S}{\partial t} + \frac{\partial Q_j}{\partial V_D}\frac{\partial V_D}{\partial t} + \frac{\partial Q_j}{\partial V_B}\frac{\partial V_B}{\partial t}$$

4) non-reciprocal capacitances

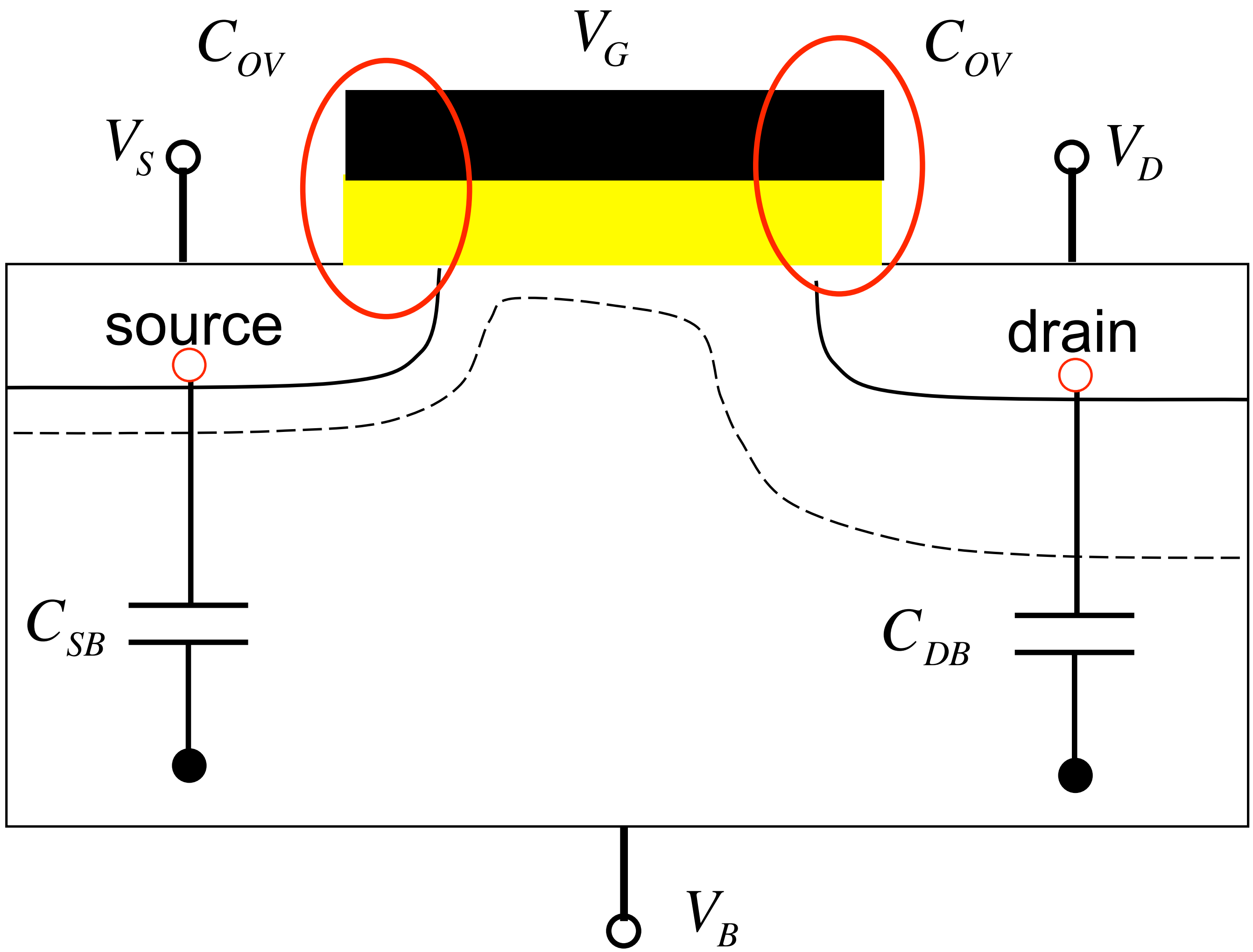
5) short channel effects

6) non-quasi-static effects

more information on capacitances

MOSFET Models for VLSI Circuit Simulation,
Narain Arora, Springer-Verlag, 1993

extrinsic MOSFET C's



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